Stability Analysis of A Time Delay Three Species Ecological Model

GNVSR C Murty, A V Paparao

***Abstract:** This paper deals with the stability analysis of three species Ecological model consists of a Prey (N_1), a predator (N_2) and a competitor (N_3). A Time delay (continuous type) is incorporated in the interaction of predator and competitor species. The model is studied by the system of integro- differential equations. Stability Analysis is studied at co-existing state. Numerical simulation is carried out in support of stability*

***Keywords:** Equilibrium points, local stability, Global stability, Numerical simulation.*

I. INTRODUCTION

The stability analysis of ecological systems is interesting &quite complex in nature. The origin of this theory is studied by Lotka[1] and Volterra[2]. The modeling of ecological systems is widely studied by ordinary, partial, delay, differential equations. Many authors [3-6] contributed significantly for this field. Delays are natural in ecological systems. The authors [7-9] describe the system dynamics using delay differential equations. The delays in three species models are complex in nature. These delays can switch over the stable equilibrium to unstable or vice versa.

The three species ecological models with different interactions are widely studied by paparao [10-14] .In spite of those models we proposed three species model with distributed delay interaction among predator and competitor species with two types of delay kernels. The stability analysis is carried at co-existing state and numerical simulation is carried out using MATLAB simulation.

II. MATHEMATICAL MODEL

We consider a three species Ecological model consisting of a Prey (N_1), predator (N_2) and a competitor (N_3) .The competitor (N_3) is competing with the Predator Species (N_2) and neutral with the prey (N_1). In this model the third species is competing with the predator for food other than the prey (N_1). In addition to that, the death rates, carrying capacities of all three species are also considered for investigation. A continuous type of delay is imposed in the interaction of predator and competitor species. The mathematical model for the above system can be formulated as a set of integro-differential equations is given by

$$\frac{dN_1}{dt} = a_1 N_1\left[1-\frac{N_1}{c_1}\right] - \alpha_{12}N_1N_2 - d_1N_1$$

$$\frac{dN_2}{dt} = a_2 N_2\left[1-\frac{N_2}{c_2}\right] + \alpha_{21}N_1N_2 - \alpha_{23}N_2\int_{-\infty}^{t} k_1(t-u)N_3(u)du - d_2N_2$$

$$\frac{dN_3}{dt} = a_3 N_3\left[1-\frac{N_3}{c_3}\right] - \alpha_{32}N_3\int_{-\infty}^{t} k_2(t-u)N_2(u)du - d_3N_3$$

(2.1)

A. Nomenclature

S. No	Parameter	Description
1	N_1, N_2 & N_3	Populations of the prey, predator and competitor respectively
2	a_1, a_2, a_3	Natural growth rates of prey, predator and competitor respectively
3	c_1, c_2, c_3	Carrying capacities of prey, predator and competitor respectively
4	α_{12}	Rate of decrease of the prey due to inhibition by the predator
5	α_{21}	Rate of increase of the predator due to successful attacks on the prey
6	α_{23}	Rate of decrease of the predator due to the competition with the competitor
7	α_{32}	Rate of decrease of the competitor due to the competition with the predator
8	d_1,d_2,d_3	Death rates of prey, predator and competitor respectively
10	$k_1(t-u)$ & $k_2(t-u)$	weight factors to give the influence at 't' of N_1, N_2 of time u

Notations: $\frac{a_1}{c_1} = k_1^*, \frac{a_2}{c_2} = k_2^*, \frac{a_3}{c_3} = k_3^*$. Assume throughout the analysis $(a_i - d_i) > 0 \ (i = 1,2,3)$

By normalizing the kernels k_1 and k_2 such that with the conditions

$$\int_0^{\infty} k_1(z)dz = 1, \int_0^{\infty} k_2(z)dz = 1 \qquad (2.2)$$

$$\int_0^{\infty} zk_1(z)dz < \infty, \int_0^{\infty} zk_2(z)dz < \infty$$

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Let $(t-u)=z, dt=dz$ and using the above relations the system of equations is
(2.2) becomes

$$\frac{dN_1}{dt}=a_1N_1\left[1-\frac{N_1}{c_1}\right]-\alpha_{12}N_1N_2-d_1N_1$$

$$\frac{dN_2}{dt}=a_2N_2\left[1-\frac{N_2}{c_2}\right]+\alpha_{21}N_1N_2-\alpha_{23}N_2\int_0^\infty k_1(z)N_3(t-z)dz-d_2N_2$$

$$\frac{dN_3}{dt}=a_3N_3\left[1-\frac{N_3}{c_3}\right]-\alpha_{32}N_3\int_0^\infty k_2(z)N_2(t-z)dz-d_3N_3$$

(2.3)

III. EQUILIBRIUM STATE

E_1: The co-existing state for which the three species are exist, is given by

$$\overline{N_1}=\frac{(a_1-d_1)(k_2^*k_3^*-\alpha_{23}\alpha_{32})-\alpha_{12}\left[(a_2-d_2)k_3^*-(a_3-d_3)\alpha_{23}\right]}{k_1^*(k_2^*k_3^*-\alpha_{23}\alpha_{32})+k_3^*\alpha_{12}\alpha_{21}},$$

$$\overline{N_2}=\frac{k_2^*\left[(a_2-d_2)k_3^*-(a_3-d_3)\alpha_{23}\right]+k_3^*(a_1-d_1)\alpha_{21}}{k_1^*(k_2^*k_3^*-\alpha_{23}\alpha_{32})+k_3^*\alpha_{12}\alpha_{21}},$$

$$\overline{N_3}=\frac{k_1^*\left[(a_3-d_3)k_2^*-(a_2-d_2)\alpha_{32}\right]+\alpha_{21}\left[(a_3-d_3)\alpha_{12}-(a_1-d_1)\alpha_{32}\right]}{k_1^*(k_2^*k_3^*-\alpha_{23}\alpha_{32})+k_3^*\alpha_{12}\alpha_{21}}.$$

(3.1)

The equilibrium state exists only when $\overline{N_1}>0, \overline{N_2}>0, \overline{N_3}>0$. i.e if

$$(i)\ (a_1-d_1)(k_2^*k_3^*-\alpha_{23}\alpha_{32})>\alpha_{12}\left[(a_2-d_2)k_3^*-(a_3-d_3)\alpha_{23}\right]$$

$$(ii)\ k_2^*\left[(a_2-d_2)k_3^*-(a_3-d_3)\alpha_{23}\right]+k_3^*(a_1-d_1)\alpha_{21}>0$$

$$(iii)\ k_1^*\left[(a_3-d_3)k_2^*-(a_2-d_2)\alpha_{32}\right]+\alpha_{21}\left[(a_3-d_3)\alpha_{12}-(a_1-d_1)\alpha_{32}\right]>0$$

$$\&(iv)\ k_1^*(k_2^*k_3^*-\alpha_{23}\alpha_{32})+k_3^*\alpha_{12}\alpha_{21}>0$$

(3.1.A) Or

$$(i)\ (a_1-d_1)(k_2^*k_3^*-\alpha_{23}\alpha_{32})<\alpha_{12}\left[(a_2-d_2)k_3^*-(a_3-d_3)\alpha_{23}\right]$$

$$(ii)\ k_2^*\left[(a_2-d_2)k_3^*-(a_3-d_3)\alpha_{23}\right]+k_3^*(a_1-d_1)\alpha_{21}<0$$

$$(iii)\ k_1^*\left[(a_3-d_3)k_2^*-(a_2-d_2)\alpha_{32}\right]+\alpha_{21}\left[(a_3-d_3)\alpha_{12}-(a_1-d_1)\alpha_{32}\right]<0$$

$$\&(iv)\ k_1^*(k_2^*k_3^*-\alpha_{23}\alpha_{32})+k_3^*\alpha_{12}\alpha_{21}<0$$

(3.1.B)

IV. LOCAL STABILITY OF THE EQUILIBRIUM POINT E_1

Theorem: The positive equilibrium point $E_1\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$ is locally asymptotically stable

If $k_2^*k_3^*>\alpha_{23}\alpha_{32}k_1(\lambda)k_2(\lambda)$

Proof: The variational matrix is given by

$$J=\begin{bmatrix} a_1-2k_1^*\overline{N_1}-\alpha_{12}\overline{N_2}-d_1 & -\alpha_{12}\overline{N_1}\overline{N_2} & 0\\ \alpha_{21}\overline{N_2} & a_2-2k_2^*\overline{N_2}+\alpha_{21}\overline{N_1}-d_2 & -\alpha_{23}\overline{N_2}k_1(\lambda)\\ 0 & -\alpha_{32}\overline{N_3}k_2(\lambda) & a_3-2k_3^*\overline{N_3}-\alpha_{32}\overline{N_3}-d_3\end{bmatrix}$$

(4.1)

Here $k_1(\lambda)\,\&\,k_2(\lambda)$ are Laplace Transforms of $k_1(z)\,\&\,k_2(z)$ respectively

$$=\begin{bmatrix} -k_1^*\overline{N_1} & -\alpha_{12}\overline{N_1} & 0\\ \alpha_{21}\overline{N_2} & -k_2^*\overline{N_2} & -\alpha_{23}\overline{N_2}k_1(\lambda)\\ 0 & -\alpha_{32}\overline{N_3}k_2(\lambda) & -k_3^*\overline{N_3}\end{bmatrix}$$

(4.2)

The characteristic equation of (4.2) be $\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$ (4.3)

Where

$$b_1 = k_1^*\overline{N_1} + k_2^*\overline{N_2} + k_3^*\overline{N_3}$$

$$b_2 = \begin{Bmatrix} k_2^*k_3^*\overline{N_2}\,\overline{N_3} + k_1^*k_2^*\overline{N_1}\,\overline{N_2} + k_2^*\overline{N_2} + k_1^*k_3^*\overline{N_1}\,\overline{N_3} \\ +\alpha_{12}\alpha_{21}\overline{N_1}\,\overline{N_2} - \alpha_{23}\alpha_{32}\overline{N_2}\,\overline{N_3}k_2(\lambda)k_1(\lambda) \end{Bmatrix}$$

$$\begin{aligned} b_3 &= \overline{N_1}\,\overline{N_2}\,\overline{N_3}\left[k_1^*k_2^*k_3^* + k_1^*k_1(\lambda)k_2(\lambda)\alpha_{23}\alpha_{32} + k_3^*\alpha_{12}\alpha_{21}\right] \\ &= \overline{N_1}\,\overline{N_2}\,\overline{N_3}\left[k_1^*\left(k_2^*k_3^* + k_1(\lambda)k_2(\lambda)\alpha_{23}\alpha_{32}\right) + k_3^*\alpha_{12}\alpha_{21}\right] \end{aligned}$$

(4.4)

By Routh-Hurwitz criteria, the system is stable if $b_1 > 0$, $(b_1b_2 - b_3) > 0$ and $b_3(b_1b_2 - b_3) > 0$.

The algebraic calculations of $(b_1b_2 - b_3)$ give that

$$\begin{aligned}(b_1b_2 - b_3) &= \overline{N_1}\,\overline{N_2}\,\overline{N_3}\left(2k_1^*k_2^*k_3^* + k_3^*\alpha_{12}\alpha_{21}\right) + \overline{N_1}\,\overline{N_3}^2\left(k_1^*k_3^{*2} + k_1^*k_2^*\right) \\ &+ \overline{N_1}^2\,\overline{N_2}\left(k_1^*k_2^* + k_1^*k_2^{*2} + \alpha_{12}\alpha_{21}\right)\left(k_2^*\overline{N_2}^2\,\overline{N_3}\right)\left(k_2^*k_3^* - \alpha_{23}\alpha_{32}k_1(\lambda)k_2(\lambda)\right)\end{aligned}$$

(4.5)

From the equation (4.5) $(b_1b_2 - b_3) > 0$ if $k_2^*k_3^* > \alpha_{23}\alpha_{32}k_1(\lambda)k_2(\lambda)$

Therefore $b_3(b_1b_2 - b_3) > 0$ if $k_2^*k_3^* > \alpha_{23}\alpha_{32}k_1(\lambda)k_2(\lambda)$ (4.6)

Hence the positive equilibrium point $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is locally asymptotically stable if $k_2^*k_3^* > \alpha_{23}\alpha_{32}k_1(\lambda)k_2(\lambda)$

Case (i): Let us define thefirstkernel as follows $k_1(z) = k_2(z) = \begin{cases} \frac{1}{T}, & if\ \ u \le T \\ 0, & if\ \ u > T \end{cases}$

Then the Laplace transform of $k_1(z)\,\&\,k_2(z)$ are defined as $k_1(\lambda) = k_2(\lambda) = \int_0^\infty e^{-\lambda t}\frac{1}{T}dt = \frac{1}{\lambda T}\left(1 - e^{-\lambda T}\right)$

Then the positive equilibrium point $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is locally asymptotically stable

if $k_2^*k_3^* > \alpha_{23}\alpha_{32}\left[\frac{1}{\lambda T}\left(1 - e^{-\lambda T}\right)\right]^2$ (4.7)

Case 2: Let us define thefirstkernel **as** follows $k_1(z) = k_2(z) = ae^{-az}\ \ for\ a > 0$

Then the Laplace transform of $k_1(z)\,\&\,k_2(z)$ are defined as $k_1(\lambda) = k_2(\lambda) = \int_0^\infty e^{-\lambda t}ae^{-at}dt = \frac{a}{a+\lambda}$

Then the positive equilibrium point $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is locally asymptotically stable if $k_2^*k_3^* > \alpha_{23}\alpha_{32}\left[\frac{a}{\lambda + a}\right]^2$

(4.8)

V. GLOBAL STABILITY:

Theorem: The co-existing $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is globally asymptotically stable

Proof: The suitable Lyapunovfunction is given by

$$V\left(N_1,N_2,N_3\right)=\sum_{i=1}^{3}N_i-\overline{N_i}-\overline{N_i}\log\left(\frac{N_i}{\overline{N_i}}\right)+\frac{1}{2}\alpha_{32}\int_0^\infty k_2\left(z\right)\int_{t-z}^{t}[N_2-\overline{N_2}]^2dudz$$
$$+\frac{1}{2}\alpha_{23}\int_0^\infty k_1\left(z\right)\int_{t-z}^{t}[N_3-\overline{N_3}]^2dudz$$

(5.1)

The time derivate of V along the solutions of equations (2.1) is

$$V^1\left(t\right)=\sum_{i=1}^{3}\frac{\left[N_i-\overline{N_i}\right]}{N_i}N_i^1+\frac{1}{2}\alpha_{32}\int_0^\infty k_2\left(z\right)\left[N_2-\overline{N_2}\right]^2dz-\frac{1}{2}\alpha_{32}\int_0^\infty k_2\left(z\right)\left[N_2\left(t-z\right)-\overline{N_2}\right]^2dz$$
$$+\frac{1}{2}\alpha_{23}\int_0^\infty k_1\left(z\right)\left[N_3-\overline{N_3}\right]^2dz-\frac{1}{2}\alpha_{23}\int_0^\infty k_1\left(z\right)\left[N_3\left(t-z\right)-\overline{N_3}\right]^2dz$$

(5.2)

Using the system of equations (2.2) and the relations $\int_0^\infty k_1\left(z\right)dz=1\&\int_0^\infty k_2\left(z\right)dz=1$

$$V^1\left(t\right)=\left[N_1-\overline{N_1}\right]\left(a_1-k_1^*N_1-\alpha_{12}N_2-d_1\right)$$
$$+\left[N_2-\overline{N_2}\right]\left(a_2-k_2^*N_2+\alpha_{21}N_1-\alpha_{23}\int_0^\infty k_1\left(z\right)N_3(t-z)dz-d_2\right)$$
$$+\left[N_3-\overline{N_3}\right]\left(a_3-k_3^*N_3-\alpha_{32}\int_0^\infty k_1\left(z\right)N_2(t-z)dz-d_3\right)+\frac{1}{2}\alpha_{32}\left[N_2-\overline{N_2}\right]^2+\frac{1}{2}\alpha_{23}\left[N_3-\overline{N_3}\right]^2$$
$$-\frac{1}{2}\alpha_{32}\int_0^\infty k_2\left(z\right)\left[N_2\left(t-z\right)-\overline{N_2}\right]^2dz-\frac{1}{2}\alpha_{23}\int_0^\infty k_1\left(z\right)\left[N_3\left(t-z\right)-\overline{N_3}\right]^2dz$$

By proper choice of $a_1,a_2\,\&\,a_3$ $a_1=k_1^*\overline{N_1}+\alpha_{12}\overline{N_2}+d_1,$

$$\left(a_2=k_2^*\overline{N_2}+\alpha_{23}\int_0^\infty k_1\left(z\right)N_3(t-z)dz+d_2\right)\&\left(a_3=\alpha_{32}\int_0^\infty k_1\left(z\right)N_2(t-z)dz+k_3^*\overline{N_3}+d_3\right)$$

$$V^1\left(t\right)=-k_1^*\left(N_1-\overline{N_1}\right)^2-k_2^*\left(N_2-\overline{N_2}\right)^2-k_3^*\left(N_3-\overline{N_3}\right)^2-\alpha_{12}\left(N_2-\overline{N_2}\right)\left(N_1-\overline{N_1}\right)$$
$$+\alpha_{21}\left(N_2-\overline{N_2}\right)\left(N_1-\overline{N_1}\right)+\frac{1}{2}\alpha_{32}\left[N_2-\overline{N_2}\right]^2+\frac{1}{2}\alpha_{23}\left[N_3-\overline{N_3}\right]^2$$
$$-\frac{1}{2}\alpha_{32}\int_0^\infty k_2\left(z\right)\left[N_2\left(t-z\right)-\overline{N_2}\right]^2dz-\frac{1}{2}\alpha_{23}\int_0^\infty k_1\left(z\right)\left[N_3\left(t-z\right)-\overline{N_3}\right]^2dz$$

(5.3)Using the inequality $ab\le\frac{a^2+b^2}{2}$ we have

$$\int_0^\infty k_2\left(z\right)\left[N_2\left(t-z\right)-\overline{N_2}\right]^2\le\int_0^\infty k_2\left(z\right)dz=1,$$
$$\int_0^\infty k_1\left(z\right)\left[N_3\left(t-z\right)-\overline{N_3}\right]^2\le\int_0^\infty k_1\left(z\right)dz=1$$

which implies

$$V^1\left(t\right)\le-\left\|\left(k_1^*-\frac{1}{2}\alpha_{12}+\frac{1}{2}\alpha_{21}\right)\right\|\left(N_1-\overline{N_1}\right)^2-\left\|\left(k_2^*+\frac{1}{2}\alpha_{32}-\frac{1}{2}\alpha_{12}+\frac{1}{2}\alpha_{21}\right)\right\|\left(N_2-\overline{N_2}\right)^2$$
$$-\left\|\left(k_3^*+\frac{1}{2}\alpha_{23}\right)\right\|\left(N_3-\overline{N_3}\right)^2-\frac{1}{2}\left\|\left(\alpha_{32}+\alpha_{23}\right)\right\|$$

$$V^1\left(t\right)\le-\mu\sum_{i=1}^{3}\left[N_i-\overline{N_i}\right]^2<0$$

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Where

$$\mu = \min\left(k_1^* + k_2^* + k_2^* + \frac{1}{2}\alpha_{23} + \frac{1}{2}\alpha_{23} - \alpha_{12} + \alpha_{21}\right)$$

Hence the system is globally stable at positive equilibrium point $E_1(\overline{N}_1, \overline{N}_2, \overline{N}_3)$

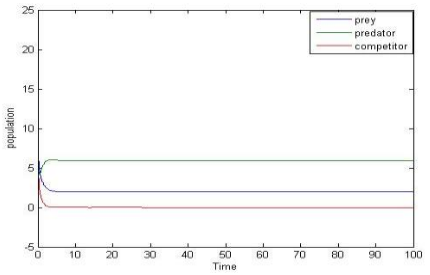
VI. NUMERICAL EXAMPLE

The systems of equations (2.1) are simulated using MATLAB using ode45.The system of equations without delay is solved with the same package we get the following results illustrated by the graphs 6.1(A), 6.1(B) for the following parametric values:

A.Graphs Description

S.No	Figures	Description
1	The figures(A)	Shows the variation of N_1, N_2 and N_3 with respect to Time (t)
2	The figures(B)	The phase portrait of N_1, N_2 and N_3

Example 1: Let a_1=2 a_2 =3, a_3=4, α_{12}=0.2, α_{21}=0.3, α_{23}=0.8, α_{32}=0.9, c_1 = 5,c_2 =5, c_3 = 5, d_1 = 0.02, d_2 = 0.02, d_3 = 0.02, N_1 =10, N_2 =15, N_3 =25.



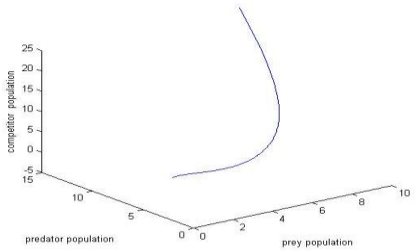


Fig: 1(A) Fig: 1(B)

For the above mentioned parametric values, the competitorspopulation is extinct. The prey and predator population are converging to the equilibrium point E(2,6, 0). Hence the system is asymptotically stable.

Case 1: Let us define thefirstkernel as follows

$$k_1(z) = k_2(z) = \begin{cases} \frac{1}{T}, \; if \;\; u \le T \\ 0 \;, if \;\; u > T \end{cases}$$

Then the Laplace transform of $k_1(z) \,\&\, k_2(z)$ are defined as $k_1(\lambda) = k_2(\lambda) = \int_0^\infty e^{-\lambda t} \frac{1}{T} dt = \frac{1}{\lambda T}\left(1 - e^{-\lambda T}\right)$

Hence the system of equations (2.3) become

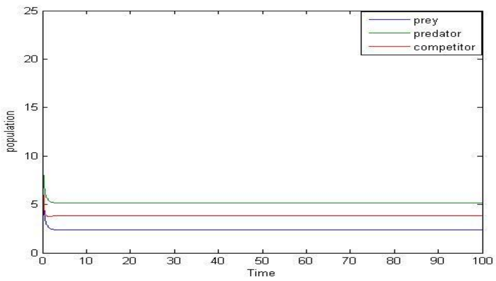
$$\frac{dN_1}{dt} = a_1 N_1\left[1 - \frac{N_1}{c_1}\right] - \alpha_{12} N_1 N_2 - d_1 N_1$$

$$\frac{dN_2}{dt} = a_2 N_2\left[1 - \frac{N_2}{c_2}\right] + \alpha_{21} N_2 N_1 - \alpha_{23} N_2 N_3 \frac{1}{\lambda T}\left(1 - e^{-\lambda T}\right) - d_2 N_2$$

$$\frac{dN_3}{dt} = a_3 N_3\left[1 - \frac{N_3}{c_3}\right] - \alpha_{32} N_2 N_3 \frac{1}{\lambda T}\left(1 - e^{-\lambda T}\right) - d_3 N_3$$

B. Nature of the System with Different Delay Kernels values λ&T

S.No	Parameters values α&β and Converging equilibrium point E	Nature of system
1	**λ =0.5 &T=10&**E (3, 6, 4)	Asymptotically stable system with a significant growth rate in competitor species.
2	**λ =0.05 &T=10&**E (2, 6, 0)	Asymptotically stable system with the competitor species is extinct.



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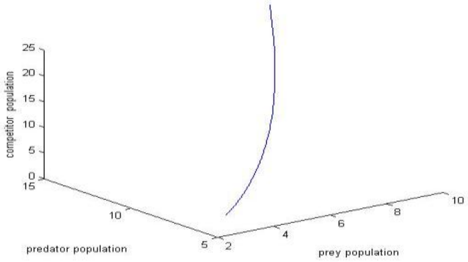
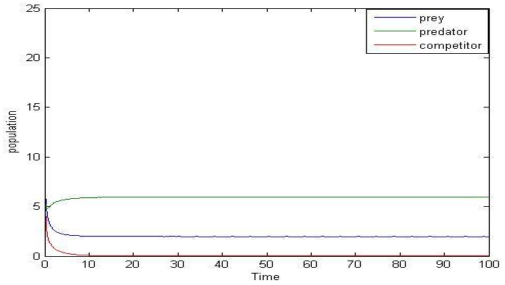


Fig 6.2.1. (A) Fig 6.2.1. (B)



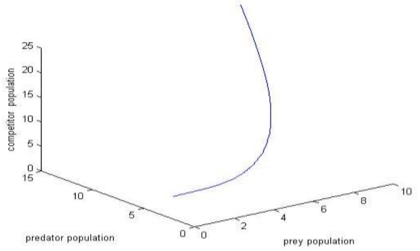


Fig 6.2.2. (A). Fig 6.2.2. (B)

Observations: For the different values of delay Kernels values λ&T are the nature of the dynamical system is asymptotically stable and observed the significant growth in competitor population, except for λ =0.05 &T=10. For this delay kernel, there is no impact of delay.

Case 2: Let us define thefirstkernel **as** follows $k_1(z)=k_2(z)=ae^{-az}\ \ for\ a>0$

Then the Laplace transform of $k_1(z)\,\&\,k_2(z)$ are defined as $k_1(\lambda)=k_2(\lambda)=\int_0^\infty e^{-\lambda t}ae^{-at}dt=\frac{a}{a+\lambda}$ Hence the system of equations (2.3) become

$$\frac{dN_1}{dt}=a_1N_1\left[1-\frac{N_1}{c_1}\right]-\alpha_{12}N_1N_2-d_1N_1$$

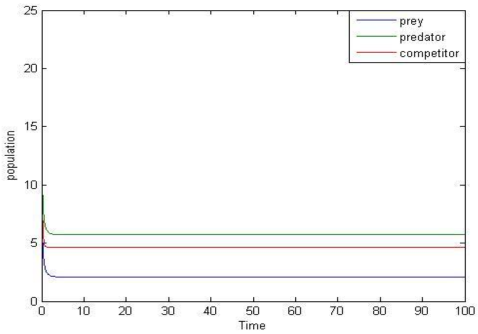
$$\frac{dN_2}{dt}=a_2N_2\left[1-\frac{N_2}{c_2}\right]+\alpha_{21}N_2N_1-\alpha_{23}N_2N_3\left(\frac{a}{a+\lambda}\right)-d_2N_2$$

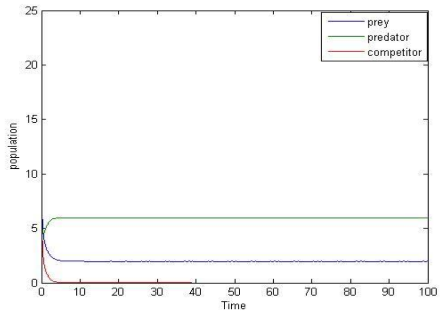
$$\frac{dN_3}{dt}=a_3N_3\left[1-\frac{N_3}{c_3}\right]-\alpha_{32}N_2N_3\left(\frac{a}{a+\lambda}\right)-d_3N_3$$

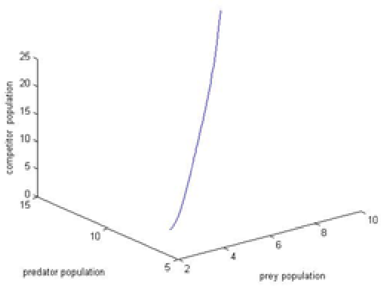
C

. Nature of the System with Different Delay Kernels values λ&a

S.No	Parameters values α&β and Converging equilibrium point E	Nature of the system
1	**λ =10 & a=0.5** E (3, 6, 5)	Asymptotically stable system with a significant growth rate in competitor species.
2	**λ =0.5 &a=5** E (2, 6, 0)	Asymptotically stable system with the competitor species is extinct







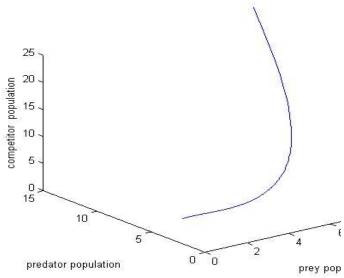


Fig 6.3.1 (A) Fig 6.3.1 (B)

Fig 6.3.2 (A)
Fig 6.3.2 (B)

VI. RESULTS

For the different values of delay Kernels values λ&T are the nature of the dynamical system is asymptotically stable and observed the significant growth in competitor population, except for λ =0.5 &a=5. For this delay kernel, there is no impact of delay.

VII. CONCLUSIONS

We study the stability analysis of three species ecological model. Co-existing state is identified.The system is locally asymptotically stable if $k_2^* k_3^* > \alpha_{23}\alpha_{32}k_1(\lambda)k_2(\lambda)$. The global stability is studied by constructing a suitable Lyapunov's function. The numerical simulation is carried out in support of the system stability analysis. We choose the suitable parametric values for the system of equations (2.1). These parameters are satisfied the conditions (3.1.A) and also local stability condition (4.6). We choose two types delay kernels square and exponential type for investigation for same set of parametric values. The simulation shows that the delay kernels are significant in the growth rate of competitor species. We compare the results with delay and without delay agreements. We identified the different delay parameters in support of stability analysis and the dynamics are shown in Table 6.2 and 6.3. For two set of parametric values λ =0.05 &T=10 for square memory kernel and λ =0.5 &a=5 for exponential kernel, the system dynamics has no influence of delay. For the remaining parameters of delay kernels, there is a significant growth in competitor species. Hence delay plays a significant role in the growth rate of competitor species.

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