The Operations Fs-Union, Fs-Intersection and Fs-Complement Operator in Fs-Set Theory- Some Results

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Abstract: ***In this Paper, we prove the collection of all FS -subsets of a given FS-set with the symmetric operation satisfies a distributive law namely FS –intersection operation distributes over the symmetric difference operation with some conditions.***

Keywords: ***Fs-set, Fs-subset, Fs-union, Fs-intersection, Fs-Complement, symmetric difference Operation.***

I. INTRODUCTION

Ever since Zadeh [8] introduced the notion of fuzzy sets in his pioneering work, several mathematicians studied numerous aspects of fuzzy sets.

Recently many researchers put their efforts in order to prove collection of all fuzzy subsets of a given fuzzy set is Boolean algebra under suitable operations [21].VaddiparthiYogeswara,G.Srinivas and BiswajitRath[11] introduced the concept of Fs-set and developed the theory of Fs-sets in order to prove collection of all Fs-subsets of given Fs-set is a complete Boolean algebra under Fs-unions, Fs-intersections and Fs-complements. The Fs-sets they introduced contain Boolean valued membership functions.In this Paper, we prove the collection of all FS -subsets of a given FS-set with the symmetric operation satisfies a distributive law namely FS –intersection operation distributes over the symmetric difference operation with some conditions. . All other relevant results are proved in the next section. We denote the largest element of a complete Boolean algebra L_A[1.1] by $M_A$or 1_A. Throughout this paper starting from section-1, For all lattice theoretic properties and Boolean algebraic properties one can refer Szasz [3], Garret Birkhoff[4],Steven Givant• Paul Halmos[2] and Thomas Jech[5].

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A. Fs-Set

Let U be a universal set, $A_1 \subseteq U$ and let $A \subseteq U$ be non-empty. A four tuple $\mathcal{A}(A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$ is said be an Fs-set if, and only if

(1) $A \subseteq A_1$

(2) L_A is a complete Boolean Algebra

(3) $\mu_{1A_1}: A_1 \longrightarrow L_A$, $\mu_{2A}: A \longrightarrow L_A$,are functions such that $\mu_{1A_1}|A \geq \mu_{2A}$

(4) $\bar{A}: A \longrightarrow L_A$ is defined by $\bar{A}x = \mu_{1A_1}x \wedge (\mu_{2A}x)^c$,for each $x \in A$

(5)

B. Fs-subset

Let $\mathcal{A}=(A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$ and $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ be a pair of Fs-sets. $\mathcal{B}$is said to be an Fs-subset of $\mathcal{A}$, denoted by $\mathcal{B} \subseteq \mathcal{A}$, if, and only if

(1) $B_1 \subseteq A_1$, $A \subseteq B$

(2) L_B is a complete subalgebra of L_A or $L_B \leq L_A$

(3) $\mu_{1B_1} \leq \mu_{1A_1}|B_1$, and $\mu_{2B}|A \geq \mu_{2A}$

C. Definition of Fs-complement of an Fs-subset:

Consider a particular Fs-set $\mathcal{A} = (A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$,$A \neq \Phi$,where

[1] $A \subseteq A_1$

[2] $L_A = [0, M_A]$, $M_A = \vee \bar{A}A = \vee_{a \in A} \bar{A}a$

[3] $\mu_{1A_1} = M_A, \mu_{2A} = 0$,
$\bar{A}x = \mu_{1A_1}x \wedge (\mu_{2A}x)^c = M_A$,for each $x \in A$

Given $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$. We define Fs-complement of $\mathcal{B}$, denoted by $\mathcal{B}^{C_\mathcal{A}}$ for B=A and $L_B = L_A$ as follows:

$\mathcal{B}^{C_\mathcal{A}} = \mathcal{D}=(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D)$, where

(a') $D_1 = C_A B_1 = B_1^c \cup A, D = B = A$

(b') $L_D = L_A$

(c') $\mu_{1D_1}: D_1 \longrightarrow L_A$, is defined by $\mu_{1D_1}x = M_A$

$\mu_{2D}: A \longrightarrow L_A$, is defined by $\mu_{2D}x = \bar{B}x = \mu_{1B_1}x \wedge (\mu_{2B}x)^c$

$\bar{D}: A \longrightarrow L_A$,is defined by $\bar{D}x = \mu_{1D_1}x \wedge (\mu_{2D}x)^c = M_A \wedge (\bar{B}x)^c = (\bar{B}x)^c$.

D. Definition

Let $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ and $\mathcal{C}=(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ be a pair of Fs-subsets of$\mathcal{A}$. Then, the Fs-union of $\mathcal{B}$ and $\mathcal{C}$, denoted by $\mathcal{B} \cup \mathcal{C}$ is defined as $\mathcal{B} \cup \mathcal{C} = \mathcal{D}=(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D)$,where

(1) $D_1 = B_1 \cup C_1$, $D = B \cap C$

(2) $L_D = L_B \vee L_C =$ The complete subalgebra generated by $L_B \cup L_C$

(3) $\mu_{1D_1}: D_1 \rightarrow L_D$ is defined by

$$\mu_{1D_1}x = (\mu_{1B_1} \vee \mu_{1C_1})x$$

$\mu_{2D} : D \to L_D$is defined by
$\mu_{2D}x = \mu_{2B}x \wedge \mu_{2C}x$and

$\bar{D}$: $D \to L_D$ is defined by

$$\bar{D}x = \mu_{1D_1}x \wedge (\mu_{2D}x)^c$$

E. Definition

Let $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1},\mu_{2B}), L_B)$ and $\mathcal{C}=(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ be a pair of Fs-subsets of$\mathcal{A}$. Then, the Fs-intersection of $\mathcal{B}$ and $\mathcal{C}$, denoted by$\mathcal{B} \cap \mathcal{C}$ is defined as
$\mathcal{B} \cap \mathcal{C}=\mathcal{D}=(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D)$,where
(1) $L_D = L_B \wedge L_C =$ The complete subalgebra generated by $L_B \cap L_C$

(2) $\mu_{1D_1}: D_1 \to L_D$ is defined by

$$\mu_{1D_1}x = \mu_{1B_1}x \wedge \mu_{1C_1}x$$

$\mu_{2D} : D \to L_D$is defined by
$\mu_{2D}x = (\mu_{2B} \vee \mu_{2C})$xand

$\bar{D}$: $D \to L_D$ is defined by

$$\bar{D}x = \mu_{1D_1}x \wedge (\mu_{2D}x)^c$$

A family $\mathfrak{G}$of all**Fs**-subsets$\mathcal{B} \subseteq \mathcal{A}$ and$\mathcal{A} = (A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$ with
$\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1},\mu_{2B}), L_B)$ with B=A and $L_B = L_A$ is a commutative group with the operation $-$ as defined below$\mathcal{B} - \mathcal{C}$ $=(\mathcal{B} \cap \mathcal{C}^{C_{\mathcal{A}}})$where $\mathcal{B}$ &$\mathcal{C}$ areFs- subsets of$\mathcal{A}$

F. Proposition

For any Fs-subsets$\mathcal{B}, \mathcal{C}$&$\mathcal{D}$ the following
$(\mathcal{B} - \mathcal{C}) - \mathcal{D}$= $(\mathcal{B} - (\mathcal{C} \cup \mathcal{D})$provided
$(\bar{C}x \vee \bar{D}x) \geq (\mu_{1D_1}x \wedge (\mu_{2C})^c)x \wedge (\mu_{1C_1}x \wedge (\mu_{2D}x)^c)$
$\mathcal{B} - \mathcal{C}=(\mathcal{B} \cap \mathcal{C}^{C_{\mathcal{A}}}) - \mathcal{D} = \mathcal{G} - \mathcal{D} = \mathcal{G} \cap \mathcal{D}^{C_{\mathcal{A}}} = \mathcal{G} \cap \mathcal{F} = \mathcal{H}$
$\mathcal{H} = (H_1, H, \bar{H}(\mu_{1H_1}, \mu_{2H}), L_H)$, where
(4)$H_1 = G_1 \cap F_1 = ((B_1 \cap C_1^c) \cup A) \cap (D_1^c \cup A) = [(B_1 \cap C_1^c) \cap D_1^c] \cup A = (B_1 - C_1) - D_1) \cup A$,
H= G∪F=A

$$(5) L_H = L_G \wedge L_F = L_A$$

(6)$\mu_{1H_1}: H_1 \to L_H$ *is defined by* $\mu_{1H_1}x = \mu_{1G_1}x \wedge \mu_{1F_1}x = \mu_{1G_1}x \wedge M_A = \mu_{1G_1}x = \mu_{1B_1}x$
$\mu_{2H} : H \to L_H$*isdefined by* $\mu_{2H}x = \mu_{2G}x \vee \mu_{2F}x = (\mu_{2B}x \vee \bar{C}x) \vee \bar{D}x$
$\bar{H}: H \to L_H$ *is definedby* $\bar{H}x = \mu_{1H_1}x \wedge (\mu_{2H}x)^c$
$= \mu_{1B_1}x \wedge (\mu_{2B}x \vee \bar{C}x \vee \bar{D}x)^c$
$= \mu_{1B_1}x \wedge (\mu_{2B}x)^c \wedge (\bar{C}x)^c \wedge (\bar{D}x)^c$
$= (\bar{B}x) \wedge (\bar{C}x)^c \wedge (\bar{D}x)^c$
R.H.S

$\mathcal{C} \cup \mathcal{D} = \mathcal{J} = (J_1, J, \bar{J}(\mu_{1J_1}, \mu_{2J}), L_J)$, where
(i) $J_1 = C_1 \cup D_1$, J=C∩ D =A
(ii) $L_J = L_C \wedge L_D = L_A$
(iii)$\mu_{1J_1} : J_1 \to L_A$is defined by$\mu_{1J_1}x = (\mu_{1C_1} \vee \mu_{1D_1})x$
$\mu_{2J} : J \to L_J$is defined by$\mu_{2J}x = \mu_{2C}x \wedge \mu_{2D}x$
$\bar{J}: J \to L_J$is defined by$\bar{J}x = \mu_{1J_1}x \wedge (\mu_{2J}x)^c$
$= (\mu_{1C_1} \vee \mu_{1D_1})x \wedge (\mu_{2C}x \wedge \mu_{2D}x)^c$
$\mathcal{J}^{C_{\mathcal{A}}} = \mathcal{K}=(K_1, K, \bar{K}(\mu_{1K_1}, \mu_{2K}), L_K)$, where
(7)$K_1 = C_A J_1 = J_1^c \cup A = (C_1 \cup D_1)^C \cup A$, K = J = A
(8)$L_K = L_J = L_A$
(9) $\mu_{1K_1}: K_1 \to L_K$, is defined by $\mu_{1K_1}x = M_A$
$\mu_{2K}: K \to L_K$, is defined by $\mu_{2K}x = \bar{J}x = \mu_{1J_1}x \wedge (\mu_{2J}x)^c$
$\bar{K}: K \to L_K$,is defined by$\bar{K}x = \mu_{1J_1}x \wedge (\mu_{2J}x)^c = M_A \wedge (\bar{J}x)^c = (\bar{J}x)^c$.

$\mathcal{B} - (\mathcal{C} \cup \mathcal{D}) = \mathcal{B} - \mathcal{J} = \mathcal{B} \cap \mathcal{J}^{C_{\mathcal{A}}} = \mathcal{B} \cap \mathcal{K} = \mathcal{L} = (L_1, L, \bar{L}(\mu_{1L_1}, \mu_{2L}), L_L)$, where
(10)$L_1 = B_1 \cap L_1 = ((B_1 \cap J_1^c) \cup A) = [B_1 \cap (C_1 \cup D_1)^C] \cup A = [(B_1 \cap C_1^c) \cap D_1^c] \cup A = (B_1 - C_1) - D_1) \cup A$,
L= B∪ J=A

$$(11) L_L = L_B \wedge L_J = L_A$$

(12) $\mu_{1L_1}: L_1 \to L_L$is defined by$\mu_{1L_1}x = \mu_{1B_1}x \wedge \mu_{1K_1}x = \mu_{1B_1}x \wedge M_A$

$\mu_{2L} : L \to L_L$ *isdefined by* $\mu_{2L}x = (\mu_{2B} \vee \mu_{2K})x = \mu_{2B}x \vee \mu_{2K}x = \mu_{2B}x \vee \bar{J}x$
$= \mu_{2B}x \vee [(\mu_{1C_1} \vee \mu_{1D_1})x \wedge (\mu_{2C}x \wedge \mu_{2D}x)^c]$
$= \mu_{2B}x \vee (\bar{C}x \vee \bar{D}x) \vee (\mu_{1D_1}x \wedge (\mu_{2C})^c)x \wedge (\mu_{1C_1}x \wedge (\mu_{2D}x)^c)$
$\geq \mu_{2B}x \vee \bar{C}x \vee \bar{D}x$
$\bar{L}: L \to L_L$ *is defined by* $\bar{L}x = \mu_{1L_1}x \wedge (\mu_{2L}x)^c$
$= \mu_{1B_1}x \wedge \{\mu_{2B}x \vee [(\mu_{1C_1} \vee \mu_{1D_1})x \wedge (\mu_{2C}x \wedge \mu_{2D}x)^c]\}^c$
As $H_1 = L_1$, H=L , $L_H = L_L = L_A$, μ_{1H_1}=μ_{1L} in H *and* L,

Sufficient to show that

$$\mu_{2H} = \mu_{2L}$$

And observe that using the condition $(\bar{C}x \vee \bar{D}x) \geq (\mu_{1D_1}x \wedge (\mu_{2C})^c)x \wedge (\mu_{1C_1}x \wedge (\mu_{2D}x)^c)$

G. Proposition

For any Fs-subsets$\mathcal{B}, \mathcal{C}$&$\mathcal{D}$ the following $(\mathcal{B} \cup \mathcal{C}) - \mathcal{D} = (\mathcal{B} - \mathcal{D}) \cup (\mathcal{C} - \mathcal{D})$
Proof:$\mathcal{D}^{C_{\mathcal{A}}} = = \mathcal{E}=(E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E}), L_E)$, where
(a') $E_1 = C_A D_1 = D_1^c \cup A, E = D = A$
(b') $L_E = L_A$
(c') $\mu_{1E_1}: E_1 \to L_E$, is defined by $\mu_{1E_1}x = M_A$
$\mu_{2E}: E \to L_E$, is defined by $\mu_{2E}x = \bar{D}x = \mu_{1D_1}x \wedge (\mu_{2D}x)^c$
$\bar{E}: E \to L_E$,is defined by$\bar{E}x = \mu_{1E_1}x \wedge (\mu_{2E}x)^c = M_A \wedge (\bar{D}x)^c = (Dx)^c$.
L.H.S $(\mathcal{B} \cup \mathcal{C}) - \mathcal{D} = (\mathcal{B} \cup \mathcal{C}) \cap \mathcal{D}^{C_{\mathcal{A}}}$
$\mathcal{B} \cup \mathcal{C} = \mathcal{F} = (F_1, F, \bar{F}(\mu_{1F_1}, \mu_{2F}), L_F)$, where
(i) $F_1 = B_1 \cup C_1$, J=B∩ C =A
(ii) $L_F = L_B \wedge L_C = L_A$
(iii)$\mu_{1F_1} : F_1 \to L_F$is defined by$\mu_{1F_1}x = (\mu_{1B_1} \vee \mu_{1C_1})x$
$\mu_{2F} : F \to L_F$is defined by$\mu_{2F}x = \mu_{2B}x \wedge \mu_{2C}x$
$\bar{F}: F \to L_F$is defined by$\bar{F}x = \mu_{1F_1}x \wedge (\mu_{2F}x)^c$
$= (\mu_{1B_1} \vee \mu_{1C})x \wedge (\mu_{2B}x \wedge \mu_{2C}x)^c$
$(\mathcal{B} \cup \mathcal{C}) - \mathcal{D} = (\mathcal{B} \cup \mathcal{C}) \cap \mathcal{D}^{C_{\mathcal{A}}} = \mathcal{F} \cap \mathcal{E} = \mathcal{G}$
$\mathcal{G} = (G_1, G, \bar{G}(\mu_{1G_1}, \mu_{2G}), L_G)$, where
(1)$G_1 = F_1 \cap E_1 = (B_1 \cup C_1) \cap (D_1^c \cup A) = [(B_1 \cup C_1) \cap D_1^c] \cup A = [(B_1 \cup C_1) - D_1] \cup A$, G= F ∪ E =A

$$(2) L_G = L_F \wedge L_E = L_A$$

(3)$\mu_{1G_1}: G_1 \to L_G$ *is defined by* $\mu_{1G_1}x = \mu_{1F_1}x \wedge \mu_{1E_1}x = \mu_{1F_1}x \wedge M_A = \mu_{1F_1}x = (\mu_{1B_1} \vee \mu_{1C_1})x$

$$\mu_{2G} : G \to L_G is defined\ by\ \mu_{2G}x = \mu_{2F}x \vee \mu_{2E}x$$
$$= (\mu_{2B}x \wedge \mu_{2C}x) \vee \bar{D}x$$
$$\bar{G}: G \to L_G\ is\ defined by \quad \bar{G}x = \mu_{1G_1}x \wedge (\mu_{2G}x)^c$$
$$= (\mu_{1B_1} \vee \mu_{1C_1})x \wedge ((\mu_{2B}x \wedge \mu_{2C}x) \vee \bar{D}x)^c$$
$$R.H.S$$

$(\mathcal{B} - \mathcal{D}) \cup (\mathcal{C} - \mathcal{D})$

$\mathcal{B} - \mathcal{D} = \mathcal{B} \cap \mathcal{D}^{C_{\mathcal{A}}} = \mathcal{B} \cap \mathcal{E} = \mathcal{H} = (\mathrm{H}_1, \mathrm{H}, \bar{\mathrm{H}}(\mu_{1\mathrm{H}_1}, \mu_{2\mathrm{H}}), \mathrm{L}_\mathrm{H})$, where

(1)$\mathrm{H}_1 = \mathrm{B}_1 \cap \mathrm{E}_1 = \mathrm{B}_1 \cap (\mathrm{D}_1^c \cup \mathrm{A}) = (\mathrm{B}_1 \cap \mathrm{D}_1^c) \cup \mathrm{A}$, G= B∪ E =A

$$(2)L_H = L_B \wedge L_E = L_A$$

$$(3)\mu_{1H_1}: H_1 \to L_H is\ defined\ by \mu_{1H_1}x = \mu_{1B_1}x \wedge \mu_{1E_1}x$$
$$= \mu_{1B_1}x \wedge M_A = \mu_{1B_1}x$$
$$\mu_{2H} : H \to L_H is defined\ by\ \mu_{2H}x$$
$$= \mu_{2B}x \vee \mu_{2E}x = \mu_{2B}x \vee \bar{D}x$$
$$\bar{H}: H \to L_H\ is\ defined by \quad \bar{H}x = \mu_{1H_1}x \wedge (\mu_{2H}x)^c$$
$$= \mu_{1B_1}x \wedge (\mu_{2B}x \vee \bar{D}x)^c$$
$$= \mu_{1B_1}x \wedge (\mu_{2B}x)^c \wedge (Dx)^c$$
$$= (\bar{B}x) \wedge (\bar{D}x)^c$$

$\mathcal{C} - \mathcal{D} = = \mathcal{C} \cap \mathcal{D}^{C_{\mathcal{A}}} = \mathcal{C} \cap \mathcal{E} = \mathcal{J} = (\mathrm{J}_1, \mathrm{J}, \bar{\mathrm{J}}(\mu_{1\mathrm{J}_1}, \mu_{2\mathrm{J}}), \mathrm{L}_\mathrm{J})$, where

(1)$\mathrm{J}_1 = \mathrm{C}_1 \cap \mathrm{E}_1 = \mathrm{C}_1 \cap (\mathrm{D}_1^c \cup \mathrm{A}) = (\mathrm{C}_1 \cap \mathrm{D}_1^c) \cup \mathrm{A}$, J= $C \cup E$ =A

$$(2)L_J = L_B \wedge L_E = L_A$$

$$(3)\mu_{1J_1}: J_1 \to L_J is\ defined\ by \mu_{1J_1}x = \mu_{1C_1}x \wedge \mu_{1E_1}x$$
$$= \mu_{1C_1}x \wedge M_A = \mu_{1C_1}x$$
$$\mu_{2J} : J \to L_J is defined\ by\ \mu_{2J}x$$
$$= \mu_{2C}x \vee \mu_{2E}x = \mu_{2C}x \vee \bar{D}x$$
$$\bar{J}: J \to L_J\ is\ defined by \quad \bar{J}x = \mu_{1J_1}x \wedge (\mu_{2J}x)^c$$
$$= \mu_{1C_1}x \wedge (\mu_{2C}x \vee \bar{D}x)^c$$
$$= \mu_{1C_1}x \wedge (\mu_{2C}x)^c \wedge (Dx)^c$$
$$= (\bar{C}x) \wedge (\bar{D}x)^c$$

$\mathcal{H} \cup \mathcal{J} = \mathcal{K} = (K_1, K, \bar{K}(\mu_{1K_1}, \mu_{2K}), L_K)$, where

(i) $K_1 = H_1 \cup J_1 = [(\mathrm{B}_1 \cap \mathrm{D}_1^c) \cup \mathrm{A}] \cup [(\mathrm{C}_1 \cap \mathrm{D}_1^c) \cup \mathrm{A}]$ = $[(\mathrm{B}_1 \cup \mathrm{C}_1) \cap \mathrm{D}_1^c] \cup \mathrm{A}$

$= [(\mathrm{B}_1 \cup \mathrm{C}_1) - \mathrm{D}_1] \cup \mathrm{A}$, K=H ∩ J =A

(ii) $L_K = L_H \wedge L_J = L_A$

(iii)$\mu_{1K_1} : K_1 \to L_A$is defined by$\mu_{1K_1}x = (\mu_{1H_1} \vee \mu_{1J_1})x = (\mu_{1B_1} \vee \mu_{1C_1})x$

$\mu_{2K} : K \to L_K$is defined by$\mu_{2K}x = \mu_{2H}x \wedge \mu_{2J}x = (\mu_{2B}x \vee \bar{D}x) \wedge (\mu_{2C}x \vee \bar{D}x)$

$= (\mu_{2B}x \wedge \mu_{2C}x) \vee \bar{D}x$

$\bar{K}: J \to L_K$is defined by$\bar{K}x = \mu_{1K_1}x \wedge (\mu_{2K}x)^c$

$= (\mu_{1B_1} \vee \mu_{1C_1})x \wedge ((\mu_{2B}x \wedge \mu_{2C}x) \vee \bar{D}x)^c$

As $G_1 = K_1$, G=K , $L_G = L_K = L_A$, $\mu_{1G_1} = \mu_{1K_1}$, $\mu_{2G} = \mu_{2K}$in G *and* K,

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