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Gallai And Anti-Gallai Graphs Of S (N, M) Graphs

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Abstract: ***This paper deals with the graph operators – Gallai and Anti -Gallaigraphs.TheGallai graph and the anti-Gallai graph of a graph G have the edges of G as their vertices. Two edges of G are adjacent in the Gallai graph of G if they are incident but do not span a triangle in G; they are adjacent in the anti-Gallai graph of G if they span a triangle in G. We discuss about the Gallai and Anti -Gallai graphs of S(n,m) graphs.***

Keywords:Anti – Gallai graphs, Gallai graphs, S(n,m) graphs.

I. INTRODUCTION

In this paper we consider the graph S(n,m) which is a quartic graph and also both Eulerian and Hamiltonian.

The graph S(n,m)[1][2] consists of n vertices denoted as v_1, v_2, ……. v_n. The edges are defined as follows:

i) v_i is adjacent to v_{i+1} and $v_n$is adjacent to v_1.

ii) v_i is adjacent to v_{i+m} if $i+m \leq n$.

iii) v_i is adjacent to v_{i+m-n} if $i+m > n$.

This paper deals with the graph operators – Gallai and Anti -Gallai graphs of the S(n,m).

The line graph[3] L(G) of a graph G has the edges of G as its vertices and two distinct edges of G are adjacent in L(G) if they are incident in G. The Gallai graph [4] Γ(G) of a graph G has the edges of G as its vertices and two distinct edges of G are adjacent in Γ(G) if they are incident in G, but do not span a triangle in G.

The anti-Gallai graph [4] Δ(G) of a graph G has the edges of G as its vertices and two distinct edges of G are adjacent in Δ(G) if they are incident in G and lie on a triangle in G It is the complement of Γ(G) in L(G).

A Hamiltonian cycle is a cycle that visits each vertex exactly once. A graph that contains a Hamiltonian cycle is called a Hamiltonian graph. An Eulerian cycle in an undirected graph is a cycle that uses each edge exactly once. A graph that contains aEulerian cycle is called a Eulerian graph.

For each natural number *n*, the edgeless graph (or empty graph) of order *n* is the graph with *n* vertices and zero edges.A triangle-free graph is an undirected graph in which no three vertices form a triangle of edges.

I. GALLAI GRAPHS OF S(N,M) GRAPHS

Let G be a S(n,m)graph ($n \geq 2m+2$) with n vertices and 2n edges. Let v_1, v_2, …….$v_n$be the vertices of the graph G . The edges of G be e_1,e_2,e_3,……,e_{2n}.The edges v_iv_{i+1} for i=1 to n-1 be e_1, e_2,…..e_{n-1} ,$v_nv_1$be e_n,v_iv_{i+m} for $i+m \leq n$be e_{n+1},e_{n+2},…$e_{2n-m+1}$and v_iv_{i+m-n} for $i+m > n$ be e_{2n-m+2},……,e_{2n}.

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Theorem 2.1:

The Gallai graph of S(n,2) ,$n \geq 7$, $n \neq 3k$ for k = 3,4,….. isEulerian and Hamiltonian.

Proof:

Let G denote the graph S(n,2) ,$n \geq 7$, $n \neq 3k$ for k = 3,4,…..

Let v_1, v_2, …….$v_n$be the vertices of the graph G .

The edges of Gbe e_1,e_2,e_3,……,e_{2n}.

Let Γ(G) be the Gallai graph of G whose vertices aree_1,e_2,e_3,……,e_{2n}.The edges of G are defined as follows:

(i) $e_i$is adjacent to e_{i+n+1} for i = 1 to n-1 and e_n is adjacent to e_{n+1}.

(ii) $e_i$is adjacent to e_{i-n+2} and e_{i+2} for i = n+1 to 2n-2.

(iii) $e_{2n-1}$is adjacent to e_1 and e_{n+1}.

(iv) $e_{2n}$is adjacent to e_2 and e_{n+2}.

With the above defined vertices and edges clearly Γ(G) is both Eulerian and Hamiltonian.

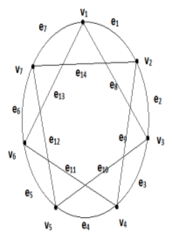

Fig. 1

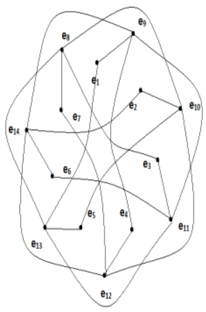

Fig .2

Fig.1 illustrates the S(7,2) graph and Fig.2 illustrates the Gallai graph of S(7,2) which is both Eulerian and Hamiltonian.

Theorem 2.2:

The Gallai graph of S(n,2) ,n = 6 ,is a 2-regular bipartite graph.

Proof:

Let G denote the graph S(n,2) ,$n = 6$.The vertices of the graph G be v_1, v_2, ……. $v_n$let the edges bee_1,e_2,e_3,……,e_{2n}.

Let Γ(G) be the Gallai graph of G whose vertices aree_1,e_2,e_3,……,e_{2n}.The edges of G are defined as follows:

(i) $e_i$is adjacent to e_{i+n+1} for i = 1 to n-1 and e_n is adjacent to e_{n+1}.

(ii) $e_i$is adjacent to e_{i-n+2} for i = n+1 to 2n-2.

(iii) $e_{2n-1}$is adjacent to e_1.

(iv) $e_{2n}$is adjacent to e_2.

With the above defined vertices and edges Γ(G) is 2-regular bipartite.

Theorem 2.3:

The Gallai graph of S(n,2) ,n=3k for k = 3,4,….. isEulerian.

Proof:

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Let G denote the graph S(n,2) ,n=3k for k = 3,4,….. .with vertices v_1, v_2, ……. v_n and edges $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

Let Γ(G) be the Gallai graph of G whose vertices are $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

The edges of G are defined as follows:

(i) e_i is adjacent to e_{i+n+1} for i = 1 to n-1 and e_n is adjacent to e_{n+1}.

(ii) e_i is adjacent to e_{i-n+2} and e_{i+2} for i = n+1 to 2n-2.

(iii) e_{2n-1} is adjacent to e_1 and e_{n+1}.

(iv) e_{2n} is adjacent to e_2 and e_{n+2}.

With the above defined vertices and edges clearly Γ(G) is an Eulerian graph.

Theorem 2.4:

The Gallai graph of S(n,m) , n≥ 2m+2, m≥3, n$\neq 3m$ is a 6-regular Eulerian and Hamiltonian graph.

Proof:

Let G denote the graph S(n,m) , n≥ 2m+2, m≥3, n$\neq 3m$ whose vertices are v_1, v_2, …….v_n and edges $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

Let Γ(G) be the Gallai graph of G whose vertices are $e_1,e_2,e_3,\ldots\ldots,e_{2n}$. The edges of G are defined as follows:

(i) e_i is adjacent to e_{i+1},e_{i+n},e_{i+n+1} for i = 1 to n-1 and e_n is adjacent to e_1, e_{n+1} and e_{2n}.

(ii) e_i is adjacent to e_{2n-m+i} and $e_{2n-m+i+1}$ for i = 1 to m-1 and e_m is adjacent to e_{2n} and e_{n+1}.

(iii) e_i is adjacent to e_{n-m+I} and $e_{n-m+i+1}$ for i = m+1 to n.

(iv) e_i is adjacent to e_{i+m} for i = n+1 to 2n - m.

(v) e_{n+i} is adjacent to e_{2n-m+i} for i = 1 to m.

With the above defined vertices and edges Γ(G) is a 6-regular Eulerian and Hamiltonian graph.

Theorem 2.5:

The Gallai graph of S(n,m), m≥3, n= $3m$ is an Eulerian and Hamiltonian graph.

Proof:

Let G denote the graph S(n,m), m≥3, n= $3m$ with vertices v_1, v_2, ……. v_n and edges $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

Let Γ(G) be the Gallai graph of G whose vertices are $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

The edges of G are defined as follows:

(i) e_i is adjacent to e_{i+1},e_{i+n},e_{i+n+1} for i = 1 to n-1 and e_n is adjacent to e_1, e_{n+1} and e_{2n}.

(ii) e_i is adjacent to e_{2n-m+i} and $e_{2n-m+i+1}$ for i = 1 to m-1 and e_m is adjacent to e_{2n} and e_{n+1}.

(iii) e_i is adjacent to e_{n-m+i} and $e_{n-m+i+1}$ for i = m+1 to n.

With the above defined vertices and edges Γ(G) is an Eulerian and Hamiltonian graph.

III. ANTI GALLAI GRAPHS OF S(N,M) GRAPHS

Let G be a S(n,m) graph (n≥ $2m+2$) with n vertices v_1, v_2, ……. v_n and 2n edges $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

The edges v_iv_{i+1} for i=1 to n-1 be e_1, e_2,…..e_{n-1} ,v_nv_1 be e_n,v_iv_{i+m} for i+m≤ n be e_{n+1},e_{n+2},…e_{2n-m+1} and v_iv_{i+m-n} for i+m> n be e_{2n-m+2},……,e_{2n}.

Theorem 3.1:

The Anti-Gallai graph of S(n,2) ,n ≥ 7is Eulerian and Hamiltonian.

Proof:

Let G denote the graph S(n,2) ,n ≥ 7.

Let v_1, v_2, …….v_n be the vertices of the graph G .

The edges of G be $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

Let Δ(G) be the Anti-Gallai graph of G whose vertices are $e_1,e_2,e_3,\ldots\ldots,e_{2n}$. The edges of G are defined as follows:

(i) e_i is adjacent to e_{i+1} and e_{i+n} for i = 1 to n-1.

(ii) e_n and e_{2n} are adjacent to e_1.

(iii) e_i is adjacent to e_{i-n+1} for i = n+1 to 2n – 1.

With the above defined vertices and edges Δ(G) is both Eulerian and Hamiltonian and also Δ(G) is isomorphic to a graph which is in the shape of polygonal pyramid net, where polygon is of n sides.

Theorem 3.2:

The Anti-Gallai graph of S(n,2) ,n=6 is quartic, Eulerian and Hamiltonian.

Proof:

Let G denote the graph S(n,2) ,n $= 6$. The vertices of the graph G be v_1, v_2, ……. v_n and the edges be $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

Let Δ(G) be the Anti-Gallai graph of G whose vertices are $e_1,e_2,e_3,\ldots\ldots,e_{2n}$. The edges of G are defined as follows:

(i) e_i is adjacent to e_{i+1} and e_{i+n} for i = 1 to n-1.

(ii) e_n is adjacent to e_1 ,e_{2n}

(iii) e_i is adjacent to e_{i-n+1} and e_{i+2} for i = n+1 to 2n – 2.

(iv) e_{2n-1} is adjacent to e_n and e_{n+1}

(v) e_{2n} is adjacent to e_1 and e_{n+2}.

With the above defined vertices and edges Δ(G) is both Eulerian and Hamiltonian and also quartic.

Theorem 3.3:

The Anti-Gallai graph of S(n,m) , n≥ 2m+2, m≥3, n$\neq 3m$ is an edgeless graph with 2n vertices.

Proof:

Let G denote the graph S(n,m) , n≥ 2m+2, m≥3, n$\neq 3m$ whose vertices are v_1, v_2, …….v_n and the edges $e_1,e_2,e_3,\ldots\ldots,e_{2n}$. Let Δ(G) be the Anti-Gallai graph of G whose vertices are $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

G is a Triangle – free graph and therefore no two incident edges of G lie on a triangle. Hence no two vertices of Δ(G) are adjacent to each other. Thus Δ(G) is an edgeless graph with 2n vertices.

Theorem 3.4:

The Anti-Gallai graph of S(n,m), m≥3, n= $3m$ is a graph with m disjoint cycles of length 3 and n isolated vertices.

Proof:

Let G denote the graph S(n,m), m≥3, n= $3m$ with vertices v_1, v_2, ……. v_n and edges $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

Let Δ(G) be the Anti-Gallai graph of G whose vertices are $e_1,e_2,e_3,\ldots\ldots,e_{2n}$.

The edges of G are defined as follows:

(i) e_i is adjacent to e_{i+m} for i = n+1 to 2n - m.

(ii) e_{n+i} is adjacent to e_{2n-m+i} for i = 1 to m.

These edges form m disjoint cycles of length 3 and the vertices e_1,e_2,….e_n are left isolated. Hence Δ(G) is a graph with m disjoint cycles of length 3 and n isolated vertices.

IV. RESULTS

In this paper I found Gallai Graphs of S(n,m). And we give the results of Gallai graphs properties and its relations to other graphs.

V. CONCLUSION

In this modern world Graph uses many places in real life. In this Gallai graph of S(N,M) graphs and its properties apply in Network theory.

REFERENCES

1. Sudha S, Manikandan K, “Total coloring of S(n. m) graph”, International Journal of Scientific and Innovative Mathematical Research, 2(1) (2014), 16-22.
2. R. Ganapathy Raman, S.Gayathri , "Upper And Lower Bounds On The Chromatic Number Of S (n,m) Graphs", International Journal Of Creative Research Thoughts (IJCRT), Issn:2320-2882, Volume.6, Issue 2, Page No pp.550-553, April 2018.
3. PrisnerE: “Graph Dynamics”, Longman, 1995.
4. S.AparnaLakshmanan;S.B.Rao;A.Vijayakumar,“Gallai and anti-Gallaigraphs of a graph”, MathematicaBohemica,Vol.132(2007),No.1,43—54.

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