An Analysis of Randomized Threshold Discrete Geo/G/1 Retrial Queue with Impatient Customers and Single Vacation

P Gunasekaran, S.Jeyakumar

Abstract: ***It is discussed in brief about randomized threshold discrete queue Geo/G/1 queue with impatient customers and single vacation in this article. Consider systems with server vacation where customers impatience are due to the absence of servers upon arrival the server begins its service, here the probability is p, this is possible if N (N- fixed count) number of customers are come together in the queue else the server will not give any service for the probability 1-p. Server will again start its service with probability p only if the length of the queue reaches N. In this model customers arrive according to geometric distribution and the service times, vacation times are generally distributed. In this article we derived PGF of the orbit by generating function technique and in the performance measure is derived an analytical expression for mean queue length. Numerical results are presented in order to illustrate the effectiveness of performance of the mean queue length.***

Keywords: Arrival, time, Geo/G/1 queue, Impatient customer, and Mean queue length, Service time, Threshold policy, Vacation time.

I. INTRODUCTION

The retrial queues are characterized by the actuality that any arriving users who discovers all servers active or down inactive or on vacation will join the retrial group/orbit and try again later. The existence of subsequent try is general characteristic of many queuing situations arising in daily life, computer networks, telecommunication etc. Initially, the work done in the retrial queues are focussed on continuous time setting.

Recently, the role of retrial queuing models with vacation has a great effect in queuing literature. This has been because of their huge application in many fields, particularly in the field of communication networks and manufacturing systems. In queuing literature customer impatience deals with mostly with regards to user forsaking the line due to either a long with priory service else a long wait anticipated upon arrival, and also customer restlessness generally happens when server is not available upon arrival. That type of model faces often situation by customer waiting in service system. Many researchers have analyzed the impatience phenomenon under various assumed points. Here, refer a few articles as follows. In earlier years [1953,1957] palm's [11] analysing queuing system with impatient clients by thinking about the boundless cradle M/M/c line and expecting that every individual clients remains in the line as long as his holding up time does not surpass an exponentially appropriated anxiety time. Altman and Yechiali [3] are discussed some queuing models such as M/M1, M/G/1 and M/M/c queue with server vacation and customer impatience where customers become impatient only when the servers where on vacation perel and Yechiali [12] have analysed M/M/c queue in a two phase (fast & slow) markovian random environment, with impatient customers. Yue et al [17] have discussed a MM/1 queuing system which is in expected maintenance time and customers without patient. Takacs [13] has extended his studies on M/G/1 queue, in this queue where customers will have a fixed time limit in the queue and can predict the limiting chances of the actual and of the virtual waiting times. Bacilli et al [4] considered G1/G/1 queue, here the customer can leave without joining in the queue since he knows the time waiting for the service is beyond the impatience threshold.

Recently, many researchers increase their attention in queuing system with N policy and this concept N policy is used to control the service in queuing system. This model is widely used for displaying purpose behind any production and manufacturing framework and also network communication and telecommunication framework. From this method it is infered that when N or more clients are available in the framework the server is enacted and deactivated when the server is empty. In earlier Yadin and Noor [16], ke and warg [8] were first introduce this policy. They focussed on a single removable server queuing system with finite capacity operating under N policy. Jain et al [7] deliberated untrustworthy MM/1/k queuing model under N and F policy with multi optional phase overhaul and start up. Wang et al [14] also has examined the concept strategic behaviour of M/M/1 constant retrial queue with N policy Moreno [9] discussed a discrete time single server queue model with general N policy and fixing close down times. Zhang and Tian [18] investigated a discrete time Geo/G/1 queue with multiple adaptive vacation.

In discrete Geo/G/1 retrial queue Jimbiac We and Wang [15] has discussed about impatient and preferred customers. This study extends by introducing two concepts namely single vacation and threshold policy and developing this new article, which is our unique contribution of this discrete queuing paper. This model has wide application in many real circumstances of our life and also in production and manufacturing system as well as network communication and telecommunication,

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which is motivation for developing this article. Here, it is investigated that one of the real application of this model as below.

This process is modelled as An Analysis of Randomized threshold discrete Geo/G/1 retrial queue with impatient customers and single vacation

II. MODEL DESCRIPTION

It is analysed that a single server discrete subsequent queue where the time axis is separated into intervals of identical length known as slots. All the queuing activities of this model could happens only at the borders of these slots. Particularly, let the time axis will be numbered as 0,1,2,,,,m….. Consider a epoch m and suppose that the exits occur in (m^-, m) and the appear and reappear occur in (m, m^+). So we consider early arrival policy or departure first policy in this queuing model.

In this model arrival time follows geometrical distribution with probability p. At the time of arrival epoch if the server is idle then, immediately the arriving customers commences his own service. Else, if the server is full of activity when arrival epoch the arriving customer interrupts the customer in service and begins his own service with probability qα (the clients are known as preferred clients) or enters into the orbit with probability $q\bar{\alpha}$ (the clients are known as repeated clients) or leaves the system completely with probability $\bar{q}$ (the client is known as impatient clients). The intermittent clients arrives into the orbit and the service of these clients start from the beginning. Since, it is proposed to examine only distribution of the numbers of clients in the system and does not touch the waiting and halt time distribution. Clients in the orbit needs to be followed FCFS discipline, because, only clients at the top of the orbit are permitted to access the server. Also, the server takes a vacation of random amount of time and the server starts its service after the vacation if at least one customer present in the system otherwise the server is idle. Here, we take systems with server unavailability where client impatience is because of the unavailability of servers upon arrival,. Also, in this model the server follows (P,N) policy, namely ; threshold policy, ie the server begins its service. Here the probability is p, it is possible if N (N- fixed count) number of customers are assembled in the queue else the server will not give any service for the probability 1-p. If the length of the queue ranges to N again server will start its service with probability p

The service times $\{s_i\}$ and vacation times $\{v_i\}$ are generally and identically distributed random variables with generating function $S(x)=\sum_{i=1}^{\infty} s_i x^i$ and $V(x)=\sum_{i=1}^{\infty} v_i x^i$ respectively. The retrial times $\{r_i\}$ are generally distributed with generating function $R(x)=\sum_{i=0}^{\infty} r_i x^i$. The inter arrival times, service times, vacation times and retrial times are expected to be mutually independent. Also, we denote $\bar{p}=1-p$ and we suppose that $0 \pi p \pi 1, 0 \pi q \le 1 and 0 \pi \alpha \le 1$

III. THE QUEUE LENGTH DISTRIBUTION

At time m^+ the system could be defined by the markov process $\{y_m; m \ge 1\}$ with $y_m = [C_m, \varepsilon_{0,m}, \varepsilon_{1,m}, \varepsilon_{2,m}, N_m]$ where, C^m denotes the state of the server 0, 1, and 2 according to the server which is on vacation, idle, and busy and N_m >0 represent the total number of recurrent customers in the orbit. If C^m =0, then $\varepsilon_{0,m}$ denotes the remaining idle time. If C^m =1, then $\varepsilon_{1,m}$ denotes the remaining retrial time and if C^m =2, then $\varepsilon_{2,m}$ denotes remaining service time of the client presently in service. It could be exposed that $\{y_m; m \ge 1\}$ is the markov process of our queuing system.

The limiting probabilities are defined as

$$\pi_{0,i,k} = \lim_{m\to\infty} \Pr\{C_m = 0, \xi_{0,m} = i, N_m = k\},$$
$$\pi_{1,i,k} = \lim_{m\to\infty} \Pr\{C_m = 1, \xi_{1,m} = i, N_m = k\}$$
$$\pi_{2,i,k} = \lim_{m\to\infty} \Pr\{C_m = 2, \xi_{2,m} = i, N_m = k\}$$

The Kolmogorov equations for the stationary distribution of the system under consideration is

$$\pi_{0,i,0} = \bar{p}\pi_{0,i+1,0} + \bar{p}\pi_{0,,0}v_i \quad , i \ge 1 \qquad (1)$$
$$\pi_{0,i,k} = \bar{p}\pi_{0,i+1,k} + \bar{p}\pi_{0,,1,k}v_i \; i \ge 1 \text{ and } 1 \le k \le N-1 \qquad (2)$$
$$\pi_{0,i,N} = \bar{p}\pi_{0,i+1,N} + \bar{p}\pi_{0,,1,N}v_i \quad , i \ge 1 \qquad (3)$$
$$\pi_{1,1,0} = \bar{p}\pi_{1,1,0} + p\pi_{0,,0} \qquad (4)$$
$$\pi_{1,i,0} = \bar{p}\pi_{1,i+1,0} + p\pi_{0,,0}v_i \quad , i \ge 1 \qquad (5)$$
$$\pi_{1,i,k} = \bar{p}\pi_{1,i+1,k} + \bar{p}r_i\pi_{1,1,k} + pr_i\pi_{0,1,k} \; i \ge 1 \text{ and } 1 \le k \le N-1 \qquad (6)$$
$$\pi_{1,i,N} = \bar{p}\pi_{1,i+1,N} + \bar{p}r_i\pi_{1,1,N} + pr_i\pi_{0,1,N} \quad , i \ge 1 \qquad (7)$$
$$\pi_{2,1,0} = \bar{p}\pi_{2,1,0} + p\pi_{0,,0} \qquad (8)$$
$$\pi_{2,i,1} = ps_i\pi_{0,1,1} + p\pi_{0,,0}s_i \; i \ge 1 \qquad (9)$$

$$\pi_{2,i,k} = ps_i\delta_{0,k}\pi_{0,0} + ps_i\pi_{0,1,k+1} + \bar{p}s_i\pi_{1,1,k+1} + (1-\delta_{0,k})_i\sum_{j=1}^{\alpha} ps_i\pi_{1,j,k} + \bar{p}r_0s_i\pi_{2,1,k+1} + (\bar{p}+p\bar{q})$$

$$\pi_{2,i+1,k} + (1-\delta_{0,k})pq\bar{\alpha}\pi_{2,i+1.k-1} + (1-\delta_{0,k})\sum_{j=2}^{\infty} pq\alpha s_i\pi_{2,j,k-1}$$

$$i \geq 1 \text{ and } \quad 1 \leq k \leq N-1 \quad (10)$$

$$\pi_{2,i,N} = ps_i\pi_{0,1,N+1} + \bar{p}s_i\pi_{1,1,N+1} + (1-\delta_{0,k})_i\sum_{j=1}^{\alpha} ps_i\pi_{1,j,N} + \bar{p}r_0s_i\pi_{2,1,N+1}$$

$$(\bar{p}+p\bar{q})\pi_{2,i=1,N} + (1-\delta_{0,k})pq\bar{\alpha}\pi_{2,i+1,N-1} + (1-\delta_{0,k})\sum_{j=2}^{\infty} pq\alpha s_i\pi_{2,j,N-1}, i \geq 1$$

(11)

$$\pi_{2,i,N+1} = ps_i\pi_{0,1,N+2} + \bar{p}s_i\pi_{1,1,N} + (1-\delta_{0,k})_i\sum_{j=1}^{\alpha} ps_i\pi_{1,j,N+1} + \bar{p}r_0s_i\pi_{2,1,N}$$

$$(\bar{p}+p\bar{q})\pi_{2,i=1,N+1} + (1-\delta_{0,k})pq\bar{\alpha}\pi_{2,i+1,N} + (1-\delta_{0,k})\sum_{j=2}^{\infty} pq\alpha s_i\pi_{2,j,N}, i \geq 1$$

(12)

$$\pi_{2,i,k} = ps_i\pi_{0,1,k+1} + \bar{p}s_i\pi_{1,1,k+1} + (1-\delta_{0,k})_i\sum_{j=1}^{\alpha} ps_i\pi_{1,j,k} + \bar{p}r_0s_i\pi_{2,1,k+1}$$

$$(\bar{p}+p\bar{q})\pi_{2,i=1,k} + (1-\delta_{0,k})pq\bar{\alpha}\pi_{2,i+1,k-1} + (1-\delta_{0,k})\sum_{j=2}^{\infty} pq\alpha s_i\pi_{2,j,k-1}, i \geq 1$$

(13)

where $\delta_{0,k}$ is the Kronecker's symbol, and the normalization condition is

$$\sum_{i=1}^{\alpha}\sum_{k=0}^{N}\pi_{0,i,k} + \sum_{i=1}^{\alpha}\sum_{k=0}^{N}\pi_{1,i,k} + \sum_{i=1}^{\alpha}\sum_{k=1}^{\alpha}\pi_{2,i,k} = 1$$

Introduce the following generating function to solve the above system of equations

$$\Phi_{0,i}(z) = \sum_{k=0}^{N}\pi_{0,i,k}z^k \quad \Phi_0(x,z) = \sum_{i=1}^{\infty}\Phi_{0,i}(z)x^i$$

$$\Phi_{1,i}(z) = \sum_{k=0}^{N}\pi_{1,i,k}z^k$$

$$\Phi_1(x,z) = \sum_{i=1}^{\infty}\Phi_{1,i}(z)x^i \quad \Phi_{2,i}(z) = \sum_{k=1}^{\infty}\pi_{2,i,k}z^k$$

$$\Phi_2(x,z) = \sum_{i=1}^{\infty}\Phi_{2,i}(z)x^i$$

Also, we introduce auxillary generating function

$$\Phi_0(z) = \sum_{k=0}^{N}\pi_{0,1,k}z^k \quad \Phi_1(z) = \sum_{k=0}^{N}\pi_{1,1,k}z^k$$

$$\Phi_2(z) = \sum_{k=0}^{N}\pi_{2,1,k}z^k$$

In equations (1), (2) and (3) multiply both sides by z^k and taking summation over k and multiply both sides by x^i and taking summation over i and after some algebraic simplification,

$$\phi_0(x,z)\left[\frac{x-\bar{p}}{x}\right] = \bar{p}\phi_0(z)[V(x)-1] + \bar{p}vV(x)\pi_{0,0}$$

(14)

Put x = $\bar{p}$ and solving for $\phi_0(z)$, obtained the result as follows

$$\phi_0(z) = \frac{V(\bar{p})}{1-V(\bar{p})}\pi_{00}$$

(15)

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Using (15) in (14) and after some simplification, obtained result as follows

$$\phi_0(x,z)=\frac{\bar{p}x\left[V(x)-V(\bar{p})\right]}{\left[1-V(\bar{p})\right]\left[x-\bar{p}\right]}\pi_{00} \quad (16)$$

In equations (4), (5), (6) and (7) multiply both sides by z^k and taking summation over k and multiply both sides by x^i and taking summation over i and after some algebraic simplification,

$$\phi_1(x,z)\left[\frac{x-\bar{p}}{x}\right]=pR(x)\pi_{00}-\bar{p}\phi_1(z)\left[R(x)-1\right]+p\phi_0(z)R(x) \quad (17)$$

Put $x=\bar{p}$ and using (15) in (17) and solving for $\phi_1(z)$. obtained result as follows

$$\phi_1(z)=\frac{px\left[R(x)-R(\bar{p})\right]}{\left[1-V(\bar{p})\right]\left[1-R(\bar{p})\right]\left[x-\bar{p}\right]}\pi_{00} \quad (18)$$

In equations (8), (9), (10) (11) and (12) multiply both sides by z^k and taking summation over k and multiply both sides by x^i and taking summation over i and after some algebraic simplification, as follows

$$\phi_2(x,z)\left[\frac{x-\beta(z)}{x}\right]=pS(x)\left[\frac{z-1}{z}\right]\pi_{00}+\frac{p}{z}S(x)\phi_0(z)+pS(x)\phi_1(z)+pS(x)\left[r_0-1\right]+pS(x)\phi_1(1,z)$$

$$+pq\alpha z\phi_2(1,z)+\phi_2(z)\left[\frac{\bar{p}r_0+pz-pq\alpha z^2}{z}S(x)-\beta(z)\right] \quad (19)$$

where $\beta(z)=\bar{p}+P\bar{q}+pq\bar{\alpha}z$

In (19) put x = 1 and solving for $\phi_2(1,z)$, obtained result as follows

$$\phi_2(1,z)=\frac{1}{pq(1-z)}\left[p\left[\frac{zr_0-1}{z}\right]\pi_{00}+\frac{p}{z}\phi_0(z)+p\phi_1(z)+\left[\frac{\bar{p}r_0}{z}+pq(1-z)-\bar{p}\right]\phi_2(z)+p\phi_1(1,z)\right] \quad (20)$$

Put x = 1 in (18), obtained result as follows

$$\phi_1(1,z)=\frac{\pi_{00}}{\left[1-V(\bar{p})\right]} \quad (21)$$

put x = $\beta(z)$ in (19) and solving for $\phi_2(z)$, obtained result as follows

$$\phi_2(z)=\frac{zS[\beta(z)]}{\left[z\beta(z)-\left(\bar{p}r_0+pz-pq\alpha z^2\right)S[\beta(z)]\right]}\left[\begin{array}{l}p\pi_{00}\left[\frac{z-1}{z}\right]+\frac{p}{z}\phi_0(z)+p\phi_1(z)+p(r_0-1)\pi_{00}\\+p\phi_1(1,z)+pq\alpha z\phi_2(1,z)\end{array}\right] \quad (22)$$

Using (15), (18), (20), (21) and (22) in (19),

$$\phi_2(x,z)=\frac{px\left[S(x)-\beta_1(x,z)S[\beta(z)]\right](1-z+\alpha z)\pi_{00}}{(x-\beta(z))(1-z)}\left[\begin{array}{l}\frac{z-1}{z}+\frac{V(\bar{p})}{z\left(1-V(\bar{p})\right)}+\frac{pR(\bar{p})}{\left(1-V(\bar{p})\right)\left[R(\bar{p})-1\right]\bar{p}}\\+\left[r_0-1\right]+\frac{1}{\left[1-V(\bar{p})\right]}\end{array}\right] \quad (23)$$

where

$$\beta_1(x,z)=\frac{S(x)\left[\bar{p}r_0+pz\right](1-z+\alpha z)-z\left[\alpha zS(x)+\beta(z)(1-z)\right]}{(1-z)\left\{\left[\bar{p}r_0+pz-pq\alpha z^2\right]S[\beta(z)]-z\beta(z)\right\}}$$

The PGF of this model under consideration is given below

$$\phi(z)=\phi_0(1,z)+\phi_1(1,z)+\phi_2(1,z)$$

$$\phi(z)=\frac{\left[1-\bar{p}V(\bar{p})\right]\pi_{00}}{p\left[1-V(\bar{p})\right]}+\frac{p\left[1-\beta_1(z)S[\beta(z)]\right](1-z+\alpha z)\pi_{00}}{(1-\beta(z))(1-z)}\left[\begin{array}{l}\frac{z-1}{z}+\frac{V(\bar{p})}{z(1-V(\bar{p}))}+(r_0-1)+\frac{1}{(1-V(\bar{p}))}\\+\frac{pR(\bar{p})}{(1-V(\bar{p}))(R(\bar{p})-1)\bar{p}}\end{array}\right]$$

(24) Where

$$\beta_1(z)=\left[\beta_1(x,z)\right]_{x=1}=\frac{(\bar{p}r_0+pz)(1-z+\alpha z)-z[\alpha z+\beta(z)(1-z)]}{(1-z)\left[(\bar{p}r_0+pz-pq\alpha z^2)S[\beta(z)]-z\beta(z)\right]}$$

The steady state condition for the model under consideration is given below

3.1 Steady state condition

$$\pi_{00}=\frac{q\alpha\left[1-V(\bar{p})\right]}{\begin{array}{l}q\alpha\left[1-\bar{p}V(\bar{p})\right]-\left[\bar{p}\left[1-V(\bar{p})\right](r_0-1)+\left[R(\bar{p})-\bar{p}\right]\right]\left[\begin{array}{l}[1-\beta_2(1)]S(1-pq\alpha)(pq\alpha\bar{\alpha}+\alpha-1)\\-\beta_2(1)S(1-pq\alpha)\alpha\end{array}\right]\\-[1-\beta_2(1)]S(1-pq\alpha)\alpha\left[\bar{p}\left[1-V(\bar{p})\right]r_0+\left[R(\bar{p})-\bar{p}\right]\right]\end{array}}$$

Where

$$\beta_2(1)=\frac{\alpha p+(\bar{p}r_0+p)(\alpha-1)-\left[\alpha(\alpha-1)+\bar{p}+p\bar{q}+pq\bar{\alpha}-1\right]}{(1-pq\alpha)-\left[\bar{p}r_0+p-pq\alpha\right]S(1-pq\alpha)}$$

Which is obtained from PGF by substituting z =1 and to equating to one and it is clearly less than one

predictable queue length is attained below. The mean of the queue length is attained by differentiating the PGF with respect to z and then put z=1,

E(Q)=

IV.EXPECTED QUEUE LENGTH

Performance degree of the queue length distribution that is

$$\pi_{00}\left\{\begin{array}{l}\frac{V(\bar{p})\left[\begin{array}{l}S''(1-pq\alpha)(pq\bar{\alpha})^2\alpha^2(\bar{p}r_0+\bar{p})\\+S'(1-pq\alpha)pq\alpha\bar{\alpha}2(\alpha-1)(1+\bar{p}r_0+p)\\+S'(1-pq\alpha)pq\bar{\alpha}^2\alpha z(p+1)\\+S(1=pq\alpha)2(\alpha-1)(\alpha-1+\alpha p)\end{array}\right]}{2q\alpha\left[1-V(\bar{p})\right]\left[(1-pq\alpha)-(\bar{p}r_0+p-pq\alpha)S(1-pq\alpha)\right]}\\-\frac{2V[\bar{p}]\left[\begin{array}{l}S'(1-pq\alpha)pq\bar{\alpha}\alpha^2(1+\bar{p}r_0+p)+\\+3S(1-pq\alpha)\alpha(\alpha-1)+S(1-pq\alpha)2(1+p)\\+S(1-pq\alpha)2(\alpha-1)(\alpha-1+\alpha p)\end{array}\right]}{2q\alpha\left[1-V(\bar{p})\right]\left[(1-pq\alpha)-(\bar{p}r_0+p-pq\alpha)S(1-pq\alpha)\right]}+\frac{(V(\bar{p})-1)^2+V(\bar{p})(\alpha-1)}{(V(\bar{p})-1)q\alpha}\end{array}\right\}$$

When the vacation is zero and there is no threshold policy the PGF is reduced into

4.1 Particular case

$$\phi(z)=\frac{R(\bar{p})\left\{S[\beta(z)]\left(\bar{p}+pz+\bar{p}\alpha z\right)q+\beta(z)[1-qz-S(\beta(z))]\right\}}{\left[z=\bar{p}R(\bar{p})(1-\bar{\alpha}z)\right]S[\beta(z)]-z\beta(z)}\frac{\pi_{00}}{q}$$

preferred and impatient clients by J, Wu and J.wang [15]

Which is the PGF of a discrete Geo/G/1 retrial queue with

IV. NUMERICAL EXAMPLES

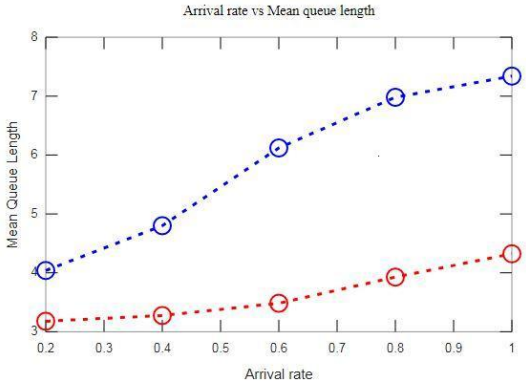
The arithmetical examples are momentarily discussed in two type of cases. In case (i), compared the influence on two mean queue lengths, when arrival rate increases and in case (ii) compared the influence on other two mean queue lengths, when service rate increases.

Case (I): In this case comparison made influence on two mean

queue lengths, when arrival rate increases. In the first mean queue length $E(Q^1)$ arrival times trail geometrical distribution and vacation times, service times and impatience times are generally distributed and in the second mean queue length $E(Q^2)$ arrival times and follows poisson distribution and Service times, vacation times and impatience times are commonly dispersed when the arrival rate increases the effect on these two mean queue lengths are shown in Table and figure below

Table 1.1 Arrival rate vs. mean queue length.

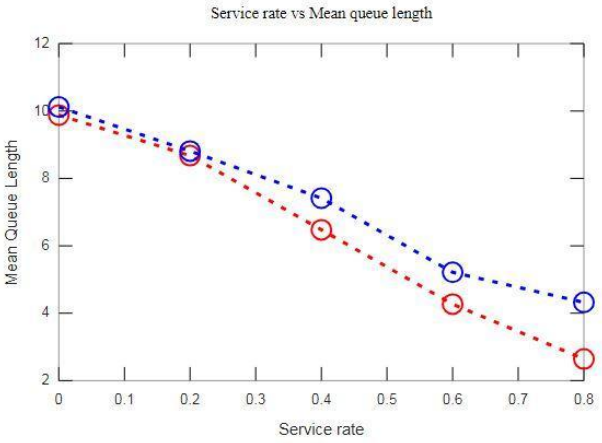
Arrival rate	π_{00}	$E(Q^1)$	π_{00}	$E(Q^2)$
0.2	0.427	3.198	0.432	4.04
0.4	0.514	3.276	0.501	4.80
0.6	0.546	3.484	0.568	6.12
0.8	0.578	3.978	0.624	6.98
1.0	0.612	4.187	0.712	7.224



In the above chart and table it is found that mean queue length will increase if arrival rate increases thereby comparing these two mean queue lengths It is concluded that the first mean queue length is shorter than second mean queue mean length.

Case (iI): . In this case we compared influence on two mean queue lengths, when service rate increases. In the first mean queue length $E(Q^1)$ arrival times trail geometrical dispersal and vacation times, service times and impatience times are commonly distributed and in the second mean queue length $E(Q^2)$ arrival times and follows poison distribution and Service times, vacation times and impatience times are generally distributed When the service rate increases the influence on mean queue length is shown in Table and Figure below

Service rate	π_{00}	$E(Q^1)$	π_{00}	$E(Q^2)$
0.0	0.901	9.878	0.987	10.120
0.2	0.719	8.674	0.874	8.812
0.4	0.624	6.475	0.681	7.412
0.6	0.537	4.271	0.542	5.214
0.8	0.457	2.641	0.388	4.321



From the above graph and table we observe that mean queue length decrease as service rate increases thereby comparing these two mean queue lengths it is concluded that the first mean queue length is shorter than second mean queue mean length.

VI. CONCLUSION

In this model it is discussed briefly a randomized threshold discrete Geo/G/1 retrial queue with single vacation. By generating function technique to obtain probability generating function of the system. In performance measure it is derived an analytic expression for mean queue length. In arithmetical examples it is deliberated and observed the result of mean queue length in several ways. The outcomes from these numerical examples are useful to decide the threshold value N of situation that was discussed in real example of this article

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