A Markovian Working Vacation Queue with Server State Dependent Arrival Rateand with Partial Breakdown

R. Kalyanaraman, A. Sundaramoorthy

Abstract:A single server Markovianqueueing system with the system alternates between regular busy state, repair state and working vacation state has been considered. If the system is busy, it functions as a single server Markovian queue. When it is on repair/vacation, again it functions as a single server Markovian queue but with different arrival and service rates. The vacation policy is multiple vacation policy and the vacation period follows negative exponential. In addition, during service the server may break down, the repair of the server starts immediately. The breakdown period and repair period follows two different negative exponential distributions. During the repair period the server serves the customer with service period follows negative exponential of different service rate. Some illustrative examples are also provided.

Keywords: Working vacation, State dependent arrival rate, Matrix-Geometric method.
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I. INTRODUCTION

Some features which are common in the manufacturing system, the production system, thetelecommunication network and the computer system are queueing and breakdown. In practice these systems with these two features can be realized through a queueing system with breakdown. Many researchers including Federgruen and Green(1986), Li et al.(1997), Tang(1997), Nakdimon and Yechiali(2003), Wang et al.(2007), Wang et al.(2008), Choudhry and Tadj(2009) have studied these models.

In day to day life, it can be seen that the server works during his vacation period, if the necessity occur, called working vacation queue. In the working vacation queues, the server works with variable service rate, in particular reduced service rate, rather than completely stops service during vacation period. Servi and Finn(2002) have first analyzed an$M/M/1$ queue with multiple working vacation, in which the vacation times are exponentially distributed. Wu and Takagi(2006) extend the work to an $M/G/1$ queue. Kim et al.(2003) analyzed the queue length distribution of the $M/G/1$ queue with working vacations. Liu et al.(2007), Examined stochastic decomposition structure of the queue length and waiting time in an $M/M/1$ working vacation queue. Xu et al.(2009) extended the $M/M/1$ working vacation queue to an $M^X/M/1$ working vacation queue. Li et al.(2009) used the matrix analytic method to analyze an $M/G/1$ queue with exponential working vacation under a specific assumption. Lin and Ke(2009) consider a multi server queue with single working vacation. Jain and Jain(2010) investigated a single working vacation model with server break down. Ke et al.(2010) have given a short survey on vacation models in recent years.

In many real life situations, it can be observed that the arrival of customers as well as service of a customer depends on current state of the system, that is, number of customers in the system, etc., Yechiali and Naor(1971) have considered a single-server exponential queueing model with arrival state depending on operational state or breakdown state of the server. Fond and Ross(1977) analyzed the same model with the assumption that any arrival finding the server busy is lost, and they obtained the steady-state proportion of customer's lost. Shogan(1979) has deals a single server queueing model with arrival rate dependenting on server state. Shanthikumar(1982) has analyzed a single server Poisson queue with arrival rate depending on the state of the server. Jayaraman et al.(1994) analyzed a general bulk service queue with arrival rate dependent on server breakdowns. Tian and Yue(2002) discussed the queueing system with variable arrival rate. The authores studied the model by using the principle of quasi-birth and death process(QBD) and matrix-geometric method. Furthermore, they calculated some performance measures, such as the number of customers in the system in steady-state, etc., Matrix-geometric method approach is a useful tool for solving more complex queueing problems. Matrix-geometric method has been applied by many researchers to solve various queueing problems in different frameworks. Neuts(1981) explained various matrix geometric solutions of stochastic models. Matrix-geometric approach is utilized to develop the computable explicit formula for the probability distributions of the queue length and other system characteristic.

As a new direction, in this paper we assume that during breakdown period, the server works but with slower rate. Also the arrival rates are state dependent. Also the server take vacation whenever the system empty and continuous the vacation until at least one customer waits for service. During the vacation period, the server serves the customers, if any one arrives, with slower service rate.

In this paper, we consider an $M/M/1$ queuewith multiple working vacation and with partial breakdown. The arrival rate dependents on the server

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states. The model has been analyzed using matrix geometric method. The rest of this paper is organized as follows: In section 2, we give the model description, establish its quasi-birth-death process. In section 3, we present the steady state solution using matrix geometric method. In section 4, we present some system performance measures. In section 5, wegives some particular models. and In section 6, we carried out a numerical study.

II. THE MODEL

We consider a single-server queueing system with the following characteristics:

1. The system alternate between two states, up state and down state. In the up state it is either in regular state or in working vacation state. In the down state it is in the repair state.
2. Arrival process follows Poisson.
3. When the system is in regular busy period it serves customers based on exponential distribution with rate μ.
4. During the regular busy period the arrival parameter is λ.
5. The server takes vacation, if there are no customer in the queue at a service completion point.
6. During vacation, the arrival rate is$\lambda_1 (\lambda_1 < \lambda)$.
7. Vacation period follows negative exponential distribution with rate θ and the vacation policy is multiple vacation policy.
8. When the server is in vacation, if customer arrives, the server serves the customer using exponential distribution with rate$\mu_1 (\mu_1 < \mu)$. The server may break down during a service and the break downs are assumed to occur according to a Poisson process with rate α.
9. Once the system break downs, the customer whose service is interrupted goes to the head of the queue and the repair to server starts immediately.
10. Duration of repaired period follows negative exponential with rate β.
11. During repair period customers arrive according to Poisson process with rate λ_2 $(\lambda_2 < \lambda_1)$.
12. During repair period the server serves the customers, and the service period follows negative exponential with rateμ_2 $(\mu_2 < \mu_1 < \mu)$.
13. The first come first served (FCFS) service rule is followed to select the customer for service.

A. *The Quasi-Birth-And-Death (QBD) Process*

The model defined in this article can be studied as a QBD process. The following notations are necessary for the analysis:

Let $L(t)$ be the number of customers in the queue at time t and let

$$J(t) = \begin{cases} 0, if the server is on working vacation \\ 1, if the server is busy \\ 2, if the server is on partial breakdown \end{cases}$$

be the server state at time t.

Let$X(t) = (L(t), J(t))$, then $\{(X(t)): t \geq 0\}$ is a Continuous time Markov chain (CTMC) with state space $S = \{(i,j): i \geq 0; i = 0,1,2\}$, where i denotes the number of customer in the queue and j denotes the server state.

Using lexicographical sequence for the states, the rate matrix$Q$, is the infinitesimal generator of the Markov chain and is given by

$$Q = \begin{bmatrix} B_0 A_0 & & & & \\ & A_2 A_1 A_0 & & & \\ & & A_2 A_1 A_0 & & \\ & & & A_2 A_1 A_0 & \\ & & & & \ddots \end{bmatrix}$$

where the sub-matrices A_0, A_1 and A_2 are of order 3×3 and are appearing as

$$A_0 = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -(\lambda_1 + \mu_1 + \theta) & \theta & 0 \\ 0 & -(\lambda + \mu + \alpha) & \alpha \\ 0 & \beta & -(\lambda_2 + \mu_2 + \beta) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu_2 \end{bmatrix}$$

and the boundary matrix is defined by

$$B_0 = \begin{bmatrix} -(\lambda_1 + \theta) & \theta & 0 \\ \mu & -(\lambda + \mu + \alpha) & \alpha \\ 0 & \beta & -(\lambda_2 + \beta) \end{bmatrix}$$

We define the matrix$A$ as$A = A_0 + A_1 + A_2$. This matrix A is a 3×3 matrix and it is of the form

$$A = \begin{bmatrix} -\theta & \theta & 0 \\ 0 & -\alpha & \alpha \\ 0 & \beta & -\beta \end{bmatrix}$$

III.THE STEADY STATE SOLUTION

Let $P = (p_0, p_1, p_2, \dots)$ be the stationary probability vector associated with$Q$, such that $PQ = 0$ and $Pe = 1$, where e is a column vector of 1's of appropriate dimension.

Let $p_0 = (p_{00}, p_{01}, p_{02})$ and $p_i = (p_{i0}, p_{i1}, p_{i2})$ for$i \geq 1$.

If the steady state condition is satisfied, then the sub vectors p_i are given by the following equations:

$$p_0 B_0 + p_1 A_2 = 0 \quad (1)$$

$$p_i A_0 + p_{i+1} A_1 + p_{i+2} A_2 = 0, i \geq 0 \quad (2)$$

$$p_i = p_0 R^i; i \geq 1 \quad (3)$$

where R is the rate matrix, is the minimal non-negative solution of the matrix quadratic equation (see Neuts(1981)).

$$R^2 A_2 + R A_1 + A_0 = 0, \quad (4)$$

Substituting the equation (3) in (1), we get

$$p_0 (B_0 + R A_2) = 0 \quad (5)$$

and the normalizing condition is

$$p_0 (I - R)^{-1} e = 1 \quad (6)$$

Theorem: 3.1

The queueing system described in section 2 is stable if and only if$\rho < 1$, where

$$\rho = \frac{\lambda_2 \alpha + \lambda \beta}{\mu \beta + \mu_2 \alpha}$$

Theorem: 3.2

If $\rho < 1$, the matrix equation (4) has the minimal non-negative solution

$$R = -A_0 A_1^{-1} - R^2 A_2 A_1^{-1}$$

Proof

Since the matrix A is reducible. The analysis present in Neuts(1978) is not applicable. In

Lucantoni(1979), similar reducible matrix is treated for the case when the elements are probabilities.
Equation (4) can be written as

$$A_0A_1^{-1} + RA_1A_1^{-1} + R^2A_2A_1^{-1} + 0.A_1^{-1}$$

Since $A_1$is non-singular, A_1^{-1} exists and

$$R = -A_0A_1^{-1} - R^2A_2A_1^{-1} \qquad (7)$$

where,

$$A_1^{-1} = \begin{bmatrix} \dfrac{-1}{(\lambda_1+\mu_1+\theta)} & \dfrac{S_0(\lambda_2+\mu_2+\beta)\theta}{(\lambda_1+\mu_1+\theta)} & \dfrac{S_0\alpha\theta}{(\lambda_1+\mu_1+\theta)} \\ 0 & S_0(\lambda_2+\mu_2+\beta) & S_0\alpha \\ 0 & S_0\beta & S_0(\lambda+\mu+\alpha) \end{bmatrix}$$

$$S_0 = \frac{1}{[\alpha\beta - (\lambda+\mu+\alpha)(\lambda_1+\mu_1+\beta)]}$$

Using Nuets and Lucantoni(1979), the matrix R is numerically computed by using the recurrence relation with $R(0) = 0$ in equation (7).

Theorem: 3.3

If $\rho < 1$, the stationary probability vectors $p_0 = (p_{00}, p_{01}p_{02})$ and$p_i = (p_{i0}, p_{i1}p_{i2}); i \geq 1$ are

$$p_{00} = \frac{1}{S_1 + S_2[(\lambda_1+\theta) - \mu_1 r_0] - S_3[(\mu r_{01}+\theta) + \frac{1}{\mu}[(\lambda_1+\theta) - \mu_1 r_0][\mu r_1 - (\lambda+\mu+\alpha)]]}$$

$$p_{01} = \frac{1}{\mu}[(\lambda_1+\theta) - \mu_1 r_0]p_{00}$$

$$p_{02} = \frac{-1}{(\beta+\mu r_{21})}[(\mu r_{01}+\theta) + \frac{1}{\mu}[(\lambda_1+\theta) - \mu_1 r_0][\mu r_1 - (\lambda+\mu+\alpha)]]p_{00}$$

and$p_i = p_0R^i; i \geq 1$
where,

$$S_1 = \frac{1}{1-r_0} + \frac{r_{21}r_{02} + r_{01}(1-r_2) + r_{01}r_{12} + r_{02}(1-r_1)}{(1-r_0)[(1-r_1)(1-r_2) - r_{21}r_{12}])}$$

$$S_2 = \frac{1 - r_2 + r_{12}}{\mu[(1-r_1)(1-r_2) - r_{21}r_{12}]}$$

$$S_3 = \frac{1 - r_1 + r_{21}}{(\beta+\mu r_{21})[(1-r_1)(1-r_2) - r_{21}r_{12}]}$$

Proof

p_{00}, p_{01}andp_{02}follows from the equations (5) and (6).

Remark: 3.1

Even though R in Theorem 3.2 has a nice structure which enables us to make use of the properties like $R^n = \begin{bmatrix} r_0^n & r_{01}\sum_{j=0}^{n-1} r_0^j r_1^{n-j-1} \\ 0 & r_1^n \end{bmatrix}$, for $n \geq 1$, due to the form of r_0&r_{01}, it may not be easy to cary out the computation required to calculate the p_i and the performance measures.

$$R(0) = 0 \qquad (8)$$

$R(n+1) = -A_0A_1^{-1} - [R(n)]^2A_2A_1^{-1}$for n≥ 0 (9)
and is the limit of the monotonically increasing sequence of matrices $\{R(n), n \geq 0\}$.

IV. PERFORMANCE MEASURES

(i) Mean queue length $E(L) = p_0R(I-R)^{-2}e$
(ii) $E(L^2) = p_0R(I+R)(I-R)^{-3}e$
(iii) Variance of queue length$L = var(L)$
$= [p_0R(I+R)(I-R)^{-3}e] - [p_0R(I-R)^{-2}e]^2$
(iv) Probability that the server is ideal $=p_0e$
(v) Mean queue length when the server is an vacation period $=\sum_{i=0}^{\infty} ip_{i0}$
(vi) Mean queue length when the server is in regular busy period $= \sum_{i=0}^{\infty} ip_{i1}$
(vii) Probability that the server is in working vacation period = pr{J=0}= $\sum_{i=1}^{\infty} p_{i0}$
(viii) Probability that the server is in regular busy period= pr{J=1}= $\sum_{i=1}^{\infty} p_{i1}$

V. PARTICULAR MODEL

In the above model, we assume that$\lambda_1 = \lambda_2 = \lambda$, and$\mu_1 = \mu_2 = \mu$, then we get

$$R = -A_0A_1^{-1} - R^2A_2A_1^{-1}$$

$$p_{00} = \frac{1}{S_1 + S_2[(\lambda+\theta) - \mu r_0] - S_3[(\mu r_{01}+\theta) + \frac{1}{\mu}[(\lambda+\theta) - \mu r_0][\mu r_1 - (\lambda}$$

$$p_{01} = \frac{1}{\mu}[(\lambda+\theta) - \mu r_0]p_{00}$$

$$p_{02} = \frac{-1}{(\beta+\mu r_{21})}[(\mu r_{01}+\theta) + \frac{1}{\mu}[(\lambda+\theta) - \mu r_0][\mu r_1 - (\lambda+\mu+\alpha)]]p_{00}$$

and$p_i = p_0R^i; i \geq 1$
where

$$A_0 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

$$A_1^{-1} = \begin{bmatrix} \dfrac{-1}{(\lambda+\mu+\theta)} & \dfrac{S_4(\lambda+\mu+\beta)\theta}{(\lambda+\mu+\theta)} & \dfrac{S_4\alpha\theta}{(\lambda+\mu+\theta)} \\ 0 & S_4(\lambda+\mu+\beta) & S_4\alpha \\ 0 & S_4\beta & S_4(\lambda+\mu+\alpha) \end{bmatrix}$$

$$S_4 = \frac{1}{[\alpha\beta - (\lambda+\mu+\alpha)(\lambda+\mu+\beta)]}$$

VI. NUMERICAL STUDY

In this section, some examples are given to show the effect of the parameters $\lambda, \lambda_1, \lambda_2, \mu, \mu_1, \mu_2, \theta, \alpha$ and β on the performance measures mean queue length, $E(L^2)$, variance of queue length$L$, probability that the server is idle, mean queue length when the server is an vacation period, mean queue length when the server is in regular busy period, probability that the server is in working vacation period and probability that the server is in regular busy period for the model analyzed in this paper. The corresponding results are presented as case(1), case(2) and case(3).

Case(i): If $\lambda = 0.7, \lambda_1 = 0.5, \lambda_2 = 0.3, \mu = 4, \mu_1 = 2, \mu_2 = 1, \theta = 0.6, \alpha = 0.3$ and$\beta = 0.5$, the matrix R is obtained using the equations (8)& (9)

$$R = \begin{bmatrix} 0.182864 & 0.031707 & 0.007446 \\ 0 & 0.166345 & 0.034620 \\ 0 & 0.026950 & 0.192199 \end{bmatrix}$$

and the invariant probability vector is $P = (p_0, p_1, p_2, \ldots)$, where

$$p_0 = (0.530459, 0.134089, 0.082768)$$

and the remaining vectors p_i 's are evaluated using the relation$p_i = p_0R^i; i \geq 1$

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$p_1 = (0.097001850605, 0.041354890913, 0.024499885738)$
$p_2 = (0.017738146707, 0.010615088977, 0.006862835493)$
$p_3 = (0.003243668471, 0.002513143932, 0.001818602788)$
$p_4 = (0.000593150151, 0.000569907250, 0.000460691052)$
$p_5 = (0.000108465807, 0.000126023864, 0.000112691145)$
$p_6 = (0.000019834491, 0.000027439592, 0.000026829708)$
$p_7 = (0.000003627014, 0.000005916392, 0.000006254289)$
$p_8 = (0.000000663250, 0.000001267717, 0.000001433900)$
$p_9 = (0.000000121285, 0.000000270552, 0.000000324421)$
$p_{10} = (0.000000022179, 0.000000057594, 0.000000072623)$

For the chosen parameters $p_{10} \to 0$, and the sum of the steady state probabilities is found to be 0.955029

The performance measures are

(i) Mean queue length $E(\mathrm{L}) = 0.24836$
(ii) $E(L^2) = 0.410295$
(iii) Variance of queue length $L = var(L) = 0.348612$
(iv) Probability that the server is ideal =0.747316
(v) Mean queue length when the server is an vacation period =0.010906
(vi) Mean queue length when the server is regular busy period =0.037819
(vii) Probability that the server is in working vacation period $= pr\{J = 0\}$ =0.010221
(viii) Probability that the server is in regular busy period= $pr\{J = 1\}$ =0.031502

Case(ii): If $\lambda = 0.6, \lambda_1 = 0.4, \lambda_2 = 0.2, \mu = 5, \mu_1 = 3, \mu_2 = 1, \theta = 0.7, \alpha = 0.2$ and $\beta = 0.6$, the matrix R is obtained using the equations (8)& (9)

$$R = \begin{bmatrix} 0.105742 & 0.016102 & 0.002201 \\ 0 & 0.116943 & 0.014977 \\ 0 & 0.013937 & 0.121081 \end{bmatrix}$$

and the invariant probability vector is $P = (p_0, p_1, p_2, \dots)$, where

$$p_0 = (0.677333, 0.121695, 0.021559)$$

and the remaining vectors p_i's are evaluated using the relation $p_i = p_0 R^i; i \geq 1$

$p_1 = (0.082196749747, 0.027048461139, 0.006143921055)$
$p_2 = (0.008691648953, 0.004572287668, 0.001329931896)$
$p_3 = (0.000919072365, 0.000693185197, 0.000248638971)$
$p_4 = (0.000097184551, 0.000099327342, 0.000042510168)$
$p_5 = (0.000010276489, 0.000013772967, 0.000006848702)$
$p_6 = (0.000001086657, 0.000001871574, 0.000001058144)$
$p_7 = (0.000000114905, 0.000000251112, 0.000000158543)$
$p_8 = (0.000000012150, 0.000000033426, 0.000000023210)$
$p_9 = (0.000000001285, 0.000000004428, 0.000000003338)$
$p_{10} = (0.000000000136, 0.000000000585, 0.000000000473)$

For the chosen parameters $p_{10} \to 0$, and the sum of the steady state probabilities is found to be 0.95269546

The performance measures are

(i) Mean queue length $E(\mathrm{L}) = 0.135078$
(ii) $E(L^2) = 0.174399$
(iii) Variance of queue length $L = var(L) = 0.156153$
(iv) Probability that the server is ideal =0.820587
(v) Mean queue length when the server is an vacation period =0.011128
(vi) Mean queue length when the server is regular busy period =0.080998
(vii) Probability that the server is in working vacation period $= pr\{J = 0\}$ =0.010102
(viii) Probability that the server is in regular busy period= $pr\{J = 1\}$ =0.054003

Case(iii): If $\lambda_1 = \lambda_2 = \lambda = 0.6, \mu_1 = \mu_2 = \mu = 4, \theta = 0.7, \alpha = 0.2$ and $\beta = 0.6$, the matrix R is obtained using the equations (8)& (9)

$$R = \begin{bmatrix} 0.125 & 0.023709 & 0.001291 \\ 0 & 0.143042 & 0.006959 \\ 0 & 0.020876 & 0.129125 \end{bmatrix}$$

and the invariant probability vector is $P = (p_0, p_1, p_2, \dots)$, where

$$p_0 = (0.625033, 0.151007, 0.056039)$$

and the remaining vectors p_i's are evaluated using the relation $p_i = p_0 R^i; i \geq 1$

$p_1 = (0.078129127622, 0.037589121610, 0.009093811736)$
$p_2 = (0.009766140953, 0.007419028785, 0.001536685857)$
$p_3 = (0.001220767619, 0.001324858051, 0.000262661662)$
$p_4 = (0.000152595952, 0.000223936848, 0.000044711884)$
$p_5 = (0.000019074494, 0.000036583675, 0.000007528800)$
$p_6 = (0.000002384312, 0.000005842410, 0.000001251367)$
$p_7 = (0.000000298039, 0.000000918363, 0.000000205318)$
$p_8 = (0.000000037255, 0.000000142717, 0.000000033287)$
$p_9 = (0.000000004657, 0.000000021993, 0.000000005339)$
$p_{10} = (0.000000000582, 0.000000003368, 0.000000000849)$

For the chosen parameters $p_{10} \to 0$, and the sum of the steady state probabilities is found to be 0.988916

The performance measures are

(i) Mean queue length $E(\mathrm{L}) = 0.172750$
(ii) $E(L^2) = 0.233721$
(iii) Variance of queue length $L = var(L) = 0.203878$
(iv) Probability that the server is ideal =0.832079
(v) Mean queue length when the server is an vacation period =0.007599
(vi) Mean queue length when the server is regular busy period =0.219899

(vii) Probability that the server is in working vacation period
$= pr\{J = 0\}$ =0.007243
(viii) Probability that the server is in regular busy period= $pr\{J = 1\}$ =0.141743

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REFERENCES

1. Choudhry, G. and Tadj, L.,An $M/G/1$ queue with two phases of service subject to the server breakdown and delayed repair.*Applied mathematical modeling,* 33, 2699-2709, 2009.
2. Doshi, B.T., Queueing systems with vacations-a survey. *Queueing systems,* 1, 29-66, 1986.
3. Federgruen, A., and Green, L.,Queueing systems with service interruptions. *Operations Research,*34, 752-768, 1986.
4. Fond, S. and Ross, S., A heterogeneous arrival and service queueing loss model. *Tech-Report ORC 77-12, Operations Research Center, University of California, Berkeley, CA,* 1977.
5. Jain, M. and Jain, A., Working vacation queueing model multiple type of server breakdown. *Appl. math. modelling,* 34(1), 1-13, 2010.
6. Jayaraman, D., R. Nadarajan and M.R. Sitrarasu, A general bulk service queue with arrival rate dependent on server breakdowns, *Appl. math. modelling,* 18, 156-160, 1994.
7. Ke, J.C, Wu, C.H, and Zhang, Z.G., Recent development in vacation queueing models: A short surrey, *Int. Jr. of Oper. Res,* 7(4), 3-8, 2010.
8. Kim, J.D., Choi, D.W., and Chae, K.C., Analysis of queue-length distribution of the $M/G/1$with working vacation$(M/G/1/WV)$. *In Proceeding of the International Conference on Statistics and related, Honolulu, Hawaii, USA,* 2003.
9. Latouche, G and Neuts, M.F., Efficient algorithmic solutions to exponential tandem queues with blocking, *SIAM J. Algebraic Discrete Math.,* 1, 93-106, 1980.
10. Li, W, Shi, D., and Chao, X., Reliability analysis of $M/G/1$ queueing system with server breakdown and vacations, *Journal of applied probability,* 34, 546-555, 1997.
11. Li, J.H, Tian, N., Zhang, Z.G., and Luh, H.P., Analysis of the $M/G/1$ queue with exponenetially working vacation- a matrix analytic approach, *Queueing systems,* 61, 139-166, 2009.
12. Lin, Ch.H., and Ke, J.Ch., Multi-server system with single working vacation, *Applied Mathematical Modelling,* 33, 2967-2977, 2009.
13. Liu, W., Xu, X. and Tian, N., Stochastic decompositions in the $M/M/1$ queue with working vacation, *Oper. Res. Lett.,* 35, 595-600, 2007.
14. Lucantoni, D.M., A $GI/M/C$queue with a different service rate for customers who need not wait an algorithmic solution, *Technical Rep. Univ. of Delware,USA,* 1979.
15. Nakdimon, O., and Yechiali, U., Polling systems with breakdowns and repairs. *European Journal of Operational Research,* 149,588-613, 2003.
16. Neuts, M.F., Markov chains with applications in queueing theory which have a matrix-geometric invariant probability vector, *Adv. Appl. Probab.,* 10, 185-212, 1978.
17. Neuts, M.F, Matrix-Geometric solution in stochastic models, *Vol 2 of John Hopkins series in the Mathematical Sciences, Johns Hopkins University press, Baltimore, md, USA,* 1981.
18. Neuts, M.F. and Lucantoni, D.M., A Markovian queue with N servers subject to breakdowns and repairs, *Management Science, 2*5, 849-861, 1979.
19. Servi, L.D., and Finn, S.G.,$M/M/1$ queues with working vacations$M/M/1/WV$, *Perform. Eval,.* 50, 41-52, 2002.
20. Shanthikumar, J.G., Analysis of a single server queue with time and operational dependent server failures,*Adv. in mgnt. studies,* 1, 339-359, 1982.
21. Shogan, A.W., A single server queue with arrival rate dependent on server breakdowns, *Naval Res. Log. Quart.,* 26, 487-497, 1979.
22. Takagi, H.,Queueing Analysis-A Foundation of Performance Evaluation vacation and Priority Systems, Vol. 1, North-Holland, New York, 1991.
23. Tang, Y., Single-server $M/G/1$ queueing system subject to breakdowns-some reliability and queueing problems. *Microelectronics Reliability,*37, 315-321, 1997.
24. Tian. N and Yue. D, Quasi-birth and death process and the matrix geometric solution. *Beijing: Science Press,* 2002.
25. Wang, J., Liu, B., and Li, J., Transient analysis of an$M/G/1$ retrial queue subject to disasters and server failures. *European Journal of Operational Research,* 189, 1118-1132, 2008.
26. Wang, K., Wang, T., and Pearn, W., Optimal control of the N policy $M/G/1$Queueing systems with server breakdowns and general startup times.*Applied Mathematical Modelling,* 31, 2199-2212, 2007.
27. Wu, D.A., and Takagi, H., $M/G/1$ queue with multiple working vacations, *Perform. Eval,.* 63, 654-681, 2006.
28. Xu, X., Zhang, Z. and Tian, N. Analysis for the $M^X/M/1$working vacation queue, *Int. Jr. of Infor. &Manag. Sci.,* 20, 3, 379-394, 2009.
29. Yechiali, V and Naor. P.,Queueing problems with heterogeneous arrivals and service, *Oprs. Res.,* 19, 722-734, 1971.

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