Analysis Of Diabetes Based On Fuzzy Fractals

G. Jayalalitha &T.Sudha

Abstract: ***The main aim of this paper is to predict the Diabetes based on Fuzzy Fractals. It can be analysed for the three types of categories people such as young, middle, and senior. It focuses to assess and analyse the diabetic patients in the blood sugar level variations. Fractal Dimension (FD) is used for determining irregularity of blood sugar level in the glucose concentration. The non-integer values are obtained by Fractal Dimension to indicate the diabetes range. It is also analysed by Fuzzy - Strong Alpha Cut and Support - Set.***

Keywords: AMS classification 2000: 03E72, 28A80 Diabetes, Fuzzy, Fractal, Normalized, Strong Alpha cut, Support set.

I. INTRODUCTION

A. *Fuzzy Set*

The Fuzzy set was first proposed by Zadeh, in the year 1965. To handle uncertainty in a lot of the real world applications the Fuzzy sets are used. Fuzzy set is the extension and generalization of basic set theory. Fuzzy set is the set which allows its members to have different degree of membership value called membership function in the interval [0,1]. Vagueness is introduced in Fuzzy set by eliminating sharp boundaries. There is a gradual transition between full membership to non-membership. Fuzzy set theory can be used in a wide range of domains in which information is incomplete or imprecise such as bio informatics. In real time a system, where the information is inherent, it has various levels of precision, to solve these kinds of problems Fuzzy Sets are used. There are different kinds of the fuzzy graph like vague fuzzy graphs, fuzzy k-competition graphs, bipolar fuzzy graphs, interval-valued fuzzy graphs etc [1].

B. *Fractals*

In the year 1975 fractals were introduced by the mathematician Benoit Mandelbrot. A Fractal is a never-ending pattern. Fractals are infinitely complex patterns which are self-similar across different scales it has infinitely complex pattern. There are all examples of sets that are commonly referred to as Fractals. Certainly, any fractal has some degree of self – similarity – they are made up of parts that resemble the whole in some way. Sometimes, the resemblance may be weaker than strict geometrical similarity; for example, the similarity may be approximate or statistical.

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Fractals don't have a whole number dimension-they have a fractional dimension.[2] Based on the Fractals dimension, fractal structures are characterized Examples of fractals are Sierpinski Triangle, Cantor Set and Von Koch Curve [3].

C. *Fuzzy Fractals*

Mandelbrot defined Fractal as a special class of subsets of a complete metric space for which the Hausdorff - Besicovitch dimension strictly exceeds the topological dimension. In other words, a given set of randomized, hyperbolic, Iterated Function System (IFS) is able to generate a particular image as a fractal set. In Fuzzy Fractals, the algorithm for a specific problem can be coined by incorporating the fuzzy set and fractals together and create a new dimension by considering the data as a time series problem. The Fuzzy Fractals are used to find the relationship between the crisp values and their properties for an investigation [4].

D. *Diabetics*

Due to the unhealthy lifestyle in India, one of the fatal diseases like Diabetics is increasing at a faster rate. These diseases occur when the patient's pancreas unable to produce enough insulin or the body could not utilize the insulin produced efficiently. In the developed countries Diabetics are increasingly rapidly like an epidemic. Based on the data from International Diabetes Foundations in 2005 over 246 million people around worldwide have been treated for Diabetics [5].In common, there are three types of diabetics they are type-1, type-2 and gestational diabetics. Among these three types, Type-2 diabetics are taken into consideration for this work**.** In Section 2 Preliminaries are explained. In section 3 Methods are explained. In section 4 the results are explained.

II. PRELIMINARIES

A. *Crisp Set*

Crisp Set is a collection of elements selected from the domain, the elements satisfying some criteria. An element is either a member of a given Crisp set, *or it is not [7].*

B. *Normalized Fuzzy Set*

Let A be a Fuzzy set on U. Then the height of A is defined to be that number ht (A) which is such that:-

- A (x) $\leq$ ht (A) for all x in supp(A) and
- A(x) = ht(A) for at least one x in supp(A). [6]

A Fuzzy set A is said to be Normalized if Height (A) =1

In this finite case, where the supremum is a maximum, this means that at

least one element of the set in the Fuzzy Set has full membership. A non-empty fuzzy set A may be normalized with result $\tilde{A}$ by dividing the membership function of the fuzzy set by its height.

For all x∈ U: $\mu\bar{A}_{(x)} = \mu A_{(x)}$ / Height(A).

C. α – Cut or α -Level Set [6]

For any Fuzzy Set $\bar{A}$, $\bar{A}_{\alpha} = \{x / \mu\bar{A}_{(x)} \geq \alpha\}$ α is arbitrary, α belongs to [0, 1] is called α – Cut or α-Level Set.

D. Fuzzy Set [7].

Any set which allows its members to have membership of different grades in the interval [0, 1] is a Fuzzy Set

E. Support of Fuzzy Set

The support of a fuzzy set F is a crisp set of all points in the Universe of Discourse U (range of all possible values for an input to a Fuzzy system) such that the membership function of F is non-zero [6].

F. Convex Fuzzy Set [7]

A real fuzzy set $\bar{A}$ is said to be convex, if and only if for all $x_1, x_2 \in \bar{A}$ for all $\lambda \in [0, 1]$

$\mu\bar{A}\ (\lambda_{x1} + (1–\lambda)\ x_2\ ) \geq [(\min\ (\mu\bar{A}_{(x1)}, \mu\bar{A}_{(x2)}\ :\ x_1, x_2 \in \bar{A}]$ (1)

G. Strong α – Cut [6]

If $\bar{A}$ is a Fuzzy Set and $\bar{A}_{\alpha} = \{x / \mu\bar{A}_{(x)} > \alpha\}$ Where α ∈ [0, 1] and α is arbitrary so called as Strong α – Cut.

*III.*METHODS

Here there are two methods are explained such as Fuzzy Fractals and HarFA Fractal Analysis.

A. Fuzzy Fractals

Based on the definitions of Fuzzy and its Membership Values it can be categorized three types such as Young, Middle, Senior diabetics

Case (i) young diabetic aged between 15 to 35

$\overline{Sy}$ = {23, 22, 25, 27, 32, 18, 27, 25, 28, 30, 25, 28} (2)

Based on Fuzzy Support Set II. E, equation (2) Support set of young diabetic is formed. It represents $\overline{S(y)}$ is a Crisp Set because it has boundary from 18 to 30 and its Membership value is non- zero.

By the definition of Normalized Fuzzy sets II. B Table 1 shows Fuzzy subsets of young diabetic values are normalized because height of the set is one (at least one element has full Membership Value)

Table 1. Young Diabetic Aged between 15 to 35 years

Age	Membership values
25	0.2
18	0.2
28	0.8
30	0
27	0.3
25	0.4
28	1
32	0.6
25	0
22	0.1
28	0.8

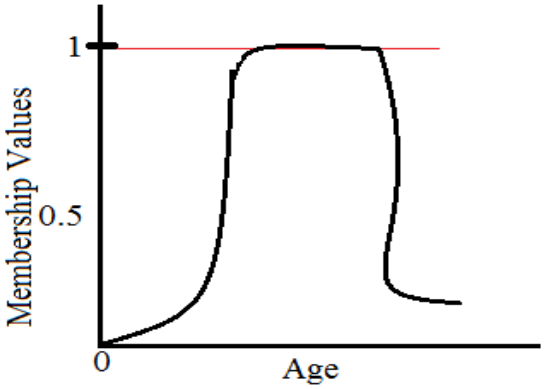


Figure 1. Normalized Young Diabetic Fuzzy Set

Based on the definition of Convex Fuzzy set II .F, equation (1) formed it represents the young diabetic has Convex Fuzzy set.

For example Let x_1 = young $_{0.2}$ = {25, 18},
x_2 = young $_{0.8}$ = {28}

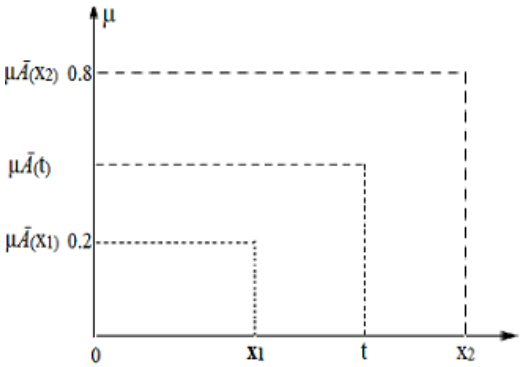


Figure. 2 Convex Young Diabetic Fuzzy set

Where $t = \lambda_{x1} + (1 – \lambda)_{x2}$
$\mu\bar{A}(t) \geq \min\ (0.2, 0.8)$

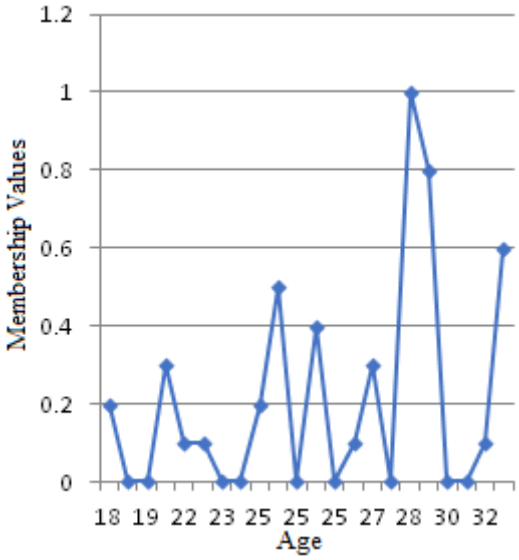


Figure. 3 Line Graph Young Diabetic

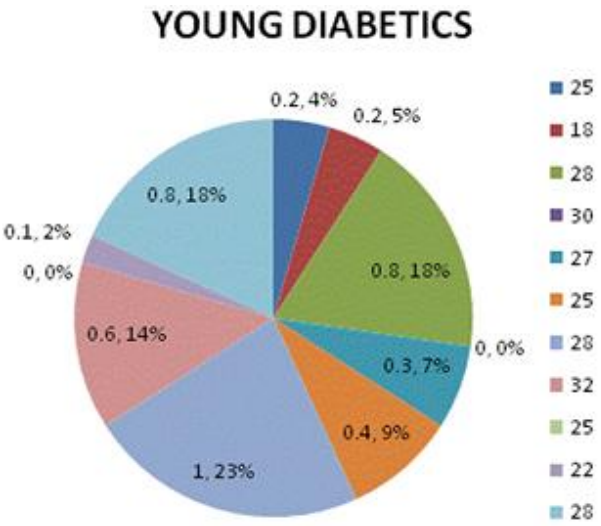


Figure 4.

Case (ii) middle diabetic aged between 35 to 60

$\bar{S}_{(m)}$={38,36,32,40,43,44,49,50,52,55,52} (3)

Based on Fuzzy Support Set II .E. Equation (3) Support set of middle diabetics formed. It represents $\overline{S_{(m)}}$ is a Crisp Set because its boundary value from 32 to 52 is sharp and Membership Values are non zero.

By the definition of Normalized Fuzzy set II. B, Table 2. shows Fuzzy subsets of middle diabetic values are normalized because height of the set is one. (at least one element has full Membership Value).

Table 2. Middle Diabetic Aged between 35 – 60 years

Age	Membership values
52	0.8
55	0.7
43	0.2
37	0
36	0.5
38	0.1
43	1
50	0.5
44	0.5
49	0.2
47	1
38	1
51	1
38	0.2
40	1

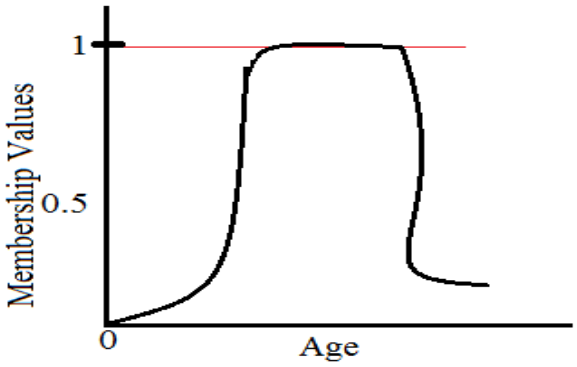


Figure 5. Normalized middle diabetic

Based on the definition of Convex Fuzzy set II .F, equation (1) formed it represents the middle diabetic has Convex Fuzzy set

Let x_1 = middle$_{0.2}$ = {43, 49, 38}, x_2 = middle$_{0.8}$ = {52}

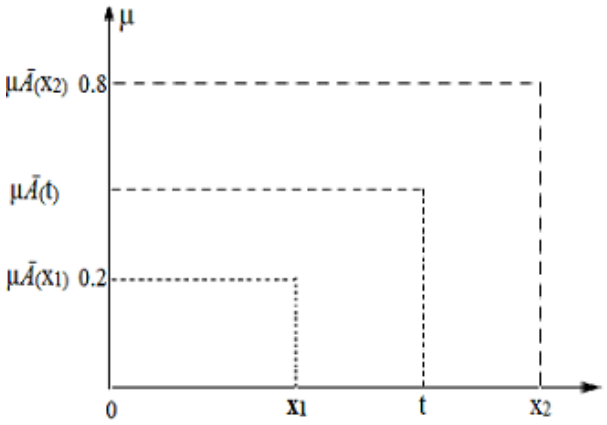


Figure. 6 Convex middle Diabetic Fuzzy set

Where $t = \lambda_{x1} + (1 - \lambda)_{x2}$

$\bar{A}(t) \geq \min (0.2, 0.8)$

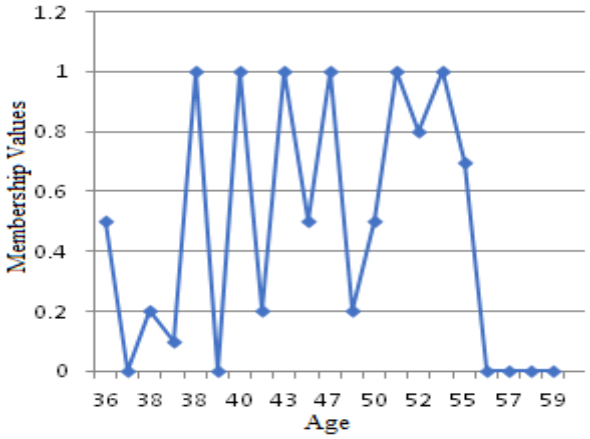


Figure.7 Line Graph middle Age Diabetic

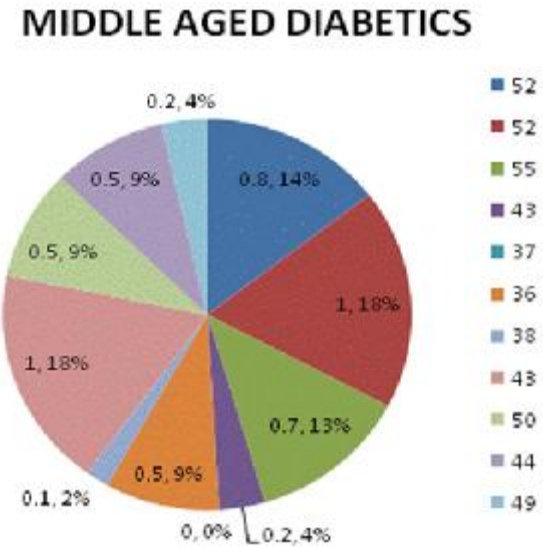


Figure.8

Case (iii) Senior Diabeticc aged between 60 to 85

$\bar{S}(s)$={61, 62, 64, 65, 60, 66, 75, 77, 74, 70, 82} (4)

Based on definition of Fuzzy Support set II.E, Equation (4) Support set of Senior diabetic formed and it represents the $\bar{S}(s)$ is a Crisp Set because its boundary value from 60 to 82 is sharp, Membership Values are non zero.
By the definition of Normalized Fuzzy set II. B, Table 3. shows Fuzzy subsets of senior diabetic values are normalized because height of the set is one. (at least one element has full Membership Value.)

Table3. Senior Diabetic Aged between 60 to 85 years

Age	Membership values
65	1
64	1
61	1
75	1
77	1
60	0.7
62	0.3
66	0.8
74	0.5
82	1
70	1

62	1

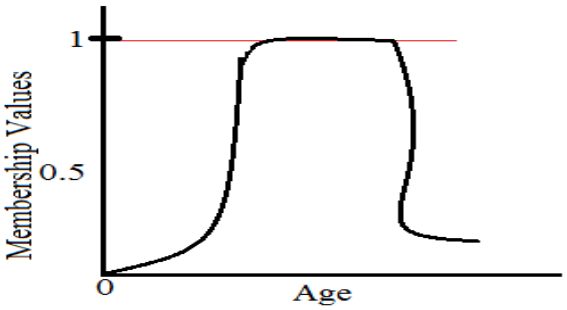


Figure.9. Normalized senior diabetic

Based on the definition of Convex Fuzzy set II.F, equation (1) formed it represents the senior diabetic has Convex Fuzzy set

Let $x_1 = \text{senior}_{0.3} = \{62\}$,
$x_2 = \text{senior}_{0.8} = 66\}$

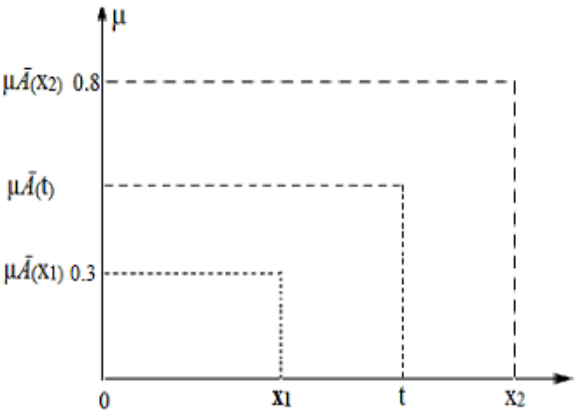


Figure.10 Convex Senior Fuzzy set

Where $t = \lambda_{x1} + (1 - \lambda)_{x2}$
$\mu\bar{A}(t) \geq \min(0.3, 0.8)$

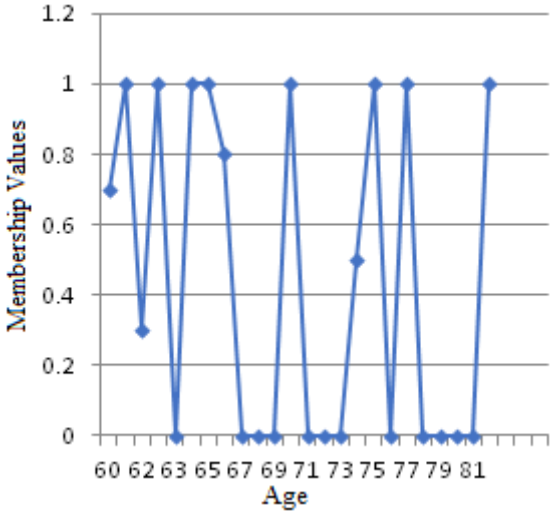


Figure.11 Line Graph for Senior Diabetic

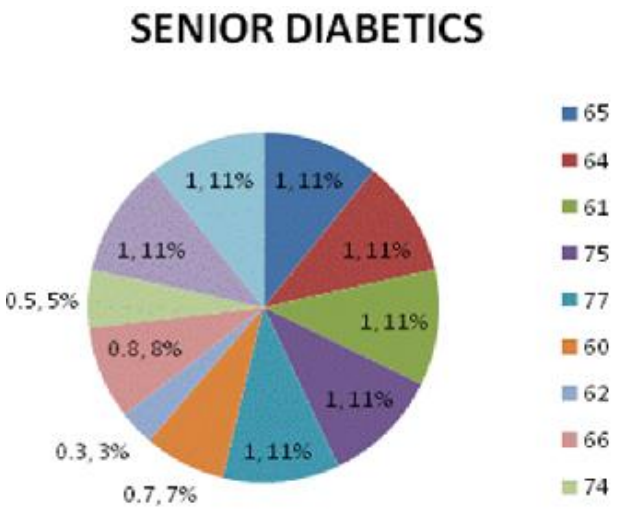


Figure.12

Table 4. Membership Values for different age group between 15 to 85

Membership Values	Age
0	30, 19, 25, 30, 19, 23, 25, 28, 25, 37
0.1	23, 27, 32, 22, 38
0.2	25, 18, 43, 49, 38
0. 3	22, 27, 62
0.4	25
0.5	25, 36, 50, 44, 74
0.6	32
0.7	55, 60
0.8	28, 52, 66
1	28, 52, 43, 47, 38, 51 ,40, 65, 64, 61, 75, 77, 62, 70, 82

Table 4. represents the Membership Values of different age group between 15 to 85 such as young, middle, senior.

B. *Fractal Dimensions*

HarFA–Fractal Nezadal et. al .[8] and Buchnicek et al [9] have implemented Box counting procedure in the software called HarFA, which was developed by the Institute of Physical and Applied Chemistry, Technical University of Bino in the Czech Republic. [10]Dimension determined by this method is called Box Dimension (D_{BBW}) . The method is often used to determine fractal box dimension of digitized images of fractal structures. HarFA analyses black and white images, Box Counting Method utilizes the covering fractal pattern with raster of boxes (squares) and than evaluating how many boxes N_{BW}, $N_{BBW} = N_B + N_{BW}$ or $N_{WBW} = N_W + N_{BW}$ of the raster are needed to cover fractal completely where

N_B – number of black squares,

N_W – number of white squares,

N_{BW} – number of black and white squares,

N_{BBW} – number of black and white and black squares, and

N_{WBW} – number of black and white and white squares.

Repeating this measurement with different sizes of boxes r =1/€ will result in logarithmical function of Box size r and number of boxes N(r) needed to completely cover fractal. The slopes of the linear functions

$$\text{In } N_{BW}(r) = \text{In}(K_{BW}) + D_{BW} \text{ In } (r) \quad (5)$$

$$\text{In } N_{BBW}(r) = \text{In } (K_{BBW}) + D_{BBW} \text{ In}(r) \quad (6)$$

$$\text{In } N_{WBW}(r) = \text{In } (K_{WBW}) + D_{WBW} \text{ In } (r) \quad (7)$$

Give D_{BW}, D_{BBW} and D_{WBW}, the fractal dimensions. D_{BW} characterizes properties of border of fractal pattern. D_{BBW} characterizes fractal pattern on the white background and D_{WBW} characterizes fractal pattern on the black background.

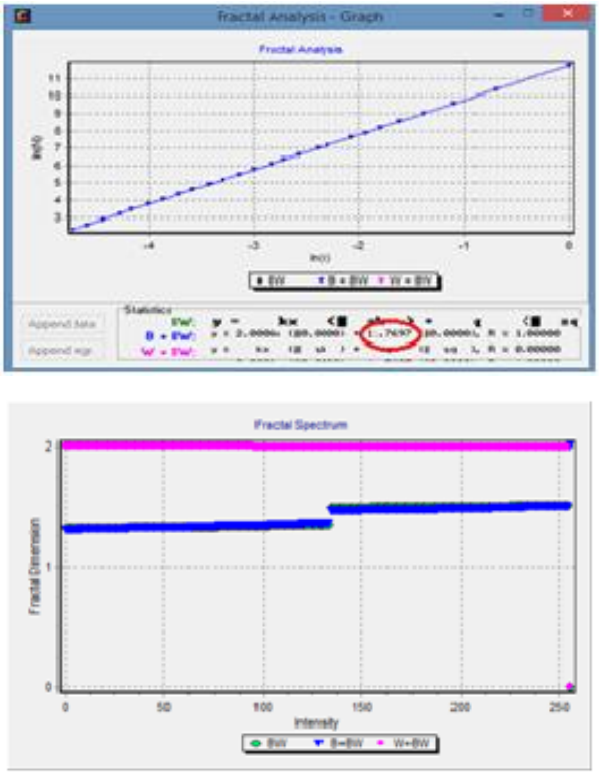


Figure.13 Fractals Dimension of Young Diabetic

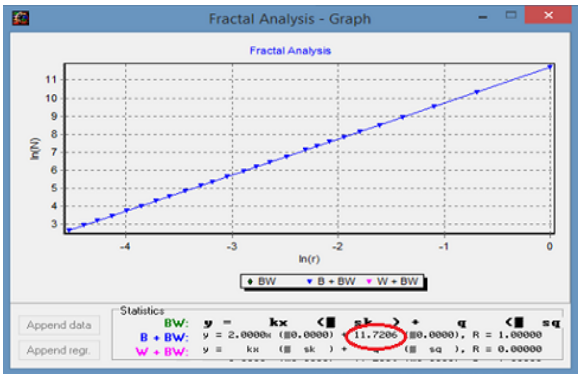
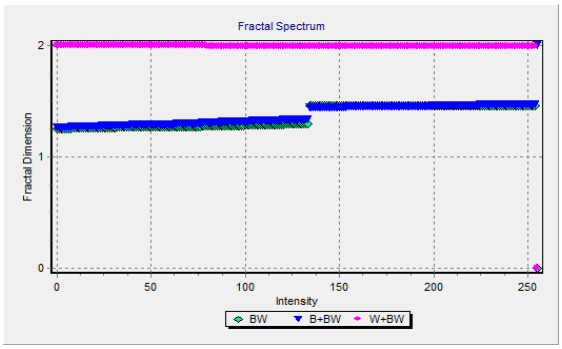
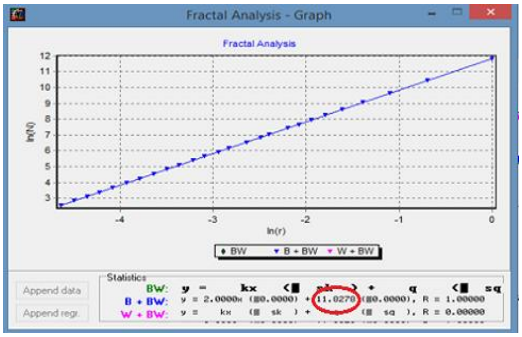


Figure. 14 Fractals Dimension of Middle Diabetic





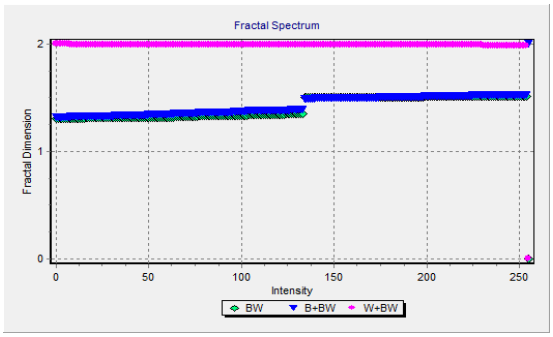


Figure.15 Fractals Dimension of Senior Diabetic

In Fig.13, 14, and 15 Fractal dimension analysed based on diabetic blood sugar level for three categories.

1V. RESULTS

From the Fig. 3 Line Graph represents the diabetic Membership Value of the young people in the age group between 15 and 35. This graph shows that at the age of 28 some of the people have the maximum value and it represents diabetic patients and at the age of 32 reaches diabetes soon. Similarly the Fig. 4 Pie Graph represents the percentage of the Membership Values and at the age 28 reached maximum percentages of 1.21% and it reaches Fractal Dimension also.

Fig. 7 the Line Graph represents middle age diabetic between 35 and 59. At the age of 52, 43, 47, 28, 51 and 40 some of the people will have diabetes. They reached the maximum Membership Values. Similarly Fig.8 represents Pie Graph shown the percentage of Membership Values and the ages 52, 43, 47, 28, 51 and 40 has maximum of 1.10 percentage and reached fractal dimension also.

Fig. 11 Line Graph represents the diabetic senior age persons. At the age of 65, 64, 61, 75, 77, 82, 70, 62 ages reached diabetic. As the age increases the diabetic level may or may not increase gradually. Similarly Fig, 12 the Pie Graph represents the percentage of Membership values of senior diabetic and most of people above 60 are maximum percentage of 1.10 and also it reaches Fractal dimensions.

By the definition of Fuzzy Alpha and Strong Alpha cut II. C and II. G table 4 all young, middle and senior diabetes of Alpha Cut set and Strong Alpha Cut set are Crisp Sets (because Membership Values [0, 1] are sharp boundary).

All young, middle , and senior diabetic sets are Normalized because its reached maximum value 1 and also Convex Sets.

[10] It is found the intensity of blood sugar by using the Box-Counting Method with the help of HarFA Fractal Analysis software. The Box Counting Method utilizes the covering fractal pattern with raster of boxes (squares) and then evaluates how many N_{BW}, N_{BBW}, and N_{WBW} of the raster are needed to cover fractal completely.

Constant values shown in the graph of Figure 13, 14, and 15 is 11.7697, 11.7206, and 11.0270 for young, middle and senior respectively. It shows that this reaches maximum diabetic and Fractal Dimension value reaches 2.

V. CONCLUSION

Diabetic Membership Values have been compared for all the three age groups like young, middle and senior. The differences are evaluated using the Fuzzy Set Graph. Fractal dimensions are estimated and drawn using HarFA. The diabetic level may or may not increases based on Membership Values as the age increases and fractal dimension reaches 2 based on the constant in the fractal graph.

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