Centralized Inventory Model with Shortages Using Lacunarity

G. Jayalalitha & K.Suganthi

Abstract: Abstract: In this paper, Inventory Model with Shortages and Screening process obtained by using Analytical Geometry and Algebraic Method. The Shortages Gap can be analyzed by the method of Lacunarity which provides the Fractals sets. It gives the maximum and minimum of the Shortage Gap. At this Shortage period the Managerial policy is provided, and its behavior can be analyzed by on Fractional Brownian Motion (FBM).

Keywords: Fractals, Fractional Brownian Motion, Inventory Model, Lacunarity, Shortages.

I. INTRODUCTION

A. Fractals

A Fractals is generally a rough or fragmented geometric shape that can be split which is reduce size copy of the whole a property called self-similarity, it was coined by Benoit Mandelbrot in 1975 and as derived from the Latin word fractal meaning “broken” or “fracture”. He was a father of Fractals[2]. It never is ending patterns. It is infinitely complex patterns that are self- similar across at different scales. Examples are trees, rivers, coastlines sea shells, Cantor set,Sierpinski Triangle,Von Koch curve[1].Fractal geometry provides a general frame work for the study of such irregular sets[3].

In section 2 preliminaries are explained. In section 3 various methods are explained. In section 4 the results are discussed.

II. PRELIMINARIES

A. Assumption and Notations [8]

The model uses the following assumption and notations

Assumptions

(i) Demand rate is uniform and constant.
(ii) Shortages are allowed for buyer only.
(iii) Buyer screened or disposed the damaged items in coordination scheme and vendor screened or disposed the damaged items in noncoordination scheme.
(iv) For benefits of buyer and vendor the system cost is formed. The system cost can be written as
(v) $TC_S = TC_B + TC_V$

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G. Jayalalitha, Professor, Department of Mathematics,
VELS Institute of Science Technology & Advanced Studies (VISTAS) Chennai, Tamil Nadu, India.
K.Suganthi, Research Scholar, Department of Mathematics,
VELS Institute of Science Technology & Advanced Studies(VISTAS) Chennai, Tamil Nadu, India.

B. Fractional Brownian Motion

The Fraction Brownian Motion, also called a Fractal Brownian Motion, is a generalization of Brownian motion, not similar to classical Brownian motion, the increments of Fractional Brownian Motion need not be independent. Fractional Brownian Motion has stationary increments that are normally distributed but no longer independent[4].The botanist Robert Brown noticed that minute particles suspended in a liquid moved on highly irregular trails[5].

C. Inventory Model

The inventory problems deal with stocking an item to meet fluctuations in demand. Keeping too much of an item on hand increases cost of capital of storage and stocking too little adversely disrupts production or sales[7].

D. Shortages in Economic Order Quantity Model

The penalty cost for running out of stock that is when an item can’t be supplied on the customer’s demand is known as Shortage cost. This cost includes the loss of potential profit through sales of item and loss of goodwill in terms of permanent loss profit in future sales the defective items are Screened or disposed[8].The Economic Order Quantity (EOQ) is that size of order which minimizes total annual costs of carrying inventory and cost of ordering[6].

Notations

D	Demand rate per time unit
r_1	Buyer’s unit setup cost per order
r_2	Vendor’s unit setup cost per order
B	Backorder level
S_1	Shortage cost per order per unit
Q	Economic Order quantity
h_b	Buyer’s unit holding cost per order per unit
h_v	Vendor’s unit holding cost per order per unit
s_c	Screening cost per order per unit
d_c	Disposal cost per order per unit
p_c	Purchase cost per order per unit
a	Percentage of defective items
b	Percentage of scrap items

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n Vendor's multiples of order with coordination

m Vendor's multiples of order with coordination

k Buyer's multiples of order with coordination

$d(k)$ Discount factor

B. Formulation of the *Model*

In this section, centralized model is developed for noncoordination and coordination strategies. That is in centralized model, integrated system cost is developed for system optimization. The integrated system cost is addition of buyer's total coast and vendor's Total cost. In noncoordination strategy, the vendor screened or disposed the damaged product for resale and the buyer having shortages. In coordination strategy, the vendor Offers quantity discount for the buyer for the large purchase. Hence, the buyer has no shortage as well as the buyer himself screened or disposed the damaged products for resale [8].

C. Centralized Model with Non Coordination

Now the Buyer's total cost is addition of ordering cost, holding cost and shortage cost

$$i.e., TC_{B1} = \frac{r_1 D}{Q} + \frac{h_b B^2}{2Q} + \frac{s_1 (Q-B)^2}{2Q}$$

The vendor's total cost is addition of setup cost, holding cost, screening cost and disposal cost

$$i.e., TC_{v1} = \frac{r_2 D}{mQ} + \frac{h_v (m-1)Q}{2} + \frac{s_c (m-1)Q}{2}$$

$$+ \frac{d_c ab (m-1)Q}{2}$$

The integrated system cost is expressed as

$$TC_{S1} = TC_{B1} + TC_{v1}$$

$$= \frac{D}{Q}\left(r_1 + \frac{r_2}{m}\right) + \frac{h_b B^2 + s_1 (Q-B)^2}{2Q} + \frac{(m-1)(h_v + s_c + d_c ab)Q}{2} \quad (1)$$

Equation (1) can be written as

$$TC_{s1} = \left(\frac{h_b + s_1}{2Q}\right) B^2 - s_1 B + \frac{s_1 Q}{2} + \frac{D}{Q}\left(r_1 + \frac{r_2}{m}\right)$$

$$+ \frac{(m-1)(h_v + s_c + d_c ab)Q}{2}$$

It is of the form $c_1 x^2 + c_2 x + c_3$ [6]

$$B = \frac{-c_2}{2c_1} = \frac{Q s_1}{h_b + s_1} \quad (2)$$

$$\text{i.e., } B^* = \frac{Q s_1}{h_b + s_1}$$

Also, Equation (1) can be written as

$$TC_{S1} = \left\{\frac{s_1 + (m-1)(h_v + s_c + d_c ab)}{2}\right\} Q$$

$$+ \left\{D\left(r_1 + \frac{r_2}{m}\right) + \frac{(h_b + s_1) B^2}{2}\right\} \frac{1}{Q} - s_1 B$$

It is of the form $c_1 x + \frac{c_2}{x} + c_3$ [6]

$$Q = \sqrt{\frac{c_2}{c_1}} = \sqrt{\frac{2D\left(r_1 + \frac{r_2}{m}\right)(h_b + s_1)}{(h_b + s_1)(m-1)(h_b + s_1)(h_v + s_c + d_c ab)}}$$

$$\text{i.e., } Q^* = \sqrt{\frac{2D\left(r_1 + \frac{r_2}{m}\right)(h_b + s_1)}{(h_b + s_1)(m-1)(h_b + s_1)(h_v + s_c + d_c ab)}} \quad (3)$$

D. Centralized Model with Coordination [8]

In coordination strategy, buyer's order size is greater than the regular size because the vendor offers quantity discount at discount factor d (K). Hence, the buyer has no shortage also the buyer screened or disposed the damaged products for resale. Now the buyer's order size is KQ_0 and the vendor's order size is KnQ_0

$$i.e., TC_{B2} = \frac{r_1 D}{Q_0} + \frac{h_b Q_0}{2} + \frac{S_c Q_0}{2} + \frac{d_c ab Q_0}{2}$$

The vendor's total cost is addition of setup cost, holding cost and buyer's quantity discount.

$$i.e., TC_{v2} = \frac{r_2 D}{KnQ_0} + \frac{hvK(n-1)Q_0}{2} + pcDd(k)$$

The integrated system cost is expressed as

$$TC_{S2} = TC_{B2} + TC_{v2}$$

$$= \frac{D}{Q_0}\left(r_1 + \frac{r_2}{kn}\right) + \frac{(h_b + (n-1)h_v + s_c + d_c ab) Q_0}{2}$$

$$+ p_c Dd(K) \qquad (4)$$

Equation (4) can be written as

$$TC_{S1} = \left\{\frac{h_b + (n-1)h_v + s_c + d_c ab}{2}\right\} Q_0$$

$$+ \left\{D\left(r_1 + \frac{r_2}{kn}\right)\right\} \frac{1}{Q_0} + p_c Dd(k)$$

It is of the form $c_1 x + \frac{c_2}{x} + c_3$ [6]

$$Q = \sqrt{\frac{c_2}{c_1}} = \sqrt{\frac{2D\left(r_1 + \frac{r_2}{kn}\right)}{h_b + K(n-1)h_v + s_c + d_c ab}}$$

$$\text{i.e., } Q_o^* = \sqrt{\frac{2D\left(r_1 + \frac{r_2}{kn}\right)}{h_b + K(n-1)h_v + s_c + d_c ab}} \quad (5)$$

E. Numerical Example

Let D = 2000 units per year, r_1= 300$ per order, r_2 = 100$ per order, h_v = 10$, h_b= 20$, s_1 = 25, s_c = 2, d_c = 1.5$, a=0.1, b=0.5, p = 3, m = 8, n = 2, K = 3, d (K) = 5%. The optimal value are Q^* = 114, B^* = 64, TC_{S1}^* = 1.0934 × 10^4,Q_o^* = 156, Tc_{S2}^* = 8.4217×10^3

F. Sensitivity Analysis for Different Parameters

The sensitivity analysis is performed by changing the value of each parameter by -50%,-25%,

+25%, +50% taking one parameter at a time and keeping the remaining parameters unchanged. The solution is highly sensitive to change in r_1, D, h_v and moderately sensitive to change in h_b and slightly sensitive to change in r_2, p, s_1,d_c. The results are shown inI (a) [8].

III. METHODS

The Shortage Gap of the different parameters in the inventory model can be analyzed by Lacunarity and Fractional Brownian Motion (FBM).

A. Lacunarity Analysis[9]

Lacunarity is another term to Mandelbrot from the Latin lacuna, which means gap. It is a counterpart to the fractals dimension that describes the pattern of fractal. It has to do with the size distribution of the holes. Roughly speaking, if a fractal has large gaps or holes, it has high Lacunarity; on the other hand, if a fractal is almost translational invariant, it has low Lacunarity. Different fractals can be constructed that have the same dimension but that look widely different because they have different Lacunarity.

It is related to the distribution of gap sizes. It is a measure of lack of rotational and translational invariance or symmetry. Low Lacunarity objects are homogeneous because all gap size is the same, whereas high Lacunarity objects are heterogeneous[9]. Lacunarity (L) can be defined in terms of the local first and second moments. It is a notion distinct and independent from D, it is not related with the topology of the fractal and needs more than one numerical variable to be fully determined

$$Lacunarity = \left[1 + \left(\frac{variance}{(mean)^2}\right)\right] \quad (6)$$

B. Fractional Brownian Motion

Fractional Brownian motion has stationary increments that are normally distributed but no longer independent [4]. A process X (t) defined on some interval is called a Gaussian process if for$0 \le t_1 \le t_2 \le \cdots \ldots\ldots \le t_m$ and scalars $\lambda_1 \ldots\ldots\ldots \lambda_m$, the random variable λ_1 X (t_1) + ... +λ_m X(t_m) has normal distribution (the vector (X_{t_1}........... X(t_m)) is multivariate normal). Since it is implicit in the above definition that the increments *X (t + h) — X (t) are stationary, that is, probability distribution independent of t.* Fractional Brownian motion of index-**α** (0 < α < 1) is defined to be a Gaussian process X: [0, ∞] → R on some probability space such that:

- *With Probability 1, X (t) continuous and X (0) = 0;For every t ≥ 0 and h > 0 the increment X (t + h) — X (t) has Normal distribution with mean zero and variance $h^{2\alpha}$, so that*

$$P(X(t+h) - X(t) \le x) =$$

$$\frac{1}{h^\alpha\sqrt{2\pi}}\int_{-\infty}^{x} exp\left(\frac{-u^2}{2h^{2\alpha}}\right) du. \quad (7)$$

C.Gaussian Condition

A time continuous stochastic process is Gaussian if and only if, for every finite set of indices$t_1 \ldots\ldots\ldots t_k$ in the index set T

$x_{t_1} \ldots\ldots x_{t_k}$= (x_{t_k} .... x_{t_k}) is a multivariate Gaussian random variable.

This can lead to discontinuous functions. For simplicity in the 1- dimensional case; analogous processes may be defined taking value in n-dimensional space.

If define an index-α fractional Brownian function X: R^2→R to be a Gaussian random function such that:

- With probability 1 X (0, 0) = 0 and is a X(x, y)continuous function of(x, y);
- For $(x, y), (h, k)$ ϵ R^2, the height increments X(x,y), (h, k) – X(x, y) are normally distributed with mean zero and variance $(h^2+k^2)^\alpha$ = $|(h,k)|^{2\alpha}$;

$$P(X(x + h, y + k) - X(x, y) \le z) =$$

$$\frac{1}{(h^2+k^2)^{\alpha/2}\sqrt{2\pi}}\int_{-\infty}^{z} exp\left(\frac{-r^2}{2(h^2+k^2)^\alpha}\right) dr \quad (8)$$

D.Graphical Method of Fractional Brownian Motion

Different Parameters of the Shortages can be analyzed by Fractional Brownian Motion. In the Figure.1 all the parameters of the shortages are formed as Normal Distribution.

By using Graphical Method of maximum and minimum Shortages are found, by Lacunarity factors. The mean value of different parameters such asD , r_1,r_2, h_v, h_b s_1, s_c, d_c,p are found. By Lacunarity, if Shortages gap is high, then the mean value of different Parameters such as Q^*, B^*, TC^*_{s1}, Q^*_0, TC^*_{S2} which can represented in the graph Figure1.At this Shortage period the Managerial policy is provided, and its behavior can be analyzed by Fractional Brownian Motion (FBM).

IV. RESULTS

Based on numerical example, Sensitivity analysis for different Parameters has been found I (a) 1 [8] and it has nine different Parameters such hasD, r_1,$r_2$$h_v$, h_b ,s_1, s_c, d_c, p.Based on the mean value of the different parameters Lacunarity analysis has been found which is given in the I (b) 2. The five cost such as, Buyer order cost(Q^*_0), Back order level (B*), Economical order quantity (Q^*), Integrated system cost ($TC_{s1}{}^*$,$Tc_{s2}{}^*$) with different parameter such asD, r_1,$r_2$$h_v$, h_b , s_1,s_c,d_c,p *the* mean is established. Based the mean value, the Lacunarity is found which gives a shortage gap. So it forms normal distribution.

In the parameter D, Lacunarity is found such as Q^*, B^*, TC^*_{s1}, Q^*_0, TC^*_{S2} based on the value of mean and variance. The maximum value of mean is 152.5 it has reached the Lacunarity 1.044, so it is a noninteger. This dimension reveals a highly reaching Shortage gap.

Similarly *the second parameter r_1*, the maximum value of mean is 153. It reached the Lacunarity 1.039, so mean value is high. This dimension also gives the high Shortage gap.

In third parameter r_2, the maximum value of mean is 156. Here Lacunarity reached to 1.0002. So this dimension gives a Shortage gap.

Fourth parameter h_v , the maximum value of men is 159.25, it reached the Lacunarity is 1.014. Then mean value is high. So this dimension gives an average Shortage gap.

Fifth parameter h_b , the maximum value of men is 157.5, it reached Lacunarity is 1.006, so it has mean value is high when Lacunarity value is high, so this dimension gives a high Shortage gap.

Sixth parameter S_1, the maximum value of mean is 156, it reached Lacunarity is 1. So this dimension gives a high Shortage gap.

Seventh parameter S_c, the maximum value of Mean is 156.25, it reached Lacunarity is 1.0001,and then mean value is high, when theLacunarity value also high. Thus dimension gives high Shortage gap.

Eight parameter dc, the maximum value of mean is 156. It reached Lacunarity is 1, hence it has mean value is high, when Lacunarity value is high, the dimension review high Shortage gap.

Ninth parameter P, the maximum value of mean is 156, it reached Lacunarity is 1. Hence this dimension only gives the low Shortage gap.

It reached the dimension value of nine parameters such as D, r_1, r_2, h_v, h_b, s_1, s_c, d_c, P. From the maximum mean value of Economical Order Quantity 156 ,157 ,159 is taken, then the Lacunarity value for the different parameters attain 1,1.04362, 1,1.0387, 1,1.0002, 1,1.014, 1.0060, 1,1.0000, 1,1.0001, 1,1.0000, 1,1.0000 This shows that when mean value is high, Lacunarity is low. This leads to give Fractal sets .Hence it is noninteger, it reaches the maximum variance value is 1. So it is a Normal Distribution.

V. CONCLUSION

In the EOQ Model the shortages gap can be analyzed by Lacunarity. Based on the mean value of different parameters in the EOQ Model can be analyzed by Fractional Brownian Motion which leads to the Normal Distribution.

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AUTHORS PROFILE

G. Jayalalitha
Professor, Department of Mathematics,
VELS Institute of Science Technology & Advanced Studies (VISTAS)
Chennai, Tamil Nadu, India.

K.Suganthi
Research Scholar,
Department of Mathematics,
VELS Institute of Science Technology & Advanced Studies(VISTAS)
Chennai, Tamil Nadu, India.

Parameters		Q*	B*	TC^*_{s1}	Q^*_0	TC^*_{S2}
D	-50	81	45	7.7313×10^3	110	5.8929×10^3
	-25	99	55	9.4688×10^3	135	7.2586×10^3
	+25	128	71	1.2224×10^3	174	9.4553×10^3
	+50	140	78	1.3391×10^4	191	1.0397×10^3
r_1	-50	82	46	7.8844×10^3	113	6.1921×10^3
	-25	100	55	9.5318×10^3	136	7.3950×10^3
	+25	127	70	1.2175×10^4	174	9.3324×10^3
	+50	139	77	1.3301×10^4	189	1.0159×10^3
r_2	-50	113	63	1.0824×10^4	154	8.3141×10^3
	-25	114	63	1.0879×10^4	155	8.3681×10^3
	+25	115	64	1.0988×10^4	157	8.4749×10^3
	+50	116	64	1.1042×10^4	158	8.5278×10^3
h_v	-50	144	80	8.7060×10^3	185	7.1529×10^3
	-25	127	70	9.8828×10^3	169	7.8141×10^3
	+25	105	58	1.1892×10^4	146	8.9869×10^3
	+50	98	54	1.2779×10^4	137	9.5175×10^3
h_b	-50	117	83	1.0704×10^4	174	7.6003×10^3
	-25	115	72	1.0834×10^4	164	8.0219×10^3
	+25	114	57	1.1013×10^4	149	8.8026×10^3
	+50	113	51	1.1077×10^4	143	9.1673×10^3
s_1	-50	116	45	1.0736×10^4	156	8.4217×10^3
	-25	115	56	1.0851×10^4	156	8.4217×10^3
	+25	114	69	1.0995×10^4	156	8.4217×10^3
	+50	114	74	1.1044×10^4	156	8.4217×10^3
S_c	-50	119	66	1.0526×10^4	158	8.3433×10^3
	-25	117	65	1.0732×10^4	157	8.3826×10^3
	+25	113	62	1.1132×10^4	155	8.4606×10^3
	+50	110	61	1.1327×10^4	155	8.4993×10^3
d_c	-50	115	64	1.0919×10^4	156	8.4188×10^3
	-25	114	64	1.0927×10^4	156	8.4203×10^3
	+25	114	64	1.0941×10^4	156	8.4230×10^3
	+50	114	63	1.0949×10^4	156	8.4246×10^3
P	-50	114	64	1.0934×10^4	156	8.2717×10^3
	-25	114	64	1.0934×10^4	156	8.3467×10^3
	+25	114	64	1.0934×10^4	156	8.4967×10^3
	+50	114	64	1.0934×10^4	156	8.8717×10^3

Parameters		Q*	B*	TC^*_{s1}	Q^*_0	TC^*_{S2}
D						
	Mean	112	62.25	4.96	152.5	5.91
	Variance	542.42	203.95	65.44	1014.42	9.53
	Lacunarity	1.043241	1.052631	3.681572	1.043619	1.272743
r_1						
	Mean	112	62	4.99	153	5.9853
	Variance	501.76	148.6	14.1376	906.01	9.4814
	Lacunarity	1.04	1.038658	1.567773	1.038703	1.2646
r_2						
	Mean	114.5	63.5	1.0933	156	8.4212
	Variance	1.2544	0.25	1.4852	4.9996	0.0196
	Lacunarity	1.0000	1.0001	2.2425	1.0002	1.0002
h_v						
	Mean	118.5	65.5	5.2639	159.25	8.3678
	Variance	331.24	104.65	16.4185	357.21	0.8717
	Lacunarity	1.02358	1.02439	1.592541	1.014	1.0124
h_b						
	Mean	114.75	65.75	1.0907	157.5	8.398
	Variance	2.1874	158.14	1.3331	149.33	0.3832
	Lacunarity	1.0001	1.03658	2.1206	1.006	1.0543
S_1						
	Mean	114.75	61	1.2821	156	8.4217
	Variance	0.6874	128.5	0.0366	0	0
	Lacunarity	1.0001	1.0345	1.0223	1	1
S_c						
	Mean	114.75	63.5	1.0929	156.25	8.4214
	Variance	12.1849	4.2497	0.0009	1.6874	0.0037
	Lacunarity	1.0009	1.001	1.000753	1.0001	1.0001
d_c						
	Mean	114.25	63.75	1.0934	156	8.4216
	Variance	0.1874	0.1874	3.5747	0	2.8723
	Lacunarity	1.0000	1.0000	3.99	1	1.0404
P						
	Mean	114	64	1.0934	156	8.4967
	Variance	0	0	0	0	0.0487
	Lacunarity	1	1	1	1	1.0006

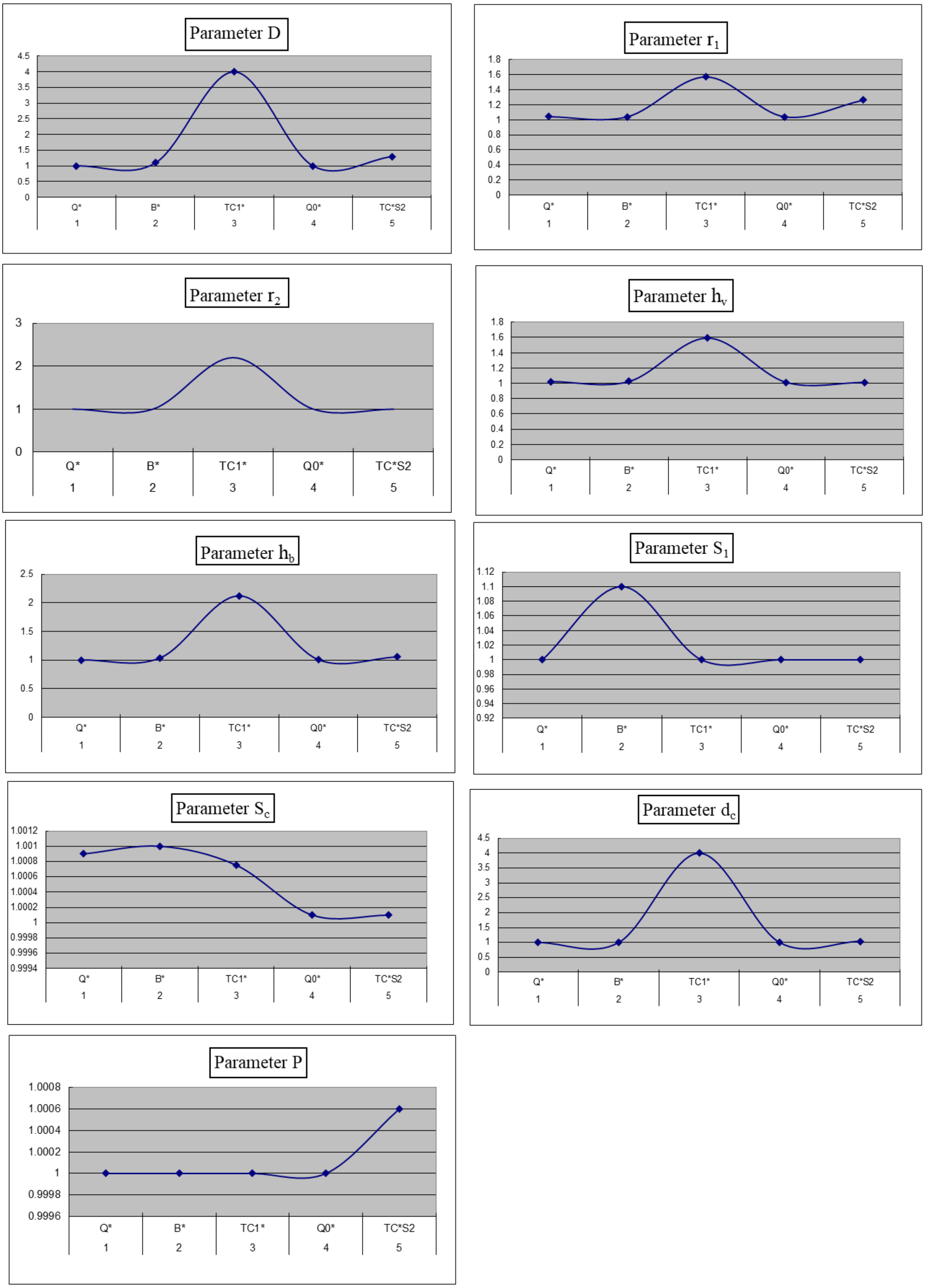

Figure.1. Graphical representation of different parameters of EOQ MODEL