A Mathematical Model for the Effect of Parathyroid Hormone using Fuzzy Log-Normal Distribution

A. Venkatesh, S. Prakasam

Abstract— The theoretical study for the effect of parathyroid hormone secretion in chronic renal failure was determined. Formulae for fuzzy mean and variance of log-normal distribution and its alpha cuts were presented. Using the fuzzy log-normal distribution model, we showed that the effect of parathyroid hormone secretion is reasonably higher when the test termination time increases.

Keywords: Log - normal distribution, Parathyroid hormone, Mean, Variance.

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1. INTRODUCTION

Models exemplify our beliefs regarding how the world functions. In mathematical modeling, we explain those attitudes to the talking of mathematics [7]. Mathematical modeling is a development by which a real world problem is described by a mathematical formulation. A main problem is to find a suitable mathematical formulation [2]. Model that is bright to correctly forecast the behavior of a system allowed engineers to program new cellular performance without having to perform large numbers of the test and error experiments [1].

Many mathematical models have been projected to describe bone remodeling development. In human, maintenance of fitting concentrations calcium ion in the extra cellular flowing requires the activity of two hormones, parathyroid hormone and a derivate of vitamin D called calcitriol [8]. We have beforehand established a consistent method of quantifying PTH secretion in vivo in normal humans and in hypo and primary hypercalcemics based on in order citrate and calcium clamping. The calcium set-point in patients with secondary hyperparathyroidism due to chronic renal failure was created to be normal compared to healthy controls, although the basal levels of PTH were much higher than normal [9]. A Calcimimetic drug has deferred the treatment options for patients with renal Hyperparathyroidism, but parathyroidectomy remains necessary for many patients [11].

Bone disease is very significant for people with kidney failure [13]. Although secondary HPT is an important difficulty of CKD with significant burdens on health and monetary resources, it remained under- recognized and under- diagnosed at previous stages of CKD [10].

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Log-normal distribution has been broadly applied to many dissimilar aspects of life sciences, including biology, ecology, geology and meteorology as well as in economics, finance, and risk analysis [4]. The two parameter log-normal distribution takes its name from the basic property that the logarithm of the random variable is distributed according to a Normal or Gaussian distribution [5]. In this paper, we study the fuzzy models to calculate the expected mean and variance for the effect of parathyroid hormone secretion in chronic renal failure using fuzzy log-normal distribution.

2. FUZZY LOG-NORMAL DISTRIBUTION

One way to quality a random variable follows a log-normal distribution is to say that its logarithm is normally distributed [6]. The log-normal distribution is given by

$$f(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{In\,x-\mu}{\sigma}\right)^2}$$

where. the variable $x > 0$ and the parameter μ and $\sigma > 0$ all are real numbers.

It is sometimes denoted $\Lambda(\mu, \sigma^2)$ in the same spirit as we often a normally distributed variable by $N(\mu, \sigma^2)$ [3]. The cumulative distribution for the log-normal distribution is given by

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^x \frac{1}{t}\, e^{-\frac{1}{2}\left(\frac{In\,x-\mu}{\sigma}\right)^2} dt$$

The coefficients of skewness $\gamma_1 = \sqrt{e^{\sigma^2} - 1}\,(e^{\sigma^2} + 2)$.

More generally algebraic moments of the log-normal distribution are given by

$$\mu_k^1 = E(x^k) = e^{k\mu + \frac{k^2\sigma^2}{2}}$$

The alpha cut of fuzzy mean life time is $\bar{E}(x) = \{\bar{E}_l(x), \bar{E}_u(x)\}$

Where, $\bar{E}_l(x) = \min\left\{e^{\bar{\mu}+\frac{\bar{\sigma}^2}{2}}\right\}$ and $\bar{E}_u(x) = \max\left\{e^{\bar{\mu}+\frac{\bar{\sigma}^2}{2}}\right\}$.

The alpha cut of fuzzy variance is $\bar{V}(x) = \{\bar{V}_l(x), \bar{V}_u(x)\}$

Where, $\bar{V}_l(x) = \min\left\{e^{2\bar{\mu}+\bar{\sigma}^2}(e^{\bar{\sigma}^2} - 1)\right\}$ and $\bar{V}_u(x) = \max\left\{e^{2\bar{\mu}+\bar{\sigma}^2}(e^{\bar{\sigma}^2} - 1)\right\}$.

3. APPLICATION

Let us consider a study by Jesper C.K, Anne Q. R, et al [9] for the effect of parathyroid hormone secretion in chronic renal failure. Blood samples for measurements of serum PTH were drawn at 15 minute interval. The levels of serum PTH obtained during 120 minutes by 12 patients (4 Females and 8 males) were taken.

Table.3.1. Level of Serum Parathyroid Hormone By 12 Patients

Time (min)	0	15	30	45	60	75	90	105	120
S-PTH (pmol/l)	20	58	23	48	46	40	24	21	14

The scale and shape parameter of log-normal distribution for Table.3.1. are

$\sigma = 0.6023$, $\mu = 3.3055$.

Let the corresponding fuzzy triangular numbers are

$\bar{\sigma}$ = [0.3878, 0.6023, 0.8512] and $\bar{\mu}$ = [2.8202, 3.3055, 3.8487]

and the corresponding α- cut are given by

$\bar{\sigma}$ = [0.3878 + 0.2145 α, 0.8512 - 0.2489 α] and

$\bar{\mu}$ = [2.8202 + 0.4853 α, 3.8487 - 0.5432 α]

4. RESULTS

Table 4.1. Fuzzy Mean value for lower and upper alpha values

α	low σ	low μ	up σ	up μ	$E_l(x)$	$E_u(x)$
0	0.3878	2.08202	0.8512	3.8487	8.647017	67.42196
0.1	0.40925	2.13055	0.82631	3.79438	9.154932	62.53798
0.2	0.4307	2.17908	0.80142	3.74006	9.697142	58.04375
0.3	0.45215	2.22761	0.77653	3.68574	10.27619	53.90587
0.4	0.4736	2.27614	0.75164	3.63142	10.89483	50.09401
0.5	0.49505	2.32467	0.72675	3.5771	11.55603	46.58054
0.6	0.5165	2.3732	0.70186	3.52278	12.26299	43.34034
0.7	0.53795	2.42173	0.67697	3.46846	13.0192	40.35052
0.8	0.5594	2.47026	0.65208	3.41414	13.8284	37.59023
0.9	0.58085	2.51879	0.62719	3.35982	14.69465	35.04047
1	0.6023	2.56732	0.6023	3.3055	15.62235	32.6839

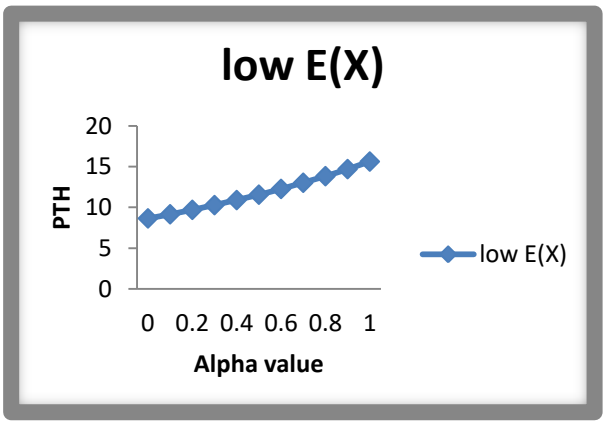

Fig.4.1. Fuzzy Mean value for lower alpha values

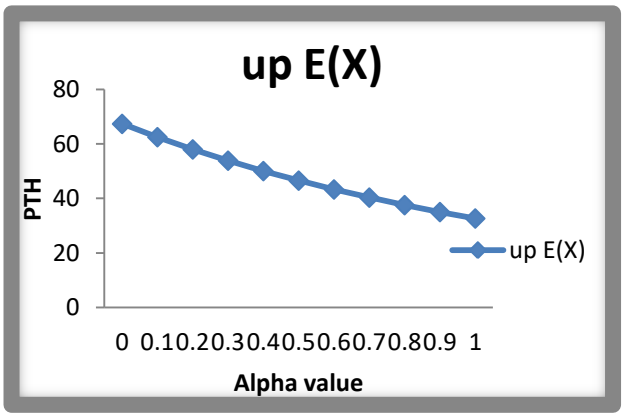

Fig.4.2. Fuzzy Survival rate for upper alpha values

Table 4.2 Fuzzy Variance value for lower and upper alpha values

α	low σ	low μ	up σ	up μ	$V_l(x)$	$V_u(x)$
0	0.3878	2.08202	0.8512	3.8487	12.13428	4835.667
0.1	0.40925	2.13055	0.82631	3.79438	15.28144	3830.39
0.2	0.4307	2.17908	0.80142	3.74006	19.16643	3034.859
0.3	0.45215	2.22761	0.77653	3.68574	23.95406	2404.85
0.4	0.4736	2.27614	0.75164	3.63142	29.84557	1905.601
0.5	0.49505	2.32467	0.72675	3.5771	37.08676	1509.75
0.6	0.5165	2.3732	0.70186	3.52278	45.97795	1195.738
0.7	0.53795	2.42173	0.67697	3.46846	56.88621	946.5576
0.8	0.5594	2.47026	0.65208	3.41414	70.2604	748.775
0.9	0.58085	2.51879	0.62719	3.35982	86.64966	591.7703
1	0.6023	2.56732	0.6023	3.3055	106.7261	467.1386

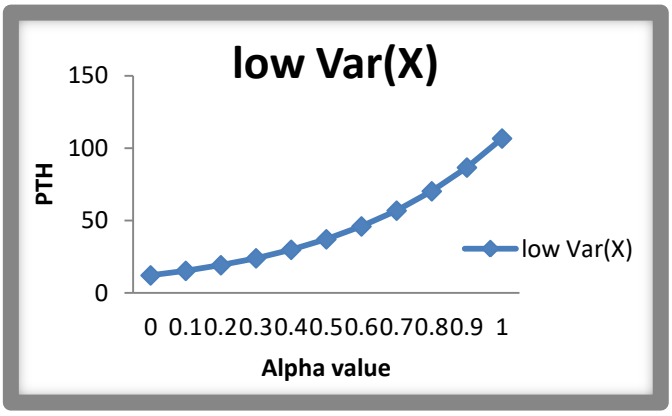

Fig.4.3.Fuzzy Variance value for Lower alpha values

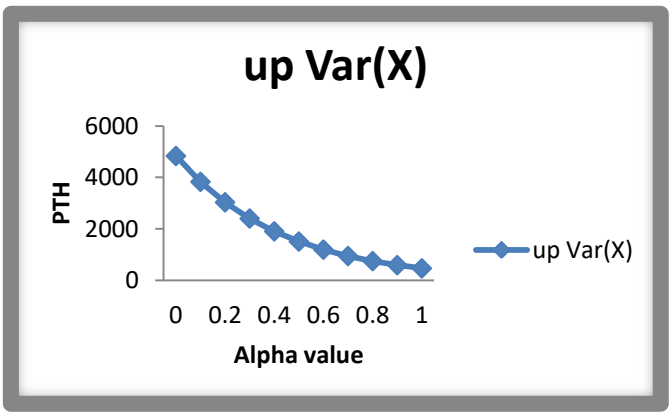

Fig.4.4.Fuzzy Variance value for upper alpha values

5. CONCLUSION

We exhibit the assessment of parathyroid hormones dynamics in chronic renal failure of 12 patients by the guesstimate of expectation and variance with two parameter log-normal distribution. The result shows that the fuzzy mean increases in lower alpha cuts and decreases in upper alpha cuts and also the fuzzy variance are decreases in lower alpha cuts and increases in upper alpha cuts for the effect of 12 patients parathyroid hormone secretion in chronic renal failure.

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