A Mathematical Model for Nutrient Requirements Using Fuzzy Optimization

A. Venkatesh, P. Manikandan

Abstract—In this paper, we discussed about the nutrient requirements of human beings from the vegetables which contains vitamins. We considered the vitamin levels in terms of trapezoidal fuzzy numbers and the fuzzy transportation problem was constructed using the cost of nutrients and edible portions of food stuffs. The optimal solution of fuzzy transportation problem has been obtained for the food stuffs that have vitamins with minimal cost.

Keywords: Fuzzy transportation problem, Trapezoidal fuzzy number, ranking function, Nutrients, Thiamine, Biotin

1. INTRODUCTION

The transportation problem is to find the cost to be from each source to each destination such that the total transportation cost is minimum. Purshoth Kumar and Ananthanarayanan [5] discussed a new ranking method for solving transportation of Fuzzy problems. Ranking methods map fuzzy numbers directly into the real number. Ranking function is used in various areas of fuzzy transportation problem[6]. Malini and Ananthanarayanan [3], [4], proposed fuzzy assignment problem using ranking of general trapezoidal fuzzy numbers.

Nutrition is a methodical discipline in which food is a major focus of interest. The simplest definition of nutrition is analysis of what happens to food once it enters the mouth and thereafter. A formal definition of nutrition is a processes by which the breathing organs receives and utilizes the resources necessary for growth; renewal and maintenance of body components. All the food contains some essential substances which perform important functions in our body. These essential substances to contribute by our food are called nutrients. Sarah Ohlhorst and Robert Russell et al [7] studied the effects and influence of nutritions in food and healthy life span. Nutrient values are usually assessed using a food composition data base. Food composition information is needed when calculating the composition of menus and recipes.

Transportation problem assign with the transportation of a commodity from ‘m’ sources to ‘n’ destinations. We consider the transportation of a different food as origins to number of vitamins as destinations. In a transportation problem using fuzzy, all the notations are fuzzy numbers. In this article, we use commonly used vegetables which are grouped together and the composition of food supplement contributions are given and a method is proposed for the ranking of general fuzzy trapezoidal numbers. Ranking of general fuzzy numbers is used in fuzzy transportation problem to minimize the cost of buying good healthy food. We have attained an optimal solution for the fuzzy transportation problem of minimal cost, using the fuzzy triangular membership function, by North West Corner rule, for the food stuffs that have rich vitamins with low cost. This study helps to increase the vitamins level in human body through the food items in which the transportation cost is minimized. This will helps the people to have a proper food supplement which is a part of their health and welfare.

2. PRELIMINARIES

Definition: 2.1
Let A be a classical set and μA(x) be a function from A to [0, 1]. A fuzzy set A with membership function[5] μA(x) is defined by A = {x, μA(x) | x ∈ A and μA(x) ∈ [0, 1]}.

Definition: 2.2
A fuzzy set A is defined on set of real numbers R is said to be fuzzy number if its membership functions μA : R → [0, 1] has the following characteristic.
A is normal. Its means that there exists an x ∈ R such that μA(x) = 1.
A is convex. Its means that for every x1, x2 ∈ R,
μA(θ x1 + (1 − θ) x2) ≥ min(μA(x1), μA(x2)), θ ∈ [0, 1]
μA is upper semi – continuous.

Definition: 2.3
A generalized fuzzy number A = (v1, v2, v3, v4, ω) is called as generalized trapezoidal fuzzy number if its membership function is given by

\[ μ_A(x) = \begin{cases} \omega (x - v_1) / (v_2 - v_1) ; & v_1 \leq x \leq v_2 \\ v_2 - v_1 ; & v_2 \leq x \leq v_3 \\ \omega (v_4 - x) / (v_4 - v_3) ; & v_3 \leq x \leq v_4 \\ 0 ; & \text{otherwise} \end{cases} \]

Definition: 2.4 Fuzzy Solutions
A set of allocation xij which satisfies the row and column restriction is called a fuzzy solution[8].

Definition: 2.5 Fuzzy optimal costs
A Fuzzy possible solution is said to be a fuzzy optimal solution if it decreases the total fuzzy transportation cost.
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Definition: 2.6 Ranking of Trapezoidal Fuzzy Numbers

We define another ranking method of general trapezoidal number is suggested using trapezoidal as reference point. Ranking method map fuzzy number directly into the real number. Let V be a general trapezoidal fuzzy number. The ranking of V is denoted by R(V) and it is calculated as follows:

\[ R(V) = \frac{1}{2} \left( \alpha (v_1 + v_2) + \frac{1-\alpha}{2}(2v_2 + v_3) \right) \]

\[ R(V) = \frac{1}{4} (\alpha (v_1 + v_4) + 2(1-\alpha)(v_2 + v_3)), \]

Where \( \alpha \in (0, 1) \).

3. MATHEMATICAL FORMULATION OF A FUZZY TRANSPORTATION PROBLEM

We consider a transportation problem based on fuzzy "m" origins and "n" fuzzy destinations. Assume that the transportation cost of single unit of outcome from \( i^{th} \) origin to \( j^{th} \) destination be denoted by \( c_{ij} = [c_1^{ij}, c_2^{ij}, c_3^{ij}, c_4^{ij}] \) the availability of commodity at origin

\[ s_i = [s_1^i, s_2^i, s_3^i, s_4^i] \] , commodity needed at the destinations \( j \) be \( d_j = [d_1^j, d_2^j, d_3^j, d_4^j] \)

The quantity transported from \( i^{th} \) origin to \( j^{th} \) destination

\[ x_{ij} = [x_1^{ij}, x_2^{ij}, x_3^{ij}, x_4^{ij}] \]

The linear programming problem indicating the fuzzy transportation is given by

Minimize \( Z = \frac{1}{4} \sum_{i=1}^{m} \sum_{j=1}^{n} [c_1^{ij}, c_2^{ij}, c_3^{ij}, c_4^{ij}] \cdot [x_1^{ij}, x_2^{ij}, x_3^{ij}, x_4^{ij}] \)

Subject to the constraints

\[ \sum_{i=1}^{m} [x_1^{ij}, x_2^{ij}, x_3^{ij}, x_4^{ij}] = [s_1^j, s_2^j, s_3^j, s_4^j] \] for \( i = 1, 2, \ldots, m \)

4. APPLICATION

Nutrition is the procedure by which food is taken in and is utilized by body. Here we consider the vitamins Niacin, Riboflavin, Thiamine and Biotinin from the vegetables Brinjal, Cauliflower, Drumstick and Capsicum. The data were collected and the amount of food supplements in the food products were recorded, from the Nutritive rate of Indian foods given by National Institute of Nutrition [1], and food composition tables [2].

The cost per 100 gms of vitamins for each food item is taken as supply and edible portion of vegetables per 100 gms of each vitamin is taken as demand. The Nutrition content for every food item is given in Table 4.1.

Table 4.1: Fuzzy transportation of Nutrition content of vegetable items

<table>
<thead>
<tr>
<th>Food</th>
<th>Thiamine</th>
<th>Riboflavin</th>
<th>Niacin</th>
<th>Biotin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brinjal</td>
<td>(0.03300,0.03767,0.04234,0.04701)</td>
<td>(0.08800,0.09600,0.10400,0.11200)</td>
<td>(0.45000,0.50333,0.55666,0.60999)</td>
<td>(1.90000,2.32667,2.75334,3.18001)</td>
</tr>
<tr>
<td>Cauliflower</td>
<td>(0.03700,0.03900,0.04100,0.04300)</td>
<td>(0.06300,0.06767,0.07234,0.07701)</td>
<td>(0.29000,0.30333,0.31666,0.32999)</td>
<td>(2.18000,2.37333,2.56666,2.75999)</td>
</tr>
<tr>
<td>Drumstick</td>
<td>(0.03700,0.03933,0.04066,0.04199)</td>
<td>(0.06300,0.06767,0.07234,0.07701)</td>
<td>(0.59000,0.61000,0.63000,0.65000)</td>
<td>(3.84000,4.14000,4.44000,4.74000)</td>
</tr>
<tr>
<td>Capsicum (green)</td>
<td>(0.04300,0.04767,0.05234,0.05701)</td>
<td>(0.01900,0.02633,0.03366,0.04099)</td>
<td>(0.51000,0.54333,0.57666,0.60999)</td>
<td>(4.13000,4.43000,4.73000,5.03000)</td>
</tr>
<tr>
<td>Demand (cost of nutrition per 100 gm)</td>
<td>(0.00001,0.00001,0.00001,0.00001)</td>
<td>(0.00001,0.00001,0.00001,0.00001)</td>
<td>(0.00009,0.00009,0.00009,0.00009)</td>
<td>(0.00057,0.00063,0.00068,0.00073)</td>
</tr>
</tbody>
</table>

\[ \sum_{i=1}^{m} [x_1^{ij}, x_2^{ij}, x_3^{ij}, x_4^{ij}] = [d_1^j, d_2^j, d_3^j, d_4^j] \] for \( j = 1,2, \ldots, n \)

The given total Fuzzy transportation problem is said to be balanced if

\[ \sum_{i=1}^{m} [s_1^j, s_2^j, s_3^j, s_4^j] = \sum_{j=1}^{n} [d_1^j, d_2^j, d_3^j, d_4^j] \]

(i.e.) the total fuzzy supply is equal to fuzzy demand.
The fuzzy transportation problem for food supplement content of food items can be formulated in the following mathematical form

\[ R(V) = \frac{1}{4} \left[ \alpha (v_1 + v_3) + 2(1 - \alpha)(v_2 + v_3) \right]. \]

\[ R(V) = \frac{1}{4} [0 + 2(0.03767 + 0.04234)] = 0.04000 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.09600 + 0.10400)] = 0.10000 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.50333 + 0.55666)] = 0.53000 \]

\[ R(V) = \frac{1}{4} [0 + 2(2.32667 + 2.75334)] = 2.54000 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.03900 + 0.04100)] = 0.04000 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.67676 + 0.72340)] = 0.07000 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.30333 + 0.31666)] = 0.31000 \]

\[ R(V) = \frac{1}{4} [0 + 2(2.37333 + 2.56666)] = 2.47000 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.00009 + 0.00010)] = 0.00010 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.00006 + 0.00006)] = 0.00006 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.00039 + 0.00041)] = 0.00040 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.00020 + 0.00022)] = 0.00021 \]

\[ \text{Supply} \]

\[ \text{Demand} \]

\[ R(V) = \frac{1}{4} [0 + 2(4.73000 + 4.73000)] = 4.78000 \]

\[ R(V) = \frac{1}{4} [0 + 2(4.43000 + 4.43000)] = 4.54000 \]

\[ R(V) = \frac{1}{4} [0 + 2(4.14000 + 4.44000)] = 4.29000 \]

\[ R(V) = \frac{1}{4} [0 + 2(2.54000 + 0.57666)] = 0.56000 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.00001 + 0.00001)] = 0.00001 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.00001 + 0.00001)] = 0.00001 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.00009 + 0.00009)] = 0.00009 \]

\[ R(V) = \frac{1}{4} [0 + 2(0.00063 + 0.00063)] = 0.00066 \]

\[ \text{Brinjal} \]

\[ \text{Cauliflower} \]

\[ \text{Drumstick} \]

\[ \text{Capsicum (green)} \]

\[ \text{when} \ \tau = 1 \ \text{and} \ \alpha = 0. \ \ R(V) = \frac{1}{4} [0 + 2(v_2 + v_3)] \]

\[ , \text{Where} \ \alpha \in [0,1]. \]

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**Fig. 4.1 Nutrition content of vegetable items after applying ranking technique**
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Table 4.2: Optimum solution by North West Corner method

<table>
<thead>
<tr>
<th>Food</th>
<th>Thiamine</th>
<th>Riboflavin</th>
<th>Niacin</th>
<th>Biotin</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brinjal</td>
<td>0.00001</td>
<td>0.01000</td>
<td>0.00008</td>
<td>0.53000</td>
<td>2.54000</td>
</tr>
<tr>
<td>Cauliflower</td>
<td>0.00001</td>
<td>0.07000</td>
<td>0.00005</td>
<td>2.47000</td>
<td>0.00006</td>
</tr>
<tr>
<td>Drumstick</td>
<td>0.00001</td>
<td>0.62000</td>
<td>0.00040</td>
<td>4.29000</td>
<td>0.00040</td>
</tr>
<tr>
<td>Capsicum (green)</td>
<td>0.00001</td>
<td>0.56000</td>
<td>0.00021</td>
<td>4.59000</td>
<td>0.00021</td>
</tr>
<tr>
<td>Demand</td>
<td>0.00001</td>
<td>0.00001</td>
<td>0.00009</td>
<td>0.00066</td>
<td>0.00077</td>
</tr>
</tbody>
</table>

The total minimum cost for food supplements in balanced diet is

\[
\text{Min } Z = (0.04000)(0.00001) + (0.10000)(0.00001) + (0.53000)(0.00008) + (0.31000)(0.00001) + (2.47000)(0.00005) + (4.29000)(0.00040) + (4.59000)(0.00021)
\]

\[
\text{Min } Z = \text{Rs. 0.00285 per mg}
\]

5. CONCLUSION

In this study, we considered the vegetables having nutrient content as transportation cost, cost of nutrients per 100 gms as supply, cost of edible portion of vegetables per 100 gms as demand and each has considered as trapezoidal fuzzy numbers. The trapezoidal fuzzy numbers of nutrient content and vegetable items are converted into a real numbers after applying ranking technique. The optimal solution of the transportation problem has been obtained by using North west corner method for the vegetables that have rich nutrients with minimal cost. These data suggest that a preservative role for vegetables intake to raise vitamins in a human body and also it conserves the healthy diet.

REFERENCES