

A Fuzzy Transportation Problem Model for Well Balanced Diet Plan

A. Venkatesh, A. Britto Manoj

Abstract— In this paper, we considered each type of nutrient content and the price of food items as fuzzy sets. We proposed a fuzzy transportation problem in which hexagonal fuzzy number represents the cost, availability and demand of food items. The optimal solution for the well balanced diet plan has been obtained for the food items that have rich nutrient content with minimal cost.

Keywords: Fuzzy transportation problem, Hexagonal fuzzy number, Ranking function, human balanced diet, nutrients AMS
Subject classification: 90B06, 90C08, 90C70, 90C90, 97M40.

1. INTRODUCTION

In our daily activity fuzzy numbers create an impact in optimization. The fundamental association of study in ranking of generalized hexagonal fuzzy number in transportation gives the value of cost in generalized hexagonal fuzzy quantities. The ranking method was first discussed by Jain [2], Yager [9]. They also proposed indices which may be employed for the purpose of ordering fuzzy quantities in $[0, 1]$. Thamaraiselvi and Shanthi [6], [7], proposed a method for optimal solution of fuzzy transportation problem using hexagonal fuzzy numbers and solving fuzzy transportation problem with generalized hexagonal fuzzy numbers. Rajarajeswari and Sahayasudha [5], proposed ordering of generalized hexagonal fuzzy numbers using rank.

The dietary habits of the people vary according to the economic and life-style condition. An understanding of nutrient gaps or excesses in the dietaries would help in planning diets to overcome diet related morbidities and thus promote health of the people. Information on food composition patterns is also essential for assessing the food needs of the population groups at national and regional level. The knowledge that nutrition that we choose to eat influence our health, well-being and quality of life.

Almonds, dates, fig, groundnut, and walnut are commonly used in India and their nutritive value is so high. Almonds are a rich source of protein that is not of high biological value. Almonds are an accomplished connection of antioxidant. Dates contain valuable amount of vitamins, minerals, energy, fibre, calcium, iron, magnesium, and zinc. Dates can be consumed in both natural and dry forms. Figs contain excellent form of calcium and fibre which helps to lower cholesterol and control blood sugar. Groundnut is a healthy food, and it comes from natural source. It contains high protein content, numerous vitamins, minerals, and plant

composition. It curtails the danger of heart disease and gallstones. Groundnut is used as a component of weight loss diet. Walnut is one of the essential healthy foods used in the aspect of control the weight and to increase brain health. The intake of health foods such as almonds, dates, fig, groundnut, and walnut substantially influence the function of the body, protect against disease, restore health and as a whole to increase the virtue of life.

In this research paper, we focus on nuts, since they are a wealthy source of protein, fat and a rich source of energy. The balance of healthy food contribution such as almond, dates, fig, groundnut, and walnut are taken. The ranking of generalized hexagonal fuzzy numbers is used in fuzzy transportation problem to minimize the cost of buying this excellent nutrition food. The objective of this paper is to discuss the development of health by using nutrient recommendations from adequate to optimum nutrition.

2. PRELIMINARIES

Definition: 2.1

If x is a set of objects denoted by X , then a fuzzy set A in X is defined as a set of ordered pairs $A = \{x, \mu_A(x) / x \in X\}$ where $\mu_A(x)$ is called the membership function [4] for the fuzzy set A . The membership function maps every element of X to a membership value between 0 and 1.

Definition: 2.2

A fuzzy set A is defined on universal set of real numbers is known as generalized fuzzy number provided the membership functions satisfies the following conditions:

- $\mu_A(x) = R \rightarrow [0,1]$ is continuous.
- $\mu_A(x) = 0$ for all $x \in A(-\infty, a] \cup [d, \infty)$
- $\mu_A(x)$ is strictly increasing on $[a_1, a_2]$ and strictly decreasing on $[a_3, a_4]$.
- $\mu_A(x) = \omega$ for all $x \in [a_2, a_3]$, where $0 \leq \omega \leq 1$.

Definition: 2.3 Hexagonal Fuzzy Number

A fuzzy number \tilde{A}_H is a hexagonal fuzzy number [6] denoted by $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where a_1, a_2, a_3, a_4, a_5 and a_6 are real numbers and its membership function is given below,

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$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2} \left(\frac{x - a_1}{a_2 - a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x - a_2}{a_3 - a_2} \right), & \text{for } a_2 \leq x \leq a_3 \\ 1, & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left(\frac{x - a_4}{a_5 - a_4} \right), & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2} \left(\frac{a_6 - x}{a_6 - a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ 0, & \text{otherwise} \end{cases}$$

Definition: 2.4 Generalized Hexagonal Fuzzy number

If a generalized hexagonal fuzzy number [6] denoted by $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where a_1, a_2, a_3, a_4, a_5 and a_6 are real numbers and ω is its maximum membership degree, its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{\omega}{2} \left(\frac{x - a_1}{a_2 - a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ \frac{\omega}{2} + \frac{\omega}{2} \left(\frac{x - a_2}{a_3 - a_2} \right), & \text{for } a_2 \leq x \leq a_3 \\ \omega, & \text{for } a_3 \leq x \leq a_4 \\ \omega - \frac{\omega}{2} \left(\frac{x - a_4}{a_5 - a_4} \right), & \text{for } a_4 \leq x \leq a_5 \\ \frac{\omega}{2} \left(\frac{a_6 - x}{a_6 - a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ 0, & \text{otherwise} \end{cases}$$

Definition: 2.5 Ranking of Hexagonal Fuzzy Numbers

The ranking method[5] map fuzzy number directly into the real line. Let \tilde{A}_H be a generalized hexagonal fuzzy number. The ranking of \tilde{A}_H is denoted by $R(\tilde{A}_H)$ and it is calculated as follows:

$$R(\tilde{A}_H) = \left[\frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18} \right]$$

3. MATHEMATICAL FORMULATION OF A TRANSPORTATION PROBLEM [8]

Let us assume that there are m sources and n destinations.

Let a_i be the supply (capacity) at the source i, b_j be the demand at destination j, C_{ij} be the unit transportation cost from source i to destination j and X_{ij} be the number of units shifted from source i to destination j.

The transportation problem can be expressed mathematically as

$$\text{Minimize } Z = \sum_{i=1}^m C_{ij} \sum_{j=1}^n X_{ij}$$

Subject to the constraints

$$\sum_{j=1}^n X_{ij} = a_i, \quad i = 1, 2, 3, \dots, m.$$

$$\sum_{i=1}^m X_{ij} = b_j, \quad j = 1, 2, 3, \dots, n.$$

and $X_{ij} \geq 0$, for all i and j.

The two sets of constraints will be consistent if

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

(total supply) (total demand)

which is the necessary and sufficient condition for a transportation problem to have a feasible solution.

4. APPLICATION

In this paper, we discuss about the nutritive value of nuts such as almond, dates, fig, groundnut, and walnut which is a concentrated source of protein, fats, total fibre, carbohydrates, and calcium. Real data were collected and the amounts of health food in the food products were recorded, from the Nutritive value of Indian foods given by National Institute of Nutrition [1], IFC tables [3]. The smallest, natural and largest content of protein, fats, total fibre, carbohydrates and calcium in each food item is considered as hexagonal fuzzy number respectively. The cost per 100 gm of healthy food for each food item is taken as supply and edible portion per 100 gm of each healthy food is taken as demand which is given in the table 4.1.



Table.4.1 Nutrition content for food items

| Foods | Protein | Fat | Total Fibre | Carbo-hydrates | Calcium | Supply (cost of dible portion of food stuff per 100 gm) |
|---------------------------------------|---|---|---|---|---|---|
| Almond | (18.37, 18.39, 18.41,18.43, 18.45, 18.47) | (58.45, 58.47, 58.49,58.51, 58.53, 58.55) | (12.75, 12.87, 12.99,13.11, 13.23, 13.35) | (2.80, 2.90, 3.00, 3.10, 3.20, 3.30) | (217.80, 1.88, 225.96, 230.04, 34.12, 238.20) | (62.96, 63.14, 63.31, 63.50, 63.69, 63.85) |
| Dates | (2.21, 2.31, 2.41,2.51, 2.61, 2.71) | (0.32, 0.33, 0.34, 0.35, 0.36, 0.37) | (8.72, 8.81, 8.90, 8.99, 9.08, 9.17) | (74.39, 74.60, 74.81,75.02, 75.23, 75.44) | (66.69, 68.49, 70.29, 72.09, 73.89, 75.69) | (17.13, 17.22, 17.30, 17.38, 17.47, 17.55) |
| Fig | (1.81, 1.90, 1.99,2.08, 2.17, 2.26) | (0.31, 0.33, 0.35, 0.37, 0.39, 0.41) | (4.23, 4.39, 4.55,4.71, 4.87, 5.03) | (15.15, 15.60, 16.05,16.50, 16.95, 17.40) | (62.04, 68.63, 75.22, 81.81, 88.40, 94.99) | (14.02, 14.48, 14.96, 15.43, 15.91, 16.38) |
| Ground nut | (22.80, 23.14, 23.48,23.82, 24.16, 24.50) | (39.34, 39.46, 39.58,39.70, 39.82, 39.94) | (10.20, 10.27, 10.34,10.41, 10.48, 10.55) | (16.94, 17.07, 17.20,17.33, 17.46, 17.59) | (46.63, 49.58, 52.53, 55.48, 58.43, 61.38) | (8.03, 8.09, 8.15, 8.21, 8.27, 8.34) |
| Walnut | (14.30, 14.55, 14.80,15.05, 15.30, 15.55) | (64.22, 64.24, 64.26,64.28, 64.30, 64.32) | (5.20, 5.28, 5.36, 5.44, 5.52, 5.60) | (9.45, 9.73, 10.01,10.29, 10.57, 10.85) | (99.10, 01.46, 103.82, 06.18, 108.54, 110.90) | (74.62, 75.11, 75.63, 76.12, 76.65, 77.15) |
| Demand (cost of nutrition per 100 gm) | (27.60, 27.93, 28.24,28.56, 28.89, 29.22) | (94.93, 94.98, 95.04,95.09, 95.14, 95.20) | (18.24, 18.50, 18.79,19.06, 19.35, 19.61) | (35.71, 36.35, 36.99,37.63, 38.28, 38.90) | (0.28, 0.28, 0.29,0.30, 0.33, 0.34) | |

Let \tilde{A}_H be a generalized hexagonal fuzzy number. The ranking of \tilde{A}_H is denoted by $R(\tilde{A}_H)$ and it is calculated as follows: $R(\tilde{A}_H) = \left[\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18} \right]$

| | |
|---|--|
| $R(\tilde{A}_H) = \frac{1}{18} [331.56] = 18.42$ | $R(\tilde{A}_H) = \frac{1}{18} [186.75] = 10.38$ |
| $R(\tilde{A}_H) = \frac{1}{18} [44.28] = 2.46$ | $R(\tilde{A}_H) = \frac{1}{18} [97.20] = 5.40$ |
| $R(\tilde{A}_H) = \frac{1}{18} [36.63] = 2.04$ | $R(\tilde{A}_H) = \frac{1}{18} [54.90] = 3.05$ |
| $R(\tilde{A}_H) = \frac{1}{18} [425.70] = 23.65$ | $R(\tilde{A}_H) = \frac{1}{18} [1348.47] = 74.92$ |
| $R(\tilde{A}_H) = \frac{1}{18} [268.65] = 14.93$ | $R(\tilde{A}_H) = \frac{1}{18} [292.95] = 16.28$ |
| $R(\tilde{A}_H) = \frac{1}{18} [1053.00] = 58.50$ | $R(\tilde{A}_H) = \frac{1}{18} [310.77] = 17.27$ |
| $R(\tilde{A}_H) = \frac{1}{18} [6.21] = 0.35$ | $R(\tilde{A}_H) = \frac{1}{18} [182.70] = 10.15$ |
| $R(\tilde{A}_H) = \frac{1}{18} [6.48] = 0.36$ | $R(\tilde{A}_H) = \frac{1}{18} [4104.00] = 228.00$ |
| $R(\tilde{A}_H) = \frac{1}{18} [713.52] = 39.64$ | $R(\tilde{A}_H) = \frac{1}{18} [1281.42] = 71.19$ |

| | |
|---|--|
| $R(\tilde{A}_H) = \frac{1}{18} [1156.86] = 64.27$ | $R(\tilde{A}_H) = \frac{1}{18} [1413.27] = 78.52$ |
| $R(\tilde{A}_H) = \frac{1}{18} [234.90] = 13.05$ | $R(\tilde{A}_H) = \frac{1}{18} [972.09] = 54.01$ |
| $R(\tilde{A}_H) = \frac{1}{18} [161.01] = 8.95$ | $R(\tilde{A}_H) = \frac{1}{18} [1890.00] = 105.00$ |
| $R(\tilde{A}_H) = \frac{1}{18} [83.34] = 4.63$ | |
| Supply | Demand |
| $R(\tilde{A}_H) = \frac{1}{18} [1141.35] = 63.41$ | $R(\tilde{A}_H) = \frac{1}{18} [511.30] = 28.41$ |
| $R(\tilde{A}_H) = \frac{1}{18} [312.15] = 17.34$ | $R(\tilde{A}_H) = \frac{1}{18} [1711.14] = 95.06$ |
| $R(\tilde{A}_H) = \frac{1}{18} [273.53] = 15.20$ | $R(\tilde{A}_H) = \frac{1}{18} [340.65] = 18.93$ |
| $R(\tilde{A}_H) = \frac{1}{18} [147.26] = 8.18$ | $R(\tilde{A}_H) = \frac{1}{18} [671.59] = 37.31$ |
| $R(\tilde{A}_H) = \frac{1}{18} [1365.82] = 75.88$ | $R(\tilde{A}_H) = \frac{1}{18} [5.43] = 0.30$ |



Table 4.2: Optimum solution by Vogel's Approximation method

| Foods | Protein | Fat | Total Fibre | Carbohydrates | Calcium | Supply |
|-----------|--------------|--------------|-------------|---------------|-------------|--------|
| Almond | 18.42 | 26.10, 58.50 | 13.05 | 37.31, 3.05 | 228.00 | 63.41 |
| Dates | 2.46 | 17.34, 0.35 | 8.95 | 74.92 | 71.19 | 17.34 |
| Fig | 2.04 | 15.20, 0.36 | 4.63 | 16.28 | 78.52 | 15.20 |
| Groundnut | 23.65 | 39.64 | 7.88, 10.38 | 17.27 | 0.30, 54.01 | 8.18 |
| Walnut | 28.41, 14.93 | 36.42, 64.27 | 11.05, 5.40 | 10.15 | 105.00 | 75.88 |
| Demand | 28.41 | 95.06 | 18.93 | 37.31 | 0.30 | |

Table 4.3: Defuzzification

| Foods | Protein | Fat | Total Fibre | Carbo-hydrate s | Calcium | Supply (cost of Edible portion of food stuff per 100 gm) |
|---------------------------------------|---|--|--|---|--|--|
| Almond | (18.37, 18.39, 18.41,18.43, 18.45, 18.47) | (26.06, 26.08, 26.10, 26.12, 26.14, 26.16) (58.45, 58.47, 58.49,58.51, 58.53, 58.55) | (12.75, 12.87, 12.99,13.11, 13.23, 13.35) | (35.71, 36.35, 36.99,37.63, 38.28, 38.90) (2.80, 2.90, 3.00,3.10, 3.20, 3.30) | (217.80, 221.88, 225.96,230.04, 234.12, 238.20) | (62.96, 63.14, 63.31,63.50, 63.69, 63.85) |
| Dates | (2.21, 2.31, 2.41,2.51, 2.61, 2.71) | (17.13, 17.22, 17.30,17.38, 17.47, 17.55) (0.32, 0.33, 0.34,0.35, 0.36, 0.37) | (8.72, 8.81, 8.90,8.99, 9.08, 9.17) | (74.39, 74.60, 74.81,75.02, 75.23, 75.44) | (66.69, 68.49, 70.29,72.09, 73.89, 75.69) | (17.13, 17.22, 17.30,17.38, 17.47, 17.55) |
| Fig | (1.81, 1.90, 1.99,2.08, 2.17, 2.26) | (14.02, 14.48, 14.96,15.43, 15.91, 16.38) (0.31, 0.33, 0.35,0.37, 0.39, 0.41) | (4.23, 4.39, 4.55,4.71, 4.87, 5.03) | (15.15, 15.60, 16.05,16.50, 16.95, 17.40) | (62.04, 68.63, 75.22,81.81, 88.40, 94.99) | (14.02, 14.48, 14.96,15.43, 15.91, 16.38) |
| Ground nut | (22.80, 23.14, 23.48,23.82, 24.16, 24.50) | (39.34, 39.46, 39.58,39.70, 39.82, 39.94) | (7.70, 7.77, 7.84, 7.91, 7.98, 8.05) (10.20, 10.27, 10.34,10.41, 10.48, 10.55) | (16.94, 17.07, 17.20,17.33, 17.46, 17.59) | (0.28, 0.28, 0.29, 0.30, 0.33, 0.34) (46.63, 49.58, 52.53,55.48, 58.43, 61.38) | (8.03, 8.09, 8.15,8.21, 8.27, 8.34) |
| Walnut | (27.60, 27.93, 28.24,28.56, 28.89, 29.22) (14.30, 14.55, 14.80,15.05, 15.30, 15.55) | (36.37, 36.39, 36.41,36.43, 36.45, 36.47) (64.22, 64.24, 64.26,64.28, 64.30, 64.32) | (10.86, 10.94, 11.02, 11.10, 11.18, 11.26) (5.20, 5.28, 5.36,5.44, 5.52, 5.60) | (9.45, 9.73, 10.01,10.29, 10.57, 10.85) | (99.10, 101.46, 103.82,106.18, 108.54, 110.90) | (74.62, 75.11, 75.63,76.12, 76.65, 77.15) |
| Demand (cost of nutrition per 100 gm) | (27.60, 27.93, 28.24,28.56, 28.89, 29.22) | (94.93, 94.98, 95.04, 95.09, 95.14, 95.20) | (18.24, 18.50, 18.79, 19.06, 19.35, 19.61) | (35.71, 36.35, 36.99,37.63, 38.28, 38.90) | (0.28, 0.28, 0.29, 0.30, 0.33, 0.34) | |

The total minimum cost for health food in balanced diet is

$$\text{Min } Z = (26.10) (58.50) + (37.31) (3.05) + (17.34) (0.35) + (15.20) (0.36) + (7.88) (10.38) + (0.30) (54.01) + (28.41) (14.93) + (36.42) (64.27) + (11.05) (5.40)$$

$$\text{Min } Z = \text{Rs.}4558.54$$



5. CONCLUSION

In this study, the cost of the healthy food in the given diet for the food stuffs that have rich nutrients with minimal cost has obtained by the optimal solution of the transportation problem by using ranking of generalized hexagonal fuzzy numbers. The given diet is to develop strength and well-being, to prevent disease and to restore health in individuals, families, communities, and the population.

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