Spectral Efficient Massive MIMO Multi-Cell  
5G Cellular Environment Using Optimal Linear Processing Schemes


Abstract:--- In this paper, the optimal scheduling of UEs per cell is carried out to increase the spectral efficiency of 5G wireless networks with massive MIMO antennas in multi-cell systems. The scheduling of UEs is carried out in terms of several system parameters. The scheduling is carried out by considering a multi-objective function that optimizes of arbitrary pilot reuse, power control and random user locations. Expressions are derived to validate uplink and downlink transmission with power control and random user locations to increase the performance of UEs. The inter-cell interferences are suppressed using linear processing schemes in a coordinated beamforming fashion.

Keywords: UE, Massive MIMO Antennas, linear Processing Schemes, System Parameters

1 INTRODUCTION

Due to increasing demand in the field of cellular communication, the demand for wireless services are increasing at rapid rate. The higher throughput in such services are achieved through higher spectral efficiency, which is achievable by the application of massive multiple-input multiple-output (MIMO) concept in the field of wireless telecommunication [1]. The concept of MIMO is operated in coherent manner with a single base station, equipped with multiple antenna elements. This provides improved spatial resolution and maintains the robustness against interference by allowing multi-user MIMO communication to multiple user equipments (UEs). The existing research focusses mainly on the establishment of the properties of physical layer i.e. acquiring channel state information, however it is limited due to the presence of channel coherence block and the impacts of SE and inter-cell interference [2]. The existing studies reveal that the multiplexing increases the overall energy efficiency massive MIMO [9], [11]. On other hand, the researchers focused on the problems related to resource allocation in MAC layer. This can be defined in terms of user scheduling. Hence, it is concluded that higher spectral efficiency can be made realizable, if both the physical and MAC layers are optimized jointly.

Generally, the resource allocation in massive MIMO [6] i.e. tremendous amount of multiplexing in MIMO is considered to have limited pilot sequences, which needs to be intelligently assigned between the UEs in order to reduce the interference. This is usually achieved through capitalization of the path loss differences and spatial correlation [7].

In this paper, the resource allocation is considered as an important task that includes allocation or scheduling of UEs per cell for spectral efficiency maximization [9] in multi-cell systems. This is measured in terms of pilot allocation [10], total number of antennas and length of coherence block [11]. Hence, mathematical expressions are derived for both uplink and downlink transmission to achieve optimal performance using optimal power control and random locations of UEs. The proposed method is compared with a zero-forcing (ZF) [12] [13] scheme and maximum-ratio (MR)[14] scheme to prove its spectral efficacy, where it eliminates the inter-cell interference using a coordinated beamforming[8] [4] in a fully distributed manner [5].

2 PILOT CONTAMINATION PROBLEM

Time-multiplexed pilots allows each user to transmit the orthogonal pilot sequence of a specific length for channel and symbol transmission during uplink or downlink data. To reduce the overhead, the pilot sequence reuse is handle between the adjacent cells. Such reuse increases the contamination of pilots by the user channel vectors in its adjacent cells. This causes interference and reduces the spectral efficiency. Hence, the transmission of pilot sequences sent in a synchronized way from each user end, which is regarded as a worst-case scenario for contamination of pilot symbols.

Consider an orthogonal matrix, whose column entries has orthogonal pilot sequences for transmission. The user channel is estimated, if the pilot sequence is reused at regular instances. It is further assumed that the transmitting powers are similar to the entire cell users that employ time-time-multiplexed pilots. Further, the transmitted power variations of each user are captivated into its own path loss coefficient. It is noted observed that the estimation of channel vectors is contaminated by the remaining channel vectors. It is seen that the transmission rate per user is considered as the function of overhead and SINR loss due to the contamination of pilot sequence. A larger value of pilot reuse factor reduces the effect associated with pilot contamination and increases the value of SINR at the rate of reduced spectral efficiency during transmission [15].

3 PROPOSED METHOD

This section provides the details of derivation and analysis of spectral efficiency of massive MIMO antennas, where each UEs are placed randomly in a multi-cell
environment. The details of analytical expressions required for the estimation of channels for both uplink and downlink with pilot signaling is derived in terms of power control and random locations. The proposed method uses superimposed pilots [15] to transmit the pilot symbol and data symbol at reduced power during the uplink process. If the number of symbols is greater than the total users during uplink, then each user is assigned with a unique orthogonal pilot using superimposed pilots. The selection of pilots usually considered from the columns of orthogonal matrix.

<table>
<thead>
<tr>
<th>UL pilots</th>
<th>Uplink data</th>
<th>Downlink data</th>
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**Figure 1: The transmission frames is divided into symbols for uplink and downlink transmission**

<table>
<thead>
<tr>
<th>Figure 2: Hexagonal grid – Coordinate system</th>
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<tr>
<td><img src="image" alt="Hexagonal Grid" /></td>
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</tbody>
</table>

### 3.1 Pilot-Based Channel Estimation

Consider a base station BS(j) that utilizes its antennas for precoding transmission in downlink and combined coherent reception in uplink, where it amplifies adaptively the desired clean signal and mitigates the noisy or interfering signals. Such an utilization requires certain understanding on UE channels. The pilot signaling is used for acquiring the channel state information, where the known signals are sent through UEs in a predefined way. However, the accuracy of acquisition of channel state information is regarded to be a challenging one in case of massive MIMO placed in multi-cell systems. In this regards, the resources needed for transmission are reused between the cells, since the inter-cell inference affects drastically the pilot signaling. Such signaling leads to contamination of pilot symbols and this reduces the quality of acquisition of channel state information and it offers the ability to rejects the so-called inter-cell interference.

It is assumed in the study that the pilot symbols used are considered to be similar in all cells, which arises due to the pilot contamination effects. The proposed method derives the power control and other properties associated with massive MIMO to reuse the pilot sequence in an arbitrary way. In this case, only the subset of pilots are used by each cell. It is seen from the Figure 1 that pilot signals spans each frame with B symbols, where the range of B lies between 1 and S. Here, pilot signal is considered as a deterministic vector v∈C^B and the symbol power entails unit magnitude for all the input entries i.e. |v|_2 = 1. The value [·]_is regarded as s^{th} element that lies between 1 and B. Further, it is assumed that the fixed pilot book (V) produces the entire pilot symbols, which is defined as

\[ V = \{v_1, v_2, ..., v_B\} \]

where,

\[ v^H_b v_b = \begin{cases} B & b_1 = b_2 \\ 0 & b_1 \neq b_2 \end{cases} \quad \text{and} \]

\[ (\cdot)^H \]

is regarded as the conjugate transpose.

Hence, the pilot signals (B) is suitable for the formation of an orthogonal basis function.

Consider a cell (l) having its UE (k), where the transmission of pilot signals is denoted as v_{lk} and the range of l_{lk} lies between [1,B], which is considered as the pilot book index. The pilot signal transmission over in uplink system over B symbols gives Y_{j} \in C^{MxB}, which is the uplink received signal at the base station (j) and it is expressed:

\[ Y_j = \sum_{i \in L} \sum_{k=1}^{K} \sqrt{p_{lk}} h_{jk} v^H_{lk} + N_j \]

where N_{j} \in C^{MxB} is said to have receiver’s additive noise at the time of pilot signaling.

The estimation of MMSE at each base station j of an uplink power controlled channel h_{jk} for an UE in a cell L, is given by,

\[ h_{jk}^{\text{eff}} = \frac{d_j(z_{jk})}{d_j(z_{jk})} Y_j (\Psi_j^{-1}) v_{lk}^* \]

where, \( \Psi_j \) is the normalized covariance matrix and \( d_j(z) \) is the deterministic function that provides the variance in relation with channel attenuation from a base station in a cell L.

### 3.2 Achievable spectral efficiency for uplink

The estimation of channel (in Eq.(3)) enables semi-coherent detection at each base station to detect the UE data signals. In specific, the base station j is assumed to be applied with a combining vector of linear reception i.e. g_{jk} \in C^M over the signal received. This is usually carried out since, g^H_{jk} Y_j rejects well the interference coming out from UEs in other cells or in same cells, and the signal is amplified from the UE (k) in terms of its spatial domain.

Here, an ergodic achievable spectral efficiency [11] is to be derived for an UE, which is given by,
where, SINR is the effective signal to interference noise ratio, $E_{[i]}[\cdot]$ is the expectation w.r.t position of UE and $E_{[h]}[\cdot]$ is the expectation w.r.t channel realization. The achievable spectral efficiency is thus given by for the subset of cells LB within a cell $j$ is expressed as,

$$SE_j = K \cdot \left( 1 - \frac{B}{S} \right) \left\{ \log_2 \left( 1 + \left( I_j \right)^{-1} \right) \right\}$$

(5)

where, $I_j$ is the interference term.

The codewords tends to span over random interfering user location and Rayleigh fading at a specific distribution of UEs. For the convenience of notation, the pilot reuse factor $\beta$ is considered as an integer. Further, the cells present in set $L$ is to be split into disjoint subsets (where, $\beta \geq 1$) with different pilots in a given set. This condition is called as fractional pilot reuse.

3.3 Achievable spectral efficiency for downlink

For downlink, linear precoding is used for channel estimation (in Eq.(3)) with various channel inputs, say $M$. These channels adds the data signal in a semi-coherent manner for reducing the inter-cell interferances between the UEs. The UE for uplink or downlink in cell $j$ is defined by

$$z_{jk} = \sum_{l=L}^{K} \sum_{m=1}^{\beta} h_{jk}^T \cdot w_{lm} \cdot s_{lm} + \eta_{jk}$$

(6)

where $(\cdot)^T$ is defined as the transpose function, $s_{lm}$ is regarded as the symbol inside UE $m$ of a cell $l$, $w_{lm} E^{M}$ is defined as the precoding vector, and the allocated transmitted power at downlink is given by $\|w_{lm}\|^2$. The precoding vector is thus expressed for uplink within a UE $(k)$ is given by.

$$w_{jk} = \sqrt{q_{jk}} \frac{g_{jk}^*}{E_{[h]} \left\{ \| g_{jk} \|^2 \right\}^{1/2}}$$

(7)

where $q_{jk}$ is the average transmit power and this is considered as a function of random user location, where the value of which is $\geq 0$. The $g_{jk}$ is a vector that forms the spatial transmission directivity using the acquired CSI, and $E_{[h]}[\| g_{jk} \|^2]$ provides the analytic tractability.

4 EXPERIMENTAL RESULTS

The entire simulations are carried out using Matlab. The results are evaluated in terms of a symmetric network topology with hexagonal cells. The entire resources are allocated for transmission of payload data that are utilized in all cells. The size of pilot book is considered as $B = \beta_k$ that mitigates the pilot contamination and permits fractional pilot reuse.

The larger hexagonal grid is used to avoid the impacts of edges and it provides similar properties for all cells. The radius of each cell is $r > 0$ and a pair of integer indexes the cell in unique way $\alpha_j^{(1)}, \alpha_j^{(2)} \in Z$ with $Z$ is the location of base station $(j)$.

$$b_j = \sqrt{3} \left[ 0.5 \sqrt{3} r \right] \alpha_j^{(1)} + \left[ 0.5 \sqrt{3} r \right] \alpha_j^{(2)} \in R^2$$

(8)

The Figure 2 provides the coordinate system of $\alpha_j^{(1)}$ and $\alpha_j^{(2)}$. It is seen that each cell has six other interfering cells in the initial tier and it doubles in its second tier and so on. From earlier work [31 or 32], it is seen that the hexagonal networks limits the factors related to pilot reuse transmission and it often provides symmetric reuse patterns.

The proposed analytical study considers path loss model with channel attenuation variance as $d_j(z) = \frac{C}{\|z - b_j\|^2}$, where the value of $C$ is greater than 0 and this is considered as the reference value, $\| \|$ is regarded as the Euclidean norm and $k \geq 2$ is regarded as the pathloss exponent. This is used for computing the following expression for the distribution of UE within each cell.

$$\mu_{zlm}^{(l)} = E_{\cdot zlm} \left\{ \begin{bmatrix} d_j(z_{lm}) / d_i(z_{lm}) \\ d_i(z_{lm}) / d_j(z_{lm}) \end{bmatrix} \right\}^{\lambda \omega}$$

(9)

It is seen that both $r$ and $C$ cancels each other, since the channel is independent of it. Further, it is very necessary to use average SNR value to serve the base station between UE and antenna.
The range of spectral efficiency is optimized at each antenna M w.r.t its pilot reuse factor (β) and UEs (K). The optimization is carried out by using a suitable range or a reasonable value, where the parameters for simulation is presented in Table.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value or range</th>
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<tbody>
<tr>
<td>Coherence block length, $S$</td>
<td>400</td>
</tr>
<tr>
<td>Coherence time</td>
<td>2 ms</td>
</tr>
<tr>
<td>Coherence bandwidth</td>
<td>200 kHz</td>
</tr>
<tr>
<td>SNR</td>
<td>5 dB</td>
</tr>
<tr>
<td>Pathloss exponent, $k$</td>
<td>3.7</td>
</tr>
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</table>

The proposed analytical model is simulated under three different propagation environments, where the sternness of inter-cell interferences is varied. The three different propagation environments include the following:

- **Average case** finds the average values over UEs that are placed in uniform locations within each cell,
- Best case locates the other cell UEs to be placed at cell edges, which are placed at farthest proximity or distances from the base station, say $j$ and
- Worst case locates the other cell UEs to be placed at cell edges, which are placed at closest proximity or distances from the base station, say $j$ and

The Monte-Carlo simulations are used to compute the values of parameters $\mu_{jl}^{(1)}$ and $\mu_{jl}^{(2)}$, where the location of UEs are placed at $10^6$ distances within each cell. The average, best and worst case results are shown in Figure 3, Figure 4 and Figure 5 w.r.t to optimized spectral efficiency with optimal UEs.

The best case is considered to be optimistic due to the interfering positions of UEs are set to be different w.r.t other cells. The results from the best case is considered to be upper bound, which is made achievable through coordinated scheduling process between each cells. On contrary, the worst case is considered to be pessimistic due to the poor or worst placement of UE inside a cell w.r.t other cells at similar instant of time. Apart from these two cases, the average case is considered to be the practically deployable case, since it involves the averaging of all the values obtained from UE scheduling, UE mobility and pilot sequences switched at random manner across each UE within a cell.

It is inferred from the results that the achievable spectral efficiency is considered to be different between the cases used for analysis. This results in a conclusion that the analysis of single cell cannot be used in multi-cell massive MIMO system. This is even true during the utilization of multi-cells for single celled massive MIMO system. It is seen that in best case, the proposed pilot based estimation obtains higher spectral efficiency than MR scheme, however, the potential gain required to eliminate or suppress the intra-cell interferences are found to be very high. The proposed pilot based estimation also reduces the inter-cell interference in worst case, where the results are similar to ZF in best case scenario. The spectral efficiency in average case is similar for all the three schemes, where the value of $M$ or antennas is between 10 and 200. It is seen from the results that, as the number of antennas increases, the difference is larger. It is further noticed that at $M = 105$, the difference is smaller and it is closer to asymptotic limit and still the best case interferences requires more number of antennas.
The following inferences are made from the current study:

It is seen from the above results, that the difference between these schemes is not the optimized spectral efficiency values. The difference is based on total number of UEs and pointing out correct pilot reuse factor $\beta$.

In general, the higher value of $K$ is due to the application of large number of $M$, that results in smaller $\beta$ value. Since, the channels between UE and base station tends to be more orthogonal in terms of total number of antennas $M$.

The integer reuse factor at rapid time intervals changes as there is a change in the value of $\beta$, it is seen that value of $K$ is larger as the value of $\beta$ is smaller and vice versa.

It is concluded that MR scheme provides lower spectral efficiency per user over multiple UEs, whereas the spectral efficiency is prominently higher in proposed pilot based estimation than other schemes. This is due to the fact that MR scheme schedules more UEs than proposed pilot based estimation scheme, hence the value of reuse factor in MR scheme is lesser than proposed pilot based estimation in case of fewer antennas. It ensures that the proposed pilot based estimations scheme suppress or eliminates well the inter-cell interference in all the three cases.

6 CONCLUSIONS

In this paper, we investigate the scheduling of total number of UEs to increase the spectral efficiency per cell. It is seen from the results that the existing method depends entirely on the position of UEs that makes the optimization of $K$ to be difficult. Whereas, in the proposed method, the analytical expression defines that this method does not depend on the position of UEs with respect to averaging over random locations of UEs and its related power control. The expressions looked similar for both uplink and downlink that provides it to be suitable for joint optimization of network. Each UEs when it is made over symmetric network topology, the proposed method is independent of location of UEs offers network-wide performance and otherwise it leads to extensive Monte-Carlo simulations.

It is seen from the analytical results that the inter-cell interferences are suppressed, where these methods listens to pilot transmission from its neighborhood UE cells. Further, the asymptotic analysis reveals that the proposed scheme spends nearly half the frame on pilot signaling if the value of $M$ is large. However, in practical cases, the limits of spectral efficiency is not reached, however, allocation of larger fractions of pilots helps in achieving the limits.

In general, high spectral efficiency is achieving through UE scheduling for instantaneous transmission. It is seen from the results that the existing ZF method obtains higher performance than MR per UEs. However, the scheduling of UEs are found to be high in proposed pilot based estimation than MR and ZF. It is further seen from the simulations that the massive MIMO achieves spectral efficiency gain in terms of $10x$ when $M = 100$ and it is found to be $40x$, when $M = 500$. The results are valid for spatially correlated fading that reduces well the interference between the users and leads to higher spectral efficiency.

REFERENCES