Shape control of composite plates with piezoelectric actuators

J. S. Mohamed Ali, Munir Mohamed Mahmood, M. S. I. Shaik Dawood

Abstract: In this paper, shape control of composite plates using piezoelectric actuators is being investigated. The goal of this study was to see how composite plates behave when they are integrated with piezoelectric actuators. Modelling and simulation were done using COMSOL Multiphysics software and results were validated using previously published studies. Parametric investigations were carried out to inves-tigate the effect of patch locations and stacking sequences with respect to suppression of deflection. The obtained results showed that for uniformly distributed load considered in this work the patches worked effectively when they were placed at the center of the composite plate.

Keywords: Shape control; Piezoelectric actuators; COMSOL; Composite plate.

I. INTRODUCTION

Smart structures such as piezoelectric materials can be considered as the major innovation in the field of structural mechanics since the introduction of composites in the 1970s. The direct piezoelectric effect was discovered in 1880 by the Curie brothers [1]. Piezoelectricity refers to generation of electric charge when mechanical pressure is applied on the surface of certain crystalline materials such a quartz. Advances in technology in the future seems to be equipped with sophisticated systems that involve sensing and actuation at a miniature level, active shape control of aerodynamic surfaces such as airfoils and turbine blades; hence this initiates an urge to study the shape control of structures using piezoelectric actuators [2].

Composite structures have revolutionized the aerospace industry due to their high strength to weight ratio compared to their metal counterparts. In addition to that, they can also be tailored in order to fulfill a certain requirement. In the present work, the shape control of composite plates bonded with piezoelectric layers has been investigated. The modelling of the composite plates with the piezoelectric actuators was done in COMSOL Multiphysics software.

Previous works in the area of shape control of composite beams with piezoelectric actuators are innumerable. The shape control of composites bonded with piezoelectric layers using layerwise theory was investigated by Donthireddy and Chandrashekhara [3], while the effect of patch location on the suppression of deflection was investigated by Liew et al [4]. An analytical model was developed by Koconis et al [5] to calculate changes of shapes of beams, plates and shells when the voltages were specified or vice versa. Her & Lin [6] used ANSYS to investigate the deflection of cross-ply laminates induced by piezoelectric actuators. Tong et al. [7] used a 2D FEM approach to solve for piezoelectric actuators model which are embedded or bonded in bimorph arrangement. They generated a solution for optimal location of piezoelectric actuators surface bonded or embedded on composite laminates. Tzou [8] presented an analytical solution to determine the tip deflection of a bimorph piezoelectric thin beam.

The objective of this work is to study the static shape control of composite plates bonded with piezoelectric actuators using COMSOL Multiphysics. The test case used in this work is based on an earlier study by Liew et al [4] who used an element free Galerkin method to study the effectiveness of piezoelectric patch location on suppressing deflection of composite plates subjected to a uniformly distributed load. In the current investigation, COMSOL Multiphysics will be used as the simulation tool to model the composite and piezoelectric actuator while the investigation will focus on the effects of actuator location and stacking sequence of the composite layup.

II. FORMULATION

A piezoelectric beam is shown in Figure 1 below. According to the applied voltages the top patch will contract while the bottom patch will undergo expansion creating the bending effect.

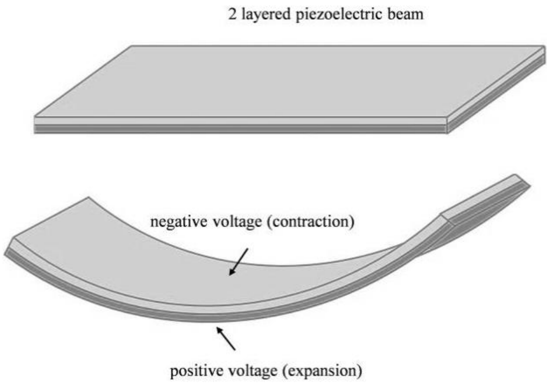

Fig. 1: Bending illustration of a piezoelectric beam

The governing equations that relate the direct piezoelectric effect can be expressed as follows;

$$D = \epsilon E + d\,\sigma \qquad (1)$$

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where D is the electrical displacement, ϵ the dielectric permittivity at zero stress, d is the piezoelectric constant and σ is the stress vector. Analogously, the expression of strain obtained from piezoelectric effect assuming a linear elastic behaviour of the piezoelectric material is;

$$\epsilon = S\,\sigma + d\,E \qquad (2)$$

where ε is the strain vector and S is the compliance matrix and E is the electric field. Assuming a plane stress condition, for a thin composite laminate the stress-strain relationship according to [9] is expressed as;

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{bmatrix} \qquad (3)$$

where

$Q_{11}=\frac{E_1}{1-\nu_{21}\nu_{12}}$, $Q_{12}=\frac{\nu_{12}E_2}{1-\nu_{21}\nu_{12}}$, $Q_{22}=\frac{E_2}{1-\nu_{21}\nu_{12}}$, $Q_{66}=G_{12}$. E_1 is the longitudinal Young’s modulus, E_2 is the transverse Young’s modulus, ν_{12} is major Poisson’s ratio, G_{12} is the in-plane shear modulus. The reduced transformed stiffness matrix for a two-dimensional angle ply lamina is;

$$[\bar{Q}]=[T]^{-1}[Q][R][T][R]^{-1} \qquad (4)$$

where $T=\begin{bmatrix}c^2 & s^2 & 2sc\\ s^2 & c^2 & -2sc\\ -sc & sc & c^2-s^2\end{bmatrix}$, $R=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 2\end{bmatrix}$

and $c = \cos\theta$, $s = \sin\theta$

The classical lamination theory assumes for a laminate, each lamina is thin, orthotropic, homogenous, and the elastic as well as the shear strains are only present in the x-y plane [9] and accordingly for a composite laminate the resulting force and moment resultants can be determined by using the *ABD* matrix and the relationship can be written as follows;

$$\begin{bmatrix}N\\ M\end{bmatrix}=\begin{bmatrix}A & B\\ B & D\end{bmatrix}\begin{Bmatrix}\epsilon\\ \kappa\end{Bmatrix} \qquad (5)$$

where

$$A_{ij}=\sum_{k=1}^{n}\left[\left(\bar{Q}_{ij}\right)\right]_k\left(h_k-h_{k-1}\right),\ \text{i,j}=1,2,6 \qquad (6)$$

$$B_{ij}=\frac{1}{2}\sum_{k=1}^{n}\left[\left(\bar{Q}_{ij}\right)\right]_k\left(h_k^2-h_{k-1}^2\right),\ \text{i,j}=1,2,6 \qquad (7)$$

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{n}\left[\left(\bar{Q}_{ij}\right)\right]_k\left(h_k^3-h_{k-1}^3\right),\ \text{i,j}=1,2,6 \qquad (8)$$

In equations 6 to 8, h_k is the distance of the top surface of the k_{th} layer from the neutral plane of the laminate.

III. MODELLING

The FEA modelling was done using COMSOL Multiphysics. A convergence study was carried out to identify the mesh dependency of the simulated models. The common steps that should be followed when modelling in COMSOL Multiphysics software include setting up the model wizard to be used, creating the geometry, specifying the material properties, defining the physics then simulating and post processing results. Modelling of composite laminates in COMSOL Multiphysics involves dividing the plate or beam into layers equal to the number of plies of laminate, defining a base vector coordinate system representing each angle of fibre orientation and entering the directional material properties of the composite.

IV. VALIDATION

To ensure the credibility of the simulation results using COMSOL Multiphysics several validations were carried out. The first test case was obtained from the ANSYS verification manual [10] which consists of a simply supported square cross ply laminate with stacking sequence of [0°/90°/90°/0°]. The geometric and material properties of the test case are shown in Figure 2.

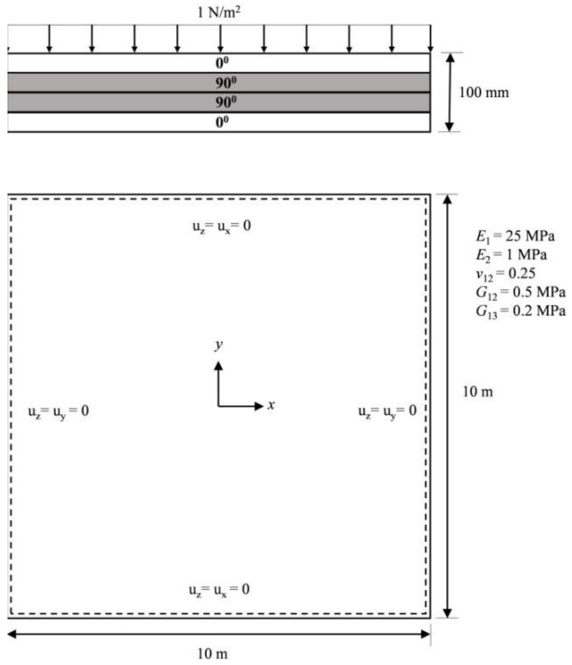

Fig. 2: Geometric and material data for test case 1 obtained from ANSYS verification manual

The deflection of the laminate under a uniformly distributed load of 1 N/m^2 is shown in Table 1. The maximum deflection obtained using COMSOL is close to the one reported in the manual with difference in result being less than 1%.

Table 1: Results from first test case

Layup	ANSYS (m)	COMSOL Multiphysics (m)	Analytical result (m)	Error %
$[0,90]_s$	0.068562	0.068578	0.0683	0.44

A mesh sensitivity analysis was conducted on the same test case and the following was observed as summarized in Table 2. The normal mesh type was observed to give satisfactory results hence used in all the other simulations as well.

Table 2: Results from first test case

Mesh type	No. of elements	Deflection (m)
Extremely coarse	6517	0.068481
Extra coarse	10200	0.068537
Normal	88284	0.068578
Fine	300796	0.068583

The second test case was obtained from the work of Tzou and Tseng [8] in which an analytical equation to find the deflection of a cantilevered bimorph piezoelectric beam was proposed and it is given as:

$$w(x) = 0.375 \times \frac{e_{31} \times V_a}{E} \times \left(\frac{x}{t}\right)^2 \qquad (9)$$

Figure 3 provides a schematic diagram with details of the geometric and material properties as well as on the actuation nature of the test case. Table 3 provides the results of the test case from different references and from the current simulation. The validation is considered successful as the errors in most cases fell below 5%.

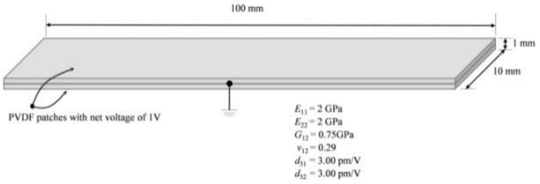

Fig. 3: Geometric and material data for test case 2 obtained from [8]

Table 3: Results from second test case

Longitudinal distance (mm)	Analytical Solution [8] (mm)	FEM solution [11] (mm)	COMSOL (mm)	Error %
20	1.40E-05	1.50E-05	1.49E-05	0.06
40	5.52E-05	5.69E-05	5.44E-05	1.45
60	1.224E-04	1.371E-04	1.302E-04	6.37
80	2.208E-04	2.351E-04	2.309E-04	4.57
100	3.451E-04	3.598E-04	3.529E-04	2.26

V. STUDY ON THE STATIC SHAPE CONTROL OF COMPOSITE LAMINATES

This section outlines the type of investigations and presents the results from the shape control analyses on composite laminates. The test cases were adopted from the work of Liew et al [4]. Figure 4 shows the test cases considered to study the effect of actuator patch locations on the deflection suppression.

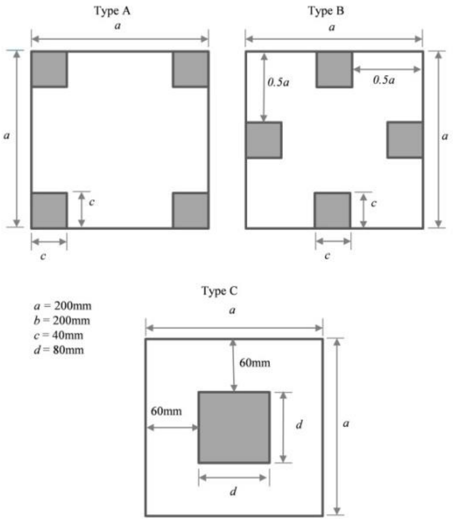

Fig. 4: Patch locations

The composite laminate is made of 4 layers of graphite/epoxy with a total thickness of 8 mm, while the actuator is of type PZT-4 with a single patch having a total thickness of 1 mm. The actuators are symmetrically bonded on the top and bottom surfaces of the laminate. Four different types of stacking sequences were considered, i.e., [-45°/45°/-45°/45°], [45°/-45°/-45°/45°], [0°/90°/0°/90°] and [30°/30°/-30°/-30°]. The plate was simply supported on all edges and a uniformly distributed load of q = 100 N/m^2 was applied on the top surface of the laminate. The material properties of the composite and piezoelectric actuator are listed in Table 4.

Table 4: Material properties of composite and PZT

Properties	Graphite/ epoxy	PZT-4
E_{11} (GPa)	132.4	81.3
E_{22} (GPa)	10.8	81.3
G_{12} (GPa)	5.6	25.6
G_{13} (GPa)	5.6	30.6
G_{23} (GPa)	3.6	25.6
v_{12}	0.24	0.33
d_{31} (pm/V)	-	-122
d_{32} (pm/V)	-	-122

Figures 5 to 7 show the effect of different patch locations and stacking sequences on the static shape control of composite plates. It was found that the configuration C was the most effective in suppression of deflection with a small amount of voltage applied. It was also found that the deflection due to load on the cross ply laminate was smallest compared to the other laminate types. It is also noticed that the suppression of deflection becomes less effective in the case of angle ply laminates.

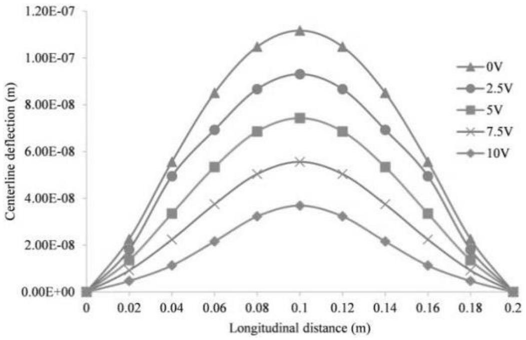

Fig 5a: [-45°/45°/-45°/45°] layup – Type A

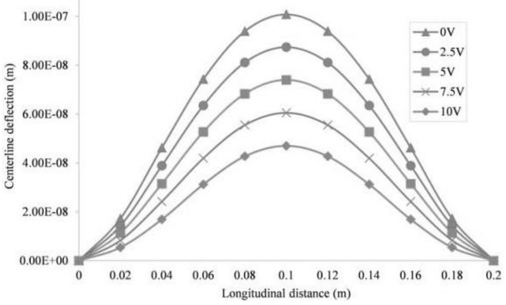

Fig 5b: [0°/90°/0°/90°] layup – Type A

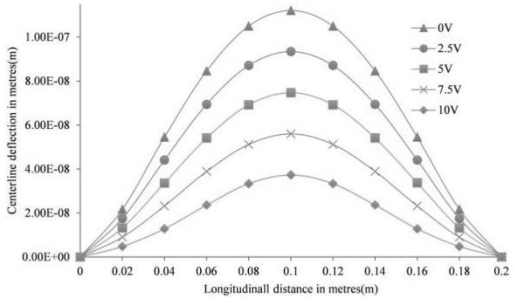

Fig 5c: [45°/-45°/-45°/45°] layup – Type A

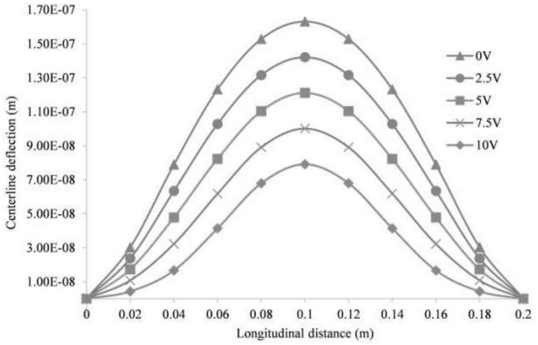

Fig 5d: [30°/30°/-30°/-30°] layup – Type A

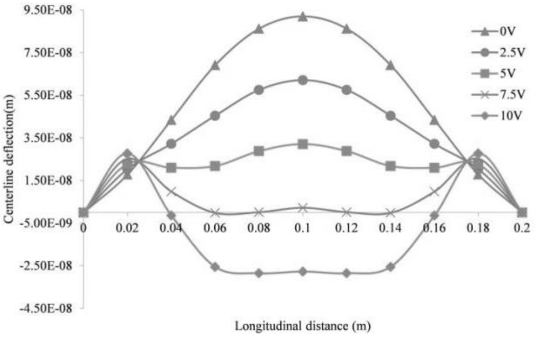

Fig 6a: [-45°/45°/-45°/45°] layup – Type B

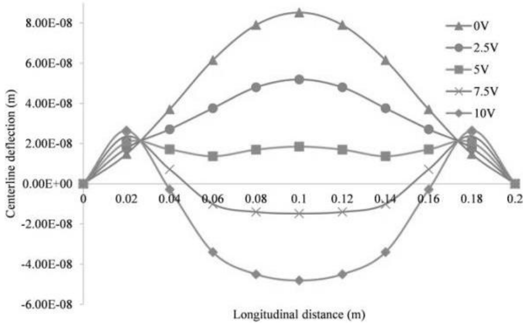

Fig 6b: [0°/90°/0°/90°] layup – Type B

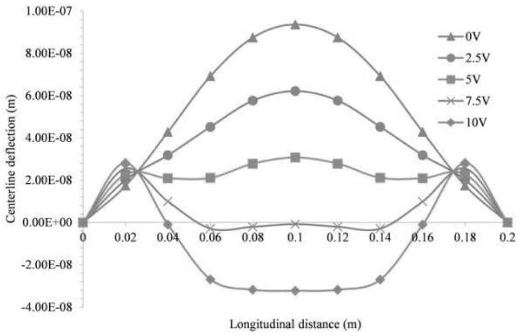

Fig 6c: [45°/-45°/-45°/45°] layup – Type B

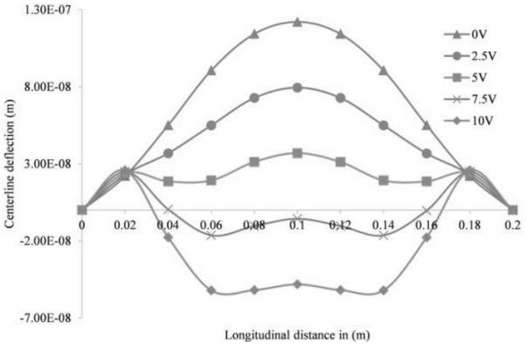

Fig 6d: [30°/30°/-30°/-30°] layup – Type B

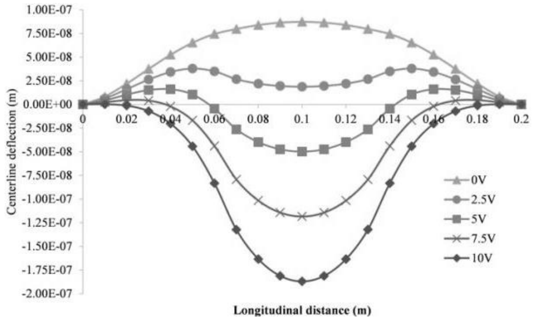

Fig 7a: [-45°/45°/-45°/45°] layup – Type C

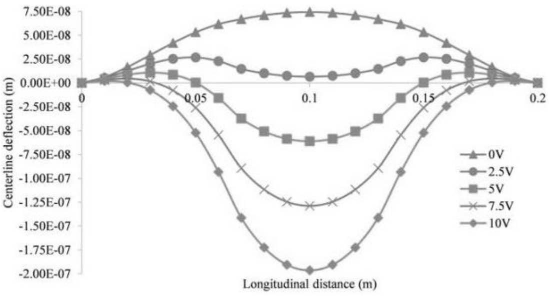

Fig 7b: [0°/90°/0°/90°] layup – Type C

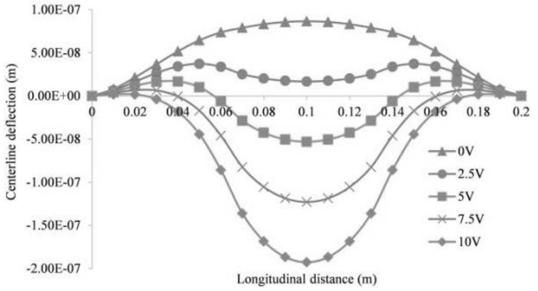

Fig 7c: [45°/-45°/-45°/45°] layup – Type C

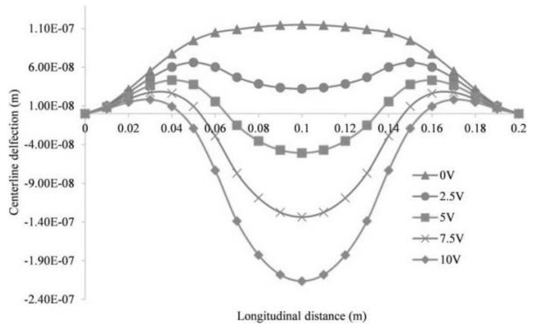

Fig 7d: [30°/30°/-30°/-30°] layup – Type C

VI. CONCLUSION

In conclusion it can be said that the shape control of composite plates is imperative to study because of its enormous significance. There are many applications in which shape distortions caused by external loads need to be corrected which otherwise will affect the performance of the component. In this study it was found that for a plate subjected to uniformly distributed load, patches located at the centre of the plate to be more effective and efficient in suppressing the deflection. Suggestion for future work may include the use of closed loop controller in controlling the deformation of the plate. Another area of interest can be on the twist control of wing, where it is well known undesirable wing twist can lead to aeroelastic phenomena such as aileron reversal. The study of shape control using shape memory alloys may be done in future as SMAs have proven to be more efficient in achieving high strains under low loads compared to piezoelectric actuators.

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REFERENCES

1. Gaudenzi P, Smart structures physical behavior, mathematical modelling and applications, John Wiley and Sons Ltd, 2009.
2. Agrawal BN & Treanor KE (1999), Shape control of a beam using piezoelectric actuators, Smart Materials and Structures 8, 729-7399.
3. Donthireddy P & Chandrashekhara K (1996), Modelling and shape control of composite beams with embedded piezoelectric actuators, Composite Structures 35, 237-244.
4. Liew KM, Lim HK, Tan MJ & He XQ (2002), Analysis of laminated composite beams and plates with piezoelectric patches using the element-free Galerkin method, Computational Mechanics 29, 486-497.
5. Koconis DB, Kollar LP & Springer GS (1994), Shape control of composite plates and shells with embedded actuators II. Desired shape specified, Journal of Composite Materials 28, 262-285.
6. Her SC & Lin CS (2010), Deflection of Cross-Ply Composite Laminates Induced by Piezoelectric Actuator, Sensors (Basel) 10, 719-733.
7. Tong D, Williams II RL & Agrawal SK (1998), Optimal shape control of composite thin plates with piezoelectric actuators, Journal of Intelligent Material Systems and Structures 9, 458-467.
8. Tzou HS & Tseng CI (1990), Distributed piezoelectric sensor/actuator design for dynamic measurement/control of distributed parameter systems: A piezoelectric finite element approach, Journal of Sound and Vibration, 138, 17-34.
9. Kaw A, Mechanics of Composite Materials, Taylor and Francis Group, 2006.
10. ANSYS Mechanical APDL Verification Manual.
11. Tseng CI, Electromechanical dynamics of a coupled piezoelectric/mechanical system applied to vibration control and distributed sensing, PhD Dissertation, University of Kentucky, Lexington, Kentucky, 1989.