

# Educational software for stress analysis of non-idealized closed thin-walled sections

J.S. Mohamed Ali, Mir Owais Ali Ibrahim, Miah Mohammed Riyadh

*Abstract---* Aerospace structures such as fuselage and wings are made of typical thin-walled closed sections and the detailed stress analysis of such closed thin-walled structures can be tedious and time consuming due to its statically indeterminate nature of the problem. In the present work, an educational software for the stress analysis of such non-idealized thin-walled closed sections has been developed that complements traditional methods of teaching and learning. The software developed is able to generate any given closed cross section which are subjected to bending, shear, and torsional loads and compute the resulting stresses on the cross section. Results from literature have been used to validate the results from the software. The software has been developed using Graphical User Interface (GUI) in MATLAB which makes the software very user friendly. The software is expected to be an effective teaching and learning tool of courses on thin-walled structures and aircraft/automotive structures.

**Keywords:** Aircraft Structures; Educational Software; Non-Idealized, Thin-walled sections; MATLAB; Stress Analysis.

## 1. INTRODUCTION

The use of contemporary and powerful software in classes is a highly useful tool for engineering students. Students are able to learn quickly and easily visualize the information given as complex formulae when they use such software as it is naturally formulated and can readily produce graphical representations [1]. As a consequence of this fact, advanced courses such as Aircraft Structures can be made simpler and more interesting to the students. In this work such an educational software was developed and used to present examples that illustrates the use of pedagogical tools to solve all typical problems related to non-idealized closed thin-walled sections which are introduced in Aircraft Structures course in Aerospace Engineering.

Traditionally, the students are taught the principles behind a phenomena and the governing equations supporting it. The students are then given some examples and are made to solve problems either from a book or other related sources. The problems concerning the Aircraft Structures course is complicated, tedious and time consuming. Moreover, the lecturers are not always available to validate the students' results during their practice sessions and this is where this software will play an important role. It is not necessary to limit undergraduate education with just reading books or problem solving with the help of calculators or simple graphing tools. They should be exposed to numerical solution process through software from the very beginning.

Moreover, students should acquire six levels of knowledge through the learning process which consist of knowledge, comprehension, application, analysis, synthesis, and evaluation [2]. The first three can be acquired from the traditional teaching and learning methods but to acquire the latter three higher levels, students have to undergo long assignments and projects with validation which can be obtained from the help of an educational software. It is believed that educational software combined with traditional lectures can be an efficient and economical alternative for students during the learning period.

For lower level engineering courses, students have been equipped with educational tools for courses such as calculus and dynamics. SECav and DYNASOFT are educational software which supports the learning and teaching process of numerical calculus and dynamics respectively [3, 4]. However, there is a lack in educational software when it comes to advanced courses such as Aircraft Structures, Machine Design, etc. These advanced courses involve a lot of complex calculations and thus a software which is associated with these advanced courses will be a complementary tool besides traditional teaching and learning methods.

The development of educational software in the field of Aerospace Engineering is not new. The software known as MS Aximer [5] solves numerical problems which are related to propulsion and aerodynamics. XFOIL and Tornado [6, 7] are also two interactive programs which involves the design and analysis of subsonic isolated airfoils and linear aerodynamic wings.

Similar software such as the one described in this paper has been developed before for thin-walled sections such as GT Shear [8]. It involves the bending and shear stress analysis for non-idealized thin-walled open sections. GT Shear is able to perform analysis on any generated section but it is limited to only non-idealized open thin-walled sections. Moreover, a software based on idealized open thin-walled sections has also been developed [9]. There is no such software available that can perform stress analysis of wings and fuselage which are non-idealized closed thin-walled sections. This work aims to develop a software that can conveniently compute stresses on typical non-idealized thin-walled closed sections under bending, shear or torsional loads. The software developed works for any general symmetric/unsymmetric cross section and the section can be auto-generated by providing the coordinates at the breaking points of the cross section of interest.

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Graphical User Interface (GUI) in MATLAB programming has been used for the development of this software. MATLAB GUIDE is chosen as it is easy to work with and has supremacy over other programming languages. For the user to navigate through a well-developed GUI, it is irrelevant for the user to understand the language underlying in the development of this software [10].

## 2. METHODOLOGY

A section can be considered thin-walled only if the ratio of the cross section dimension to thickness of section is 10 or higher. Idealization of a thin-walled cross-section simplifies the problem drastically as the booms are now responsible for carrying direct stress whereas the skins are only carrying shear stresses. This idealization may be of convenience to the user but actually both the effect of direct stresses and shear stresses are felt on the skin, thus the effective thickness of a skin in resisting bending stress can never be zero. Thus the idealization is an approximate method to simplify the analysis by replacing integration by just summation. In the present analysis non-idealized sections are considered and hence the actual integration is carried out for webs. The following sections outline the development of six general functions for integration and their use in finding shear stress distribution of a non-idealized thin-walled structure. The methodology behind finding bending stress distribution and shear stress distribution is also mentioned.

### 2.1. Section Properties

Finding the section properties of a cross-section is the first step before determining the stresses. Data such as the centroid, moments of inertia, and the shear centre are properties of interest.

#### 2.1.1 Centroid

Calculating the centroid for a thin-walled inclined sections is straight forward. To evaluate the coordinates  $x$  and  $y$  location of centroid, a reference axis is set at any point and the centroid values are relative to this point as we define the point as the origin (0,0) which can be seen in Figure 1. If the stringer areas are present at the ends (corners) of the web then their effect are also included.

$$\bar{x} = \frac{\sum \bar{x}A}{\sum A} \quad (1)$$

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} \quad (2)$$

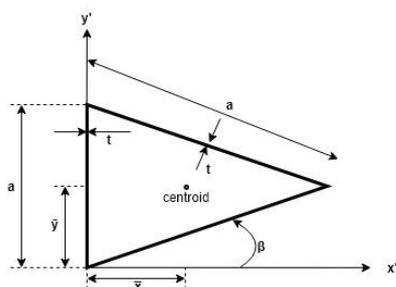


Fig. 1: Thin-walled closed section

Therefore, it is obtained by finding the sum of product of the centroid of the thin section with respect to reference point and its area divided by the sum of its area.

#### 2.1.2 Moments of Inertia

Assuming the skins shown in Figure 1 are all rectangular and at an angle makes them unsymmetrical about centroidal axes. The moment of inertia of such skins with respect to their centroids can be defined as follows [11]:

$$I_{xx}' = \frac{a^3 t \sin^2 \beta}{12} \quad (3)$$

$$I_{yy}' = \frac{a^3 t \cos^2 \beta}{12} \quad (4)$$

$$I_{xy}' = \frac{a^3 t \sin 2\beta}{24} \quad (5)$$

where,  $a$ ,  $t$ , and  $\beta$  are the length, thickness and the angle of the skin from the horizontal respectively (also shown in Figure 1). To find the moment of inertia with respect to the centroid, parallel axis theorem is used. The theorem is as follows:

$$I = I' + Ad^2 \quad (6)$$

Moreover, if the stringer areas are present at the ends (corners) of the web then their effect are also included.

#### 2.1.3 Shear Centre

The procedure for locating the shear centre for closed section is adopted from Megson [11]. Firstly one need to find the shear flow distribution and the general equation used in calculating shear flow for a closed section is given as follows [11]:

$$q_s = - \left( \frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) \left( \int_0^s t x ds + \sum_{r=1}^n B_r x_r \right) - \left( \frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) \left( \int_0^s t y ds + \sum_{r=1}^n B_r y_r \right) + q_{s,0} \quad (7)$$

To find the horizontal location of shear centre,  $x_{sc}$ ,  $S_x$  (Shear Force in  $x$  direction) is assumed to be zero. Similarly, to find the vertical location of shear centre,  $y_{sc}$ ,  $S_y$  is assumed zero. This assumption simplifies the general equation (7). To begin the analysis, any point is selected, and the closed section is cut at that point to make the section open and the equation (7) is used to determine the open section shear flow  $q_b$ . If stringers are present in the closed section then their areas  $B_r$  are taken into consideration for shear flow calculation. Once the shear flow  $q_b$  for all the webs are found then the shear flow  $q_{s,0}$  will be determined using the condition that when the load is applied at the shear centre, it will produce zero twist which lead to the following equation [11]

$$q_{s,0} = - \frac{\oint (q_b / Gt) ds}{\oint (ds / Gt)} \quad (8)$$

where,  $G$  is the Shear Modulus. Once  $q_{s,0}$  is found, then it is added to  $q_b$  values in all the webs to get final shear flow  $q$ .

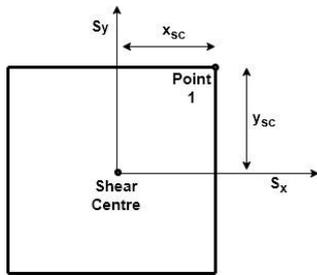


Fig. 2: Shear centre location

Then to find the shear centre, moment about any point is taken, such that for equilibrium, *External Moment = Internal Moment, about point 1*, which will lead to equations (9) and (10) from which the shear centre location can be found

$$S_x y_{sc} = S_x \oint q p d s \quad (9)$$

$$S_y x_{sc} = S_y \oint q p d s \quad (10)$$

Where p is the moment arm of the internal force.

### 2.2. General Line Equations

To integrate the functions as described above through equation (7)- (10), functions for y and x in terms of s, must be described. This section aims to find these general functions and discuss ways in which they can be integrated.

The functions found for y in terms of s (for i signifying the number of stringer) are as follows:

1. If  $y_i > \bar{y}$  and  $y_{i+1} < \bar{y}$   
 $y = -s_i \sin\beta + d$  (11)

2. If  $y_i < \bar{y}$  and  $y_{i+1} > \bar{y}$   
 $y = s_i \sin\beta - d$  (12)

3. If  $y_i > \bar{y}$  and  $y_{i+1} > \bar{y}$  and  $y_i \leq y_{i+1}$   
 $y = s_i \sin\beta + d_{min}$  (13)

4. If  $y_i > \bar{y}$  and  $y_{i+1} > \bar{y}$  and  $y_i > y_{i+1}$   
 $y = (a - s_i) \sin\beta + d_{min}$  (14)

5. If  $y_i < \bar{y}$  and  $y_{i+1} < \bar{y}$  and  $y_i \geq y_{i+1}$   
 $y = -s_i \sin\beta - d_{min}$  (15)

6. If  $y_i < \bar{y}$  and  $y_{i+1} < \bar{y}$  and  $y_i < y_{i+1}$   
 $y = -(a - s_i) \sin\beta - d_{min}$  (16)

where  $\bar{y}$ ,  $d_{min}$ , and  $a$  are the distance of the  $y_i$  point from centroid, the minimum y distance between the  $y_i$  and  $y_{i+1}$  point from the centroid, and the length of the web respectively. Upon observing Figure 3, it can be shown that equation (13) can be used to describe webs 1-2 and 2-3, since they are both above the line and are moving in the same direction. Equation (14) can be used to describe web 3-4, as it's above the centroid line but its direction is opposite to the positive y. For web 4-5, equation (11) can be used, since it is going through the centroid line but from the top to the bottom. Webs 5-6 and 6-7 can be defined by equation (15), as they are below the centroid line and moving in the same direction as negative y. For web 7-8, equation (16) must be used as it is opposing the negative y direction and is below the centroid line. Finally, equation 12 is utilized in calculating y for web 8-1 as it is going from below the centroid line to above.

Similarly, for x in terms of s, similar equations are used, only with the exception of changing  $\sin\beta$  to  $\cos\beta$  and y

distance from centroid to x distance from the centroid. As discussed in the previous sections these equations need to be integrated. Integrated once to find shear flow and it is integrated again to find shear force (which is used in moment analysis). Equations (11)-(16) are therefore, integrated and then a limit of 0 to  $s_i$  is replaced with  $y_i$  to  $y_{i+1}$  for y equations and  $x_i$  to  $x_{i+1}$  for x equations. Changing the limits, makes the computational process easier, as user defined variables will be used to solve the problem instead of derived variables which takes more time and memory to operate.

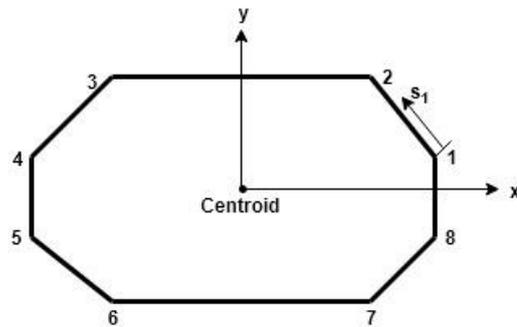


Fig. 3: Thin-walled closed section with general inclined webs

### 2.3. Stress Analysis

A typical example of a non-idealized thin-walled cantilever beam is as shown in Figure 4. The section may have stringers attached at the corners. The beam is subjected to bending loads:  $M_x$ ,  $M_y$ , shear loads,  $S_x$ ,  $S_y$  and torsion load,  $T$ . All the loads can appear simultaneously if the box shown in Figure 4 were a representation of typical torsion box of a wing. Hence, it is necessary to know the effects of these loads to design the torsion box to able to withstand maximum stresses from all kinds of loads. Thus the software is developed to find stresses due to bending, shear and torsion.

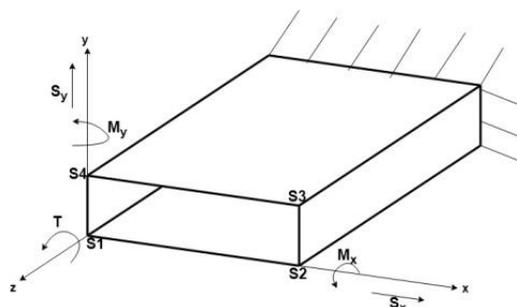


Fig. 4: A typical non-idealized thin-walled closed section with loads

#### 2.2.1 Bending Stress

Unlike the idealized sections, for a non-idealized cross-section, bending stresses are variable along the web and it varies depending on the position, which is measured relative to the centroid. The equation as taken from Megson [11] is given by equation (17).



$$\sigma_z = \left( \frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) x + \left( \frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) y \quad (17)$$

The x, y in the equation represent the positions across any web but the position is relative to the centroid.  $M_x, M_y$  here are the bending moments that cause the bending stress and  $I_{xx}, I_{yy},$  and  $I_{xy}$  are the section properties which are already described in the previous sections.

The line across the cross-section through which the bending stress remains zero can be defined as the neutral axis. Equation (18) shows the calculation of the angle of the neutral axis relative to the centroid.

$$\alpha = \tan^{-1} \left( \frac{M_y I_{xx} - M_x I_{xy}}{M_x I_{yy} - M_y I_{xy}} \right) \quad (18)$$

### 2.2.2 Shear Stresses due to Shear load

For non-idealized sections, shear stresses, just like bending stress, varies across the webs. The shear flow is first evaluated using the similar procedure as shear centre with the equation (7). Thus finding shear stress for a non-idealized section is difficult as a function must be defined for integration. The line functions are already discussed in the previous sections and will be used in the analysis. Shear stress is defined simply as shear flow of a web divided by its thickness as given by the equation (19). Shear flow equation uses external shear loads  $S_x,$  and  $S_y$  and section properties

$$\tau = q/t \quad (19)$$

Similar to the procedure for finding shear centre, a cut is made to make the section open, then the open section shear flow  $q_b$  is found for all the webs, Then moment is taken about any point to find  $q_{s,0}$  using equation (20)

$$S_x y + S_y x = \oint q_b p ds + 2A q_{s,0} \quad (20)$$

where p is the moment arm of the internal force and x, y are the moment arm of  $S_y$  and  $S_x$  to the point of moment respectively and A the enclosed area of the cross-section.

Now that  $q_{s,0}$  is known, it will be added to all the  $q_b$ 's to give shear flow, q for all the webs. Finally, to find shear stress, equation (19) is used.

### 2.2.3 Shear Stress due to Torsion

The shear flow due to torsion remains constant throughout all the webs, as it depends only on the area of the enclosed section A, and the torque applied, T. The formula for shear flow due to torsion is given in equation (21). Once the shear flow is found, stress can be found by equation (19).

$$q = \frac{T}{2A} \quad (21)$$

## 3. RESULTS AND DISCUSSION

The Graphical User Interface for the software was programmed using MATLAB GUIDE. The flowchart in Figure 5 describes the steps that the software follows in order to evaluate the stresses. Firstly the software, in the first window prompts the user to enter the number of stringers. Once it is entered, an auto-generated table pops

up, which asks the user to write the stringer position and its area (if any). This is the first step of the flowchart in Figure 5. Next, an output window showing the section properties such as the Centroid, Moments of Inertia and Shear Centre appear and another output window showing the plot of the section according to the data provided also appears. This is the second step of the flowchart. Once, all the section properties are calculated, they are stored for use in stress analysis. Before that the software asks the user to select a type of loading (bending, shear, torsion). Once the type of loading is selected, another window appears for input of the loads such as bending moment as described in previous section for bending stress, shear loads for shear stress and torsional loads for torsional shear stress. Once, the user has entered all the data then the stress distribution on the webs is plotted. The GUI designed is very user-friendly as it helps the user along the way. A user manual for the software has also been provided to guide the users. Typical software GUI windows are as shown in Figure 8.

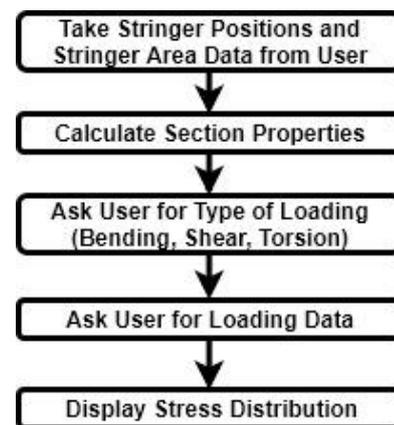


Fig. 5: Flow chart for stress analysis

## 4. VALIDATION OF RESULTS

All components of the software were checked individually and compared with results from literature to ensure that the results obtained from the software are accurate and can be used for any case. To demonstrate the capability of the software most tedious examples with inclined sections are solved for which the results are readily available in the literature.

### 4.1. Validation of Section Properties

A solved example problem from Megson [11] is used to validate the section properties. Figure 6 shows the cross-section of the problem. There is no stringer area at the points, only skins all having a thickness, t of 5 mm. The dimensions of the cross-section are given in terms of a. The value of a here is taken to be of 10mm. The objective of this problem is to find all the section properties. The section properties found using the software are then compared with the literature as shown in Table 1.

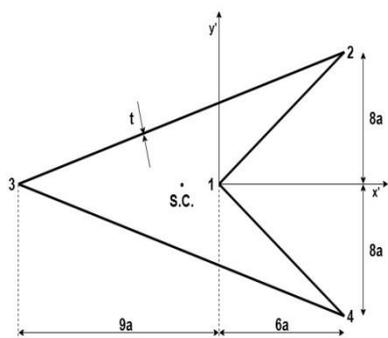


Fig. 6: Problem to validate section properties

Table 1: Section Properties

Section Properties	Software values	Literature values [11]
Centroid x (mm)	1.667	1.667
Centroid y (mm)	0	0
$I_{xx}$ (mm <sup>4</sup> )	5.76e+06	5.76e+06
$I_{yy}$ (mm <sup>4</sup> )	4.763e+06	4.763e+06
$I_{xy}$ (mm <sup>4</sup> )	0	0
$x_{sc}$ (mm)	-33.3	-33.5
$y_{sc}$ (mm)	0	0

As observed from Table 1, all the section properties match exactly. All the distances measured here are measured

from point 1 in Figure 6. This shows the capability of the software to accurately calculate the section properties of any non-idealized closed section.

#### 4.2. Validation of Shear Flow

To demonstrate the shear flow distribution, a thin-walled section comprising three points (or stringers) with no stringer areas and subjected to shear load (Megson [11]) as shown in Figure 7, is considered. The thickness of each web is  $t$ , which is taken to be 5 mm. The dimensions  $h$  and  $d$  are taken to be 200 mm and 141.42 mm respectively. Shear forces,  $S_y = 1000$  N is applied at stringer 1 and  $S_x$  is taken to be zero. Table 2 depicts the comparison between software values and literature for the shear flow distribution for each web.

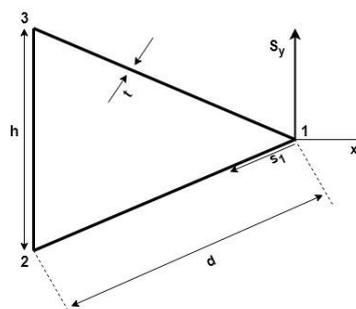
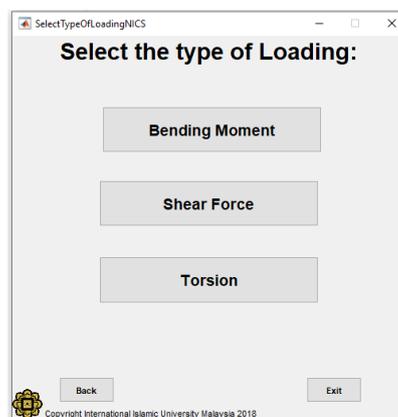
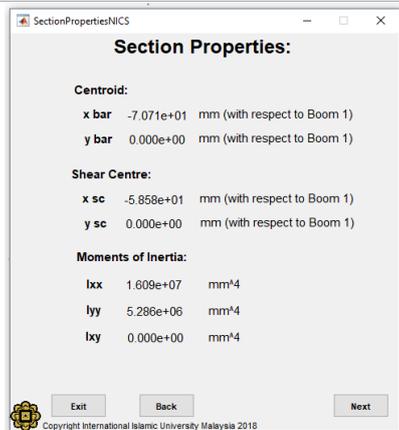
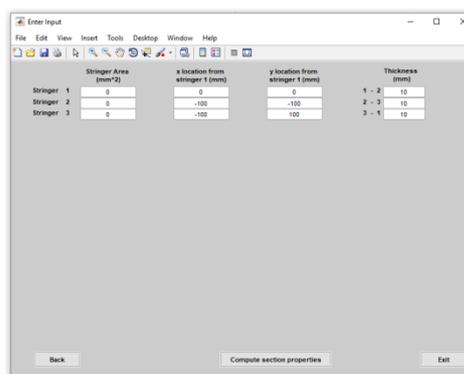
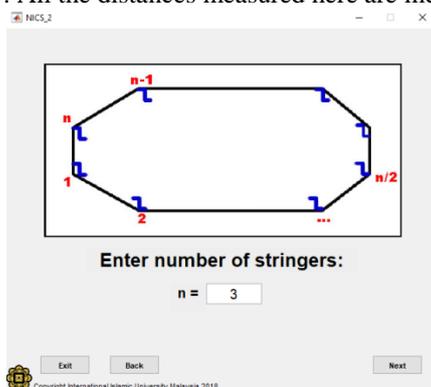


Fig. 7: Problem to validate shear flow



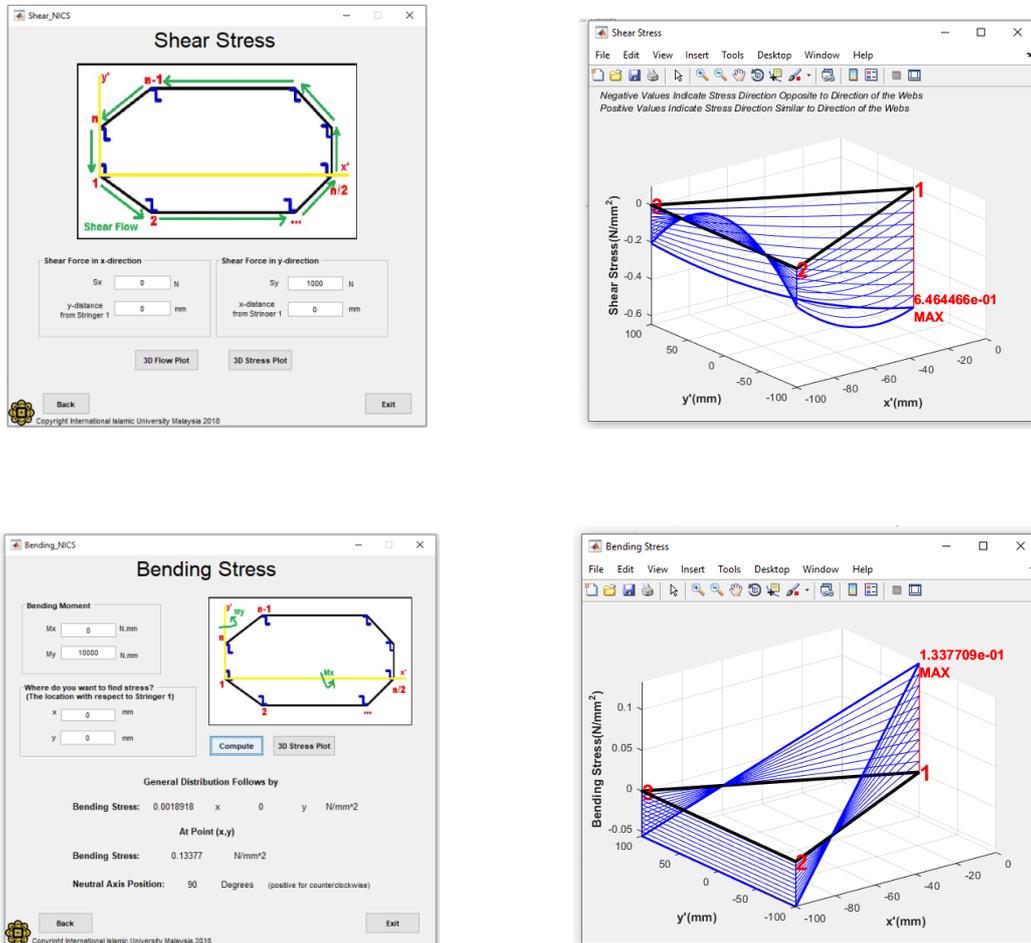


Fig. 8: Typical GUI figures for non-idealized closed section stress analysis

Table 2: Shear flow distribution

Location	Shear flow (N/mm) Software values	Shear flow (N/mm) [11]
At stringer 1	6.465	6.465
Mid of web 12	5.366	5.366
At stringer 2	2.071	2.071
Mid of web 23	1.036	1.036
At stringer 3	2.071	2.071
Mid of web 31	5.366	5.366

As seen from the Table 2, all the values for the shear flow match accurately even for a position in between stringers. Now to know the direction and overall distribution, figures for shear flow from software and shear flow from literature are compared in Figure 9.

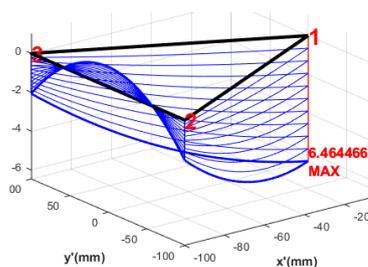


Fig. 9a

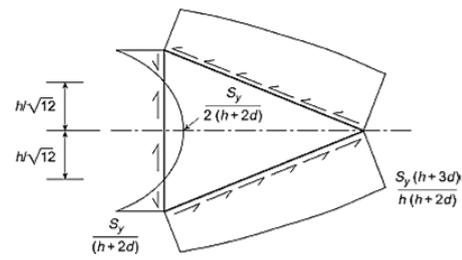


Fig. 9b

Fig. 9: Comparison of distribution of shear flow between the software (Fig 9a) and the literature [11] (Fig 9b)

In Fig 9, for webs 1-2 and 1-3, the shear flow from software has the same magnitude and distribution as reported in the literature. Similarly, it can be observed that the results from the software in web 2-3, the shear flow distribution and the change of directions of the shear flow match perfectly with literature.

Thus the results obtained from the software are identical with the literature [11]. Therefore, this software can be used to analyze any type of non-idealized thin-walled closed section including unsymmetrical section. This educational software can be used to repeat and carry out parametric study of such problems involving aircraft structures. The software can give the student the solution and guideline before they attempt to solve their assignments or homework.

## 5. CONCLUSION

An educational software for stress analysis of non-idealized thin-walled closed section has been developed. The program was developed using MATLAB GUI which is very user-friendly. The software has been tested through various case studies to ensure that it works for any type of closed cross-section and for any kind of load applied. The software is expected to be a very useful tool for effective teaching-learning of courses on aircraft structures.

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