Performance evaluation of linear quadratic regulator and linear quadratic Gaussian controllers on quadrotor platform

M Islam, M Okasha, E Sulaeman, S Fatai, A Legowo

Abstract: ***The purpose of this article is to evaluate the performances of the three different controllers such as Linear Quadratic Regulator (LQR), 1-DOF (Degree of Freedom) Linear Quadratic Gaussian (LQG) and 2-DOF LQG based on Quadrotor trajectory tracking and control effort. The basic algorithm of these three controllers are almost same but arrangement of some additional features, such as integral part and Kalman filter in the 1-DOF and 2-DOF LQG, make these two LQG controllers more robust comparing to LQR. Circular and Helical trajectories have been adopted in order to investigate the performances of the controllers in MATLAB/Simulink environment. Remarkably the 2-DOF LQG ensures its highly robust performance when system was considered under uncertainties. In order to investigate the tracking performance of the controllers, Root Mean Square Error (RMSE) method is adopted. The 2-DOF LQG significantly ensures that the error is less than 5% RMSE and maintains stable control input continuously.***

Keywords: LQR; LQG; Quadrotor; trajectory tracking; noise and disturbance rejection; controller robustness.

Nomenclature

Symbol	Description
g	Gravity
J	Moment of inertia
J_r	Inertia of motor
k_f	Thrust coefficient
k_M	Moment coefficient
k_r	Aerodynamic moment drag coefficient
k_t	Aerodynamic thrust drag coefficient
l	Arm length
m	Mass of quadcopter
Ω_i	Angular velocity of i motor
ω	Angular velocity of quadrotor that includes p, q, r

I. INTRODUCTION

Quadrotor is a remotely piloted vehicle of four independent motors that generates lift to fly. It has become popular in movie industry, military surveillance, mapping, agriculture etc. because of its agile movement, stealthy features and high accessibility at some inaccessible area [1-3]. As mostly it is being used for sophisticated applications that require highly accurate tracking, its controllability and maneuverability are the demands for improvement nowadays. As a consequence, researchers work on different types of controller to achieve the promising performance from the controller in order to maintain accurate and precise maneuverability. However, based on controller algorithm, it can be classified by three different types: linear, nonlinear and learning-based controller [4].

Considerably, linear control algorithm encourages the researchers because it is easy to design a linear model and can be applied easily on real-time platform. Since these control algorithms are developed based on linearized model, sometimes the controllers fail to offer promising performance especially in presence of uncertainty to the model [5]. LQR and LQG are considered as linear controller and developed based on linear model of the system. These controllers follow Cost function minimizing approach also known as Optimal control method in order to compute the states of the system. In literatures, it is demonstrated that LQR is quicker in response with low steady state error [6-9]. Several literatures present the significance of LQG controller as a solution of steady state error and state estimation along with one of the best options to achieve satisfactory performance from the system [10, 11]. In the meanwhile, another literature demonstrates that LQG can be adopted for collision avoidance movement of Quadrotor regardless having some of its limitations [12].

In this work, 2 DOF LQG has been introduced on Quadrotor platform for trajectory tracking with high accuracy under uncertainties to the system. In general, Quadrotors are designed based on the type of outdoor applications that implies the influence of uncertainties to the system. Remarkably, the functionalities of 2 DOF LQG such as noise and disturbances rejection with highly accurate tracking performance motivates to carry out the work.

The paper is organized as follows: Section 2 illustrates the dynamic model of quadrotor, section 3 presents control algorithms of the controllers, section 4 demonstrates the simulations with controller parameters and finally section 5 concludes the work focusing on the results and future works.

II. MATHEMATICAL MODELING

The stability and movements of a quadrotor in air depend on its mass and the inertia along with the gravitational force.

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Therefore, proper balance and counterbalance are substantially considered for a smooth flight. However, the fundamental concept should be introduced offhand such as a quadrotor has 4 motors wherein 2 motors rotates along clockwise direction and the other 2 rotates along anti-clockwise direction. The quadrotor considers two different frames of references (i.e. Body frame, B and inertia frame, E) to obtain the movement of Quadrotor along three directions, X, Y and Z axes in 3D space. In addition, the rotation along the axes respectively are known as roll (ϕ), pitch (θ) and yaw (ψ). For better understanding, figure 1 depicts all the notations regarding Quadrotor.

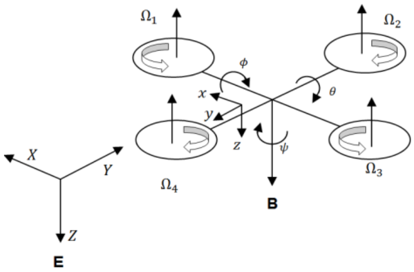

Fig. 1: Quadrotor configuration [13]

As to earlier description, the movements of Quadrotor are carried out based on the proper balance of the thrust of the motors. However, the four control inputs can be considered as:

$$u_1 = k_f(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (1)$$

$$u_2 = k_f(\Omega_4^2 - \Omega_2^2) \quad (2)$$

$$u_3 = k_f(\Omega_1^2 - \Omega_3^2) \quad (3)$$

$$u_4 = k_M(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \quad (4)$$

where, Ω_i = Angular velocity of i motor

Here, u_1 generates lift for moving upward and downward direction while u_2, u_3 and u_4 are responsible to provide roll, pitch and yaw movements respectively. Successively, a mathematical model of Quadrotor needs to be introduced to proceed on controller design. However, the dynamic equations (5)-(10) have been developed using Newton's second law and Newton-Euler equation wherein aerodynamic drag and aerodynamic moment drag have been considered to make the system more accurate [13, 14].

Several parameters have been considered in order to develop the model in MATLAB/Simulink environment as shown in Table 1.

Table 1: Parameters and initial conditions for simulation [15]

Symbol	Value	Symbol	Value	Symbol	Value
J_x	7.5×10^{-3} kg.m^2	k_{t_x}	0.1 Ns/m	k_{r_x}	0.1 Nm.s
J_y	7.5×10^{-3} kg.m^2	k_{t_y}	0.1 Ns/m	k_{r_y}	0.1 Nm.s
J_z	1.3×10^{-2} kg.m^2	k_{t_z}	0.15 Ns/m	k_{r_z}	0.15 Nm.s
J_r	6×10^{-5} kg.m^2	k_f	3.13×10^{-5} Ns^2	g	9.81 m/s^2
l	0.23 m	k_M	7.5×10^{-7} Nms^2	m	0.65 kg

$$\ddot{x} = \frac{-1}{m}[k_{t_x}\dot{x} + u_1(\sin\varphi\sin\psi + \cos\varphi\sin\theta)] \quad (5)$$

$$\ddot{y} = \frac{-1}{m}[k_{t_y}\dot{y} + u_1(\sin\varphi\cos\psi - \cos\varphi\sin\psi\sin\theta)] \quad (6)$$

$$\ddot{z} = \frac{-1}{m}[k_{t_z}\dot{z} - mg + u_1\cos\varphi\cos\theta] \quad (7)$$

$$\dot{p} = \frac{-1}{I_x}[k_{r_x}p - lu_2 - I_y qr + I_z qr + I_r q\omega_r] \quad (8)$$

$$\dot{q} = \frac{-1}{I_y}[-k_{r_y}q + lu_3 - I_x pr + I_z qr + I_r p\omega_r] \quad (9)$$

$$\dot{r} = \frac{-1}{I_z}[u_4 - k_{r_z}r + I_x pq - I_y pq] \quad (10)$$

Moreover, a kinematic relationship between Inertia frame and Body frame should be represented in order to deal with the Euler angle rates and angular velocity at the same time as follows [13, 16]. Noted that Euler angles is computed on inertia frame and angular velocity obtained on Body frame.

$$\dot{\varphi} = p + r\cos\varphi\tan\theta + q\sin\varphi\tan\theta \quad (11)$$

$$\dot{\theta} = q\cos\varphi - r\sin\varphi \quad (12)$$

$$\dot{\psi} = r\frac{\cos\varphi}{\tan\theta} + q\frac{\sin\varphi}{\cos\theta} \quad (13)$$

Subsequently, the states and control inputs of the model should be introduced where the plant has 12 states and 4 inputs respectively as $x_s = [x, y, z, \dot{x}, \dot{y}, \dot{z}, \phi, \theta, \psi, p, q, r]^T$ and $[u_1, u_2, u_3, u_4]^T$.

III. CONTROL ALGORITHM

The objective of this study is to investigate the performance of LQR and two types of LQG as 1 DOF an 2 DOF LQG on Quadrotor platform. Fundamentally, the basic algorithms are almost similar for both LQR and LQG and both follow linear control approach. Hence, a continuous state space model of the plant can be represented as follows [17].

$$\dot{x}_s(t) = Ax_s(t) + Bu(t)$$
$$y(t) = Cx_s(t) + Du(t) \quad (14)$$

where, A is considered as system matrix, B represents input matrix, C represents output matrix and D describes the feedback matrix. Apart from that, x_s represents the state matrix while y represents output matrix.

Noted that before moving towards the designing of state space feedback model, it must be ensured that the state variables, which have been necessarily chosen for computation, are measureable and able to give feedback. In order to carry out the procedure, controllability and observability can be checked if it is full ranked.

According to the first order Taylor series expansion, the linearized model has been derived from nonlinear equations as follows [18].

$$\partial\dot{x}_s(t) = A\partial x_s + B\partial u \quad (15)$$

When the system is linearized around a certain hovering point, (x_{ss}, u_{ss}) where $x_{ss} = [x_{ss}, y_{ss}, z_{ss}, \psi_{ss}]^T$ and $u_{ss} = [mg, 0, 0, 0]^T$.

Here,

$$\partial x = x_d - x_a$$

$$u = u_{ss} + \partial u \quad (16)$$

$$x = x_{ss} + \partial x \quad (17)$$

Now according to the control approach of LQR, a feedback control is required to be designed by following equation [17].

$$u = -K\partial x_{ss} + u_{ss} \quad (18)$$

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where, K is the feedback gain matrix that can be obtained by minimizing the cost function as follows.

$$J = \int_0^{\infty} (\partial x_{xx} Q \partial x_{xx} + \partial u R \partial u) \partial t \qquad (19)$$

where, Q is considered as a 12×12 semi-positive definite matrix and R is a 12×4 positive definite matrices for this quadrotor.

However, LQG is an advanced type of LQR that includes Kalman filter as an additional feature. Kalman filter functions as an estimator and helps the system to reject model uncertainties. For instance, the plant can be represented by stochastic equation as follows.

$$\dot{x} = Ax + Bu + w$$
$$y = Cx_s + Du + v \qquad (20)$$

where, w denotes process noise and v denotes measurement noise to the system.

It is noted that noises are considered as zero mean uncorrelated white Gaussian noise for simplicity. However, for the functionality of LQG, it estimates the states, $\hat{x}$ for full states of x with the help of Kalman filter where $\hat{y}$ is symbolized as estimated output. Then it formulates the error between the current states and estimated states to operate the LQR algorithm. Figure 2 and 3 illustrates the block diagram of 2 DOF and 1 DOF LQG controller respectively.

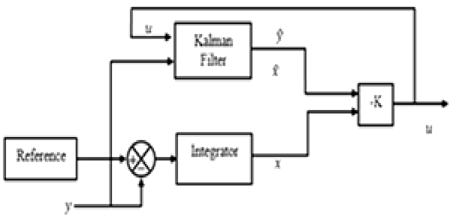

Fig 2: Block Diagram from 2 DOF LQG controller

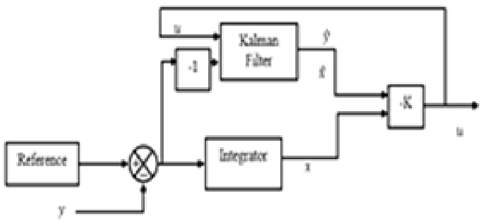

Fig 3: Block Diagram from 1 DOF LQG controller

IV. SIMULATED RESULTS

As to the goal of this work, the performance of the controllers can be juxtaposed based on their dealing capability of model infidelity. Before going directly to the comparison among the controllers, the parameters should be introduced that help to achieve the performances of the controllers.

Hence, for LQR, Q and R matrices have been shown in table 2.

Table 2. Q and R matrices for LQR controller

Q	R
$\begin{pmatrix} 10&0&0&0&0&0&0&0&0&0&0&0 \\ 0&20&0&0&0&0&0&0&0&0&0&0 \\ 0&0&80&0&0&0&0&0&0&0&0&0 \\ 0&0&0&5&0&0&0&0&0&0&0&0 \\ 0&0&0&0&5&0&0&0&0&0&0&0 \\ 0&0&0&0&0&15&0&0&0&0&0&0 \\ 0&0&0&0&0&0&10&0&0&0&0&0 \\ 0&0&0&0&0&0&0&20&0&0&0&0 \\ 0&0&0&0&0&0&0&0&100&0&0&0 \\ 0&0&0&0&0&0&0&0&0&8&0&0 \\ 0&0&0&0&0&0&0&0&0&0&5&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$	$\begin{pmatrix} 1&0&0&0 \\ 0&0.1&0&0 \\ 0&0&0.1&0 \\ 0&0&0&0.1 \end{pmatrix}$

For LQG, the process noise, Q_n and measurement noise covariance, R_n data has been plotted on table 3.

Table 3. Q_n and R_n matrices for LQG controller

Q_n	R_n
$5I_{12\times12}$	$I_{12\times12}$

Here, table 4 depicts the weighing matrix of LQG controller.

Table 4. Q_{XU}, Q_{WV} and Q_I matrices for LQG controller

Q_{XU}	Q_{WV}	Q_I
$10^{-5} I_{16\times16}$	$\begin{pmatrix} 5I_{12\times12} & 0 \\ 0 & 5\times10^{-5} I_{12\times12} \end{pmatrix}$	$I_{12\times12}$

Now, the tracking performance of the controllers can be demonstrated with the help of Root Mean Square Error method. Noted that two different trajectories such as helical and circular have been adopted in order to investigate the performance where considered noise and disturbances are respectively [1, 1, 1] along X, Y and Z axes and [4; 4; 4; 4] at control input. Figure 4 and 5 depict the performance of the controllers in both circular and helix trajectories in presence of model uncertainties in the system.

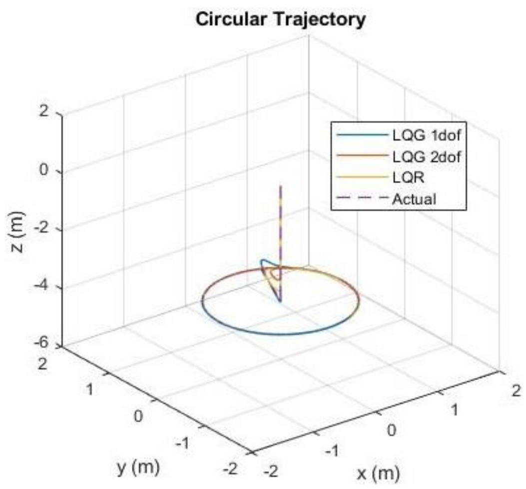

Fig 4: Circular trajectory without disturbance

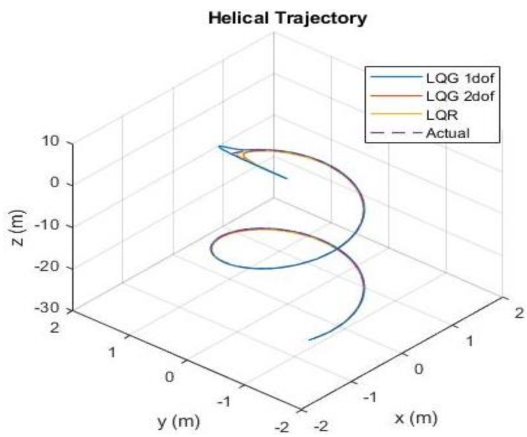

Fig 5: Helical trajectory without disturbance

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Table 5. RMS Error of the controllers for circular trajectory in three different conditions

	Case with disturbance and noise (%)			Case with disturbance (%)			Case without disturbance and noise (%)		
	x	**y**	**z**	**x**	**y**	**z**	**x**	**y**	**z**
2 DOF LQG	1.1	1.2	0.0	0.5	0.7	0	0.6	0.7	0.0
1 DOF LQG	0.4	1.6	0.6	0.9	2.2	0.7	0.9	2.3	0.7
LQR	4.2	1.1	9.8	3.7	0.1	9.8	3.2	1.1	0.8

Table 6. RMS Error of the controllers for helical trajectory in three different conditions

	Case with disturbance and noise (%)			Case with only disturbance (%)			Case without disturbance and noise (%)		
	x	**y**	**z**	**x**	**y**	**z**	**x**	**y**	**z**
2 DOF LQG	0.9	0.5	0	0.5	0.7	0	0.5	0.7	0
1 DOF LQG	0.3	2.1	1.6	0.8	2.2	1.6	0.8	2.2	1.6
LQR	3.7	2.9	1.3	3.0	1.5	1.3	0.6	1.1	0.8

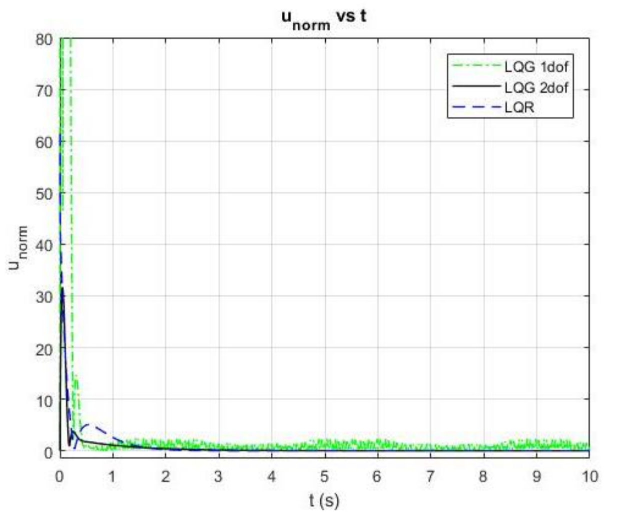

Fig 6: Control efforts against time

Tables 5 and 6 underscored significantly the performance of 2 DOF LQG as comparatively more suitable controller for Quadrotor. Less than 5% RMSE, necessarily indicates that the robustness of the controller is better than 1 DOF LQG and LQR.

A graph of control inputs norm can help to compare their performances of the controllers in terms of control effort parameter. It is considered as one of the most important concerns to controller designers because of motor safety and system stability. Figure 6 depicts the least control input norm is generated by 2 DOF LQG controller and it offers the highest stability comparing to others.

V. CONCLUSION

This study evaluates three different controllers as LQR, 1-DOF LQG and 2-DOF LQG based on two different performance indexes such as control efforts and trajectory tracking. LQR is a very simple MIMO system controller that doesn't offer the estimator or integral part and as a result, it is unable to tackle any noise, disturbance or steady state error. However, it provides the fastest response along with small settling time and overshoot that spur on researchers to work on it. Noted that angular velocity saturation limit has not been considered in this work.

In the meanwhile, the 1-DOF LQG performs comparatively better than LQR in trajectory tracking but it offers remarkably high control effort. Here, at the beginning, 1 DOF LQG estimator estimates states of the system that causes high control effort and later it becomes stable when controller starts to work on.

In addition, the novelty of this work is the introduction of the 2-DOF LQG controller. Among these three controllers, the 2 DOF LQG performs in the highest efficient manner and offered promising performance according to both evaluation factors. For tracking, it notably ensures less than 5% RMSE under disturbance and noise and least control effort as well. Hence, the 2-DOF LQG can be considered as a highly robust controller and can be applicable for any outdoor application of Quadrotor.

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