The whirling frequency of high-speed shaft with torsional effect

A. M. A. Wahab, S. A. H. Roslan, Z. A. Rasid, A. Abu, N. F. M. N. Rudin, F. Yakub

Abstract--- The study on whirling frequency of high speed shaft is necessary as the current trend of rotor system is to operate at high speed. Fur-thermore most studies on whirling frequency of shaft have developed formulations that are based on the Timoshenko beam theory that neglects the torsional effect of shaft. In this research, the effect of boundary condition and slenderness ratio on the whirling frequency behavior of high speed shaft is studied. A finite element formulation for the whirling frequency problem of shaft has been developed based on the 5 degree of freedom per node element that includes the torsional degree of freedom. The derived Mathieu-Hill equation is solved using the Bolotin's method. It was found that the effect of torsional degree of freedom on the whirling frequency behaviour is significant especially for shaft at high speed where at the spin speed of 20000 RPM, the difference between the whirling frequencies corresponds to FEM model that considers torsional degree of freedom and model that does not consider can be as high as 45.3%.

Keywords: Whirling frequency, Critical speed; Mathieu-Hill equation; Bolotin's method

1. INTRODUCTION

Shaft is a very important mechanical component of a rotor system that plays role of transferring power. In recent years, the demand for high efficient machine has led to the requirement of having high speed rotating shaft [1]. Shaft always comes with vibration problem especially at high speed that the problem must be considered even at the design stage. Whirling is an important lateral vibration of shaft that may lead to failure of machine system. It is known that resonance occurs when the spin speed is equal to whirling frequency of shaft and due to gyroscopic effect of shaft, whirling frequency is split into two while it varies with the change of the spin speed. The frequency can be the forward frequency (FF) when it increases with spin speed as whirling occurs in the rotational direction of the shaft while backward frequency (BF) is when the frequency decreases with the increase of the spin speed as the shaft whirls in the direction opposite to the shaft rotation. This behavior is usually captured in the so called Campbell diagram. Furthermore, at high rotating speed, a shaft may also carry high torque but the effect of this torque on the whirling frequency of the shaft has been hardly considered.

Shagholi et al. [2] studied the free vibration analysis of a nonlinear slender rotating shaft with simply support condition. The formulation considered the system rotary inertia and gyroscopic effect but neglected the effect of shear deformation. Utilizing the multiple scales method, it was seen that the backward and forward modes were involved as whirling frequencies of shaft. Furthermore, the study found that the values of nonlinear forward and backward frequencies were lower in the second mode compared to first mode. In their study on the combined effects of bearing and rotor asymmetry on the stability of the classical Laval rotor using analytical techniques, Mailybaev and Spelsberg [3] found that the instability region of the shaft-bearing system around whirling speed cannot completely be removed by increasing the external asymmetry. In addition, a secondary resonance zone appeared as the asymmetric in rotor and bedding were taking place together. However, as the symmetric condition was considered independently, the secondary resonance was disappeared. Shahgholi and Khadem [4,5] investigated parametric instability of simply supported rotating asymmetry shaft with large amplitude. The analytical investigation using multiple scales method showed that parametric resonances of an asymmetrical shaft were not excited at the backward whirling mode. Comparing asymmetric and symmetric shaft, bifurcation can occur for the speeds either higher or lower than linear forward frequency for asymmetric shaft. Besides that, the asymmetrical shaft showed significance of balancing more than symmetrical shaft. In a study on the backward whirl of a cracked rotor, Al-Shudeifat [6] found that experiment results and numerical calculation produced the similar result that the unstable zones of the cracked rotor have been found to only appear at the backward whirl. In a different method of analysis, free vibrations of shafts supported on resilient bearings were analyzed by Karunendiran et al. [7]. Using the Timoshenko beam theory, the exact frequency equation in the complex compact form was derived. The above excellent studies show how research in the area of whiling frequency of shaft has progressed within this decade. None of the studies however has focused on the whirling frequency of high speed rotating shaft. Furthermore the studies are clearly lacking in considering the torsional effect of shaft on its whirling frequency behavior.

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In this research, the effect of torsional motion on the whirling frequency of high speed rotor system is studied where the spin speed of up to 40000 RPM is considered. The simple rotor system considered here consists of a shaft and bearings. This study focuses on the effect of boundary conditions and slenderness ratio on the whirling frequency behavior of the shaft. The Mathieu-Hill equation that represents the whirling frequency behavior of the shaft has been developed based on the finite element method (FEM). The torsional degree of freedom (DOF) based on the Nelson's finite element model [8] is added to the typical DOF given by the Timoshenko's beam theory. Using the Bolotin's method [9] to solve the Mathieu-Hill's equation, the FF and BF of the shaft can be determined and the Campbell's diagram can be plotted. The FEM model in this study applies 5 DOF per node element that includes the torsional DOF and as such the model is called the 5DOF model. By doing so, the results can be compared to the results due to a model that uses 4 DOF per node element that is called the 4DOF model.

2. MATERIAL AND METHOD

The geometry and material properties of the shaft system are given in this section. Following that the Mathieu-Hill's equation for whirling frequency of rotor system based on FEM is derived.

2.1. Dimensions and Material Properties

Figure 1 shows the simple rotor system considered in this study that consists of a shaft and two bearings. Rigid bearings are assumed here. d is the diameter of the shaft while L is the length of the shaft. Three types of boundary conditions considered here are the simply support (P-P), the clamped support at one end (C-F) and the combination of clamped support at one end and simply support at another end (C-P). Table 1 shows material properties and dimensions of the shaft of the rotor system.

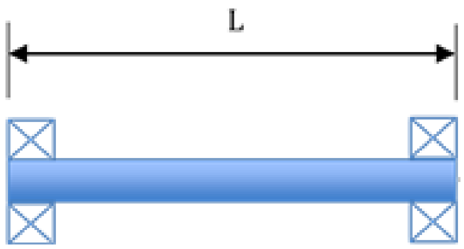

Fig. 1: The shaft understudy: L = length of shaft.

Table 1: Geometry and material properties of the shaft

Item	Value
Young's modulus, E	207 GPa
Modulus of rigidity, G	79.6 GPa
Poisson's ratio, v	0.303
Density, ρ	7833 kg/m^3
Radius, r	0.0508 m
Length, L	1.27 m
Shear correction factor, κ	0.9
Yield strength, S_y	400 MPa

2.2. The FEM formulations

Following the standard procedure in deriving the FEM formulation, the constitutive and strain equations are combined in the energy equation where the potential energy is

$$U = \frac{1}{2}\int_{-a}^{+a} EI_p(\acute{\theta}_z^2 + \acute{\theta}_y^2)\,dx + \frac{1}{2}\int_{-a}^{+a} \kappa GA(\acute{v} - \theta_z)^2 + \kappa GA(\acute{w} + \theta_y)^2 dx + \frac{1}{2}\int_{-a}^{+a} GJ\acute{\theta}_x^2\,dx - \frac{1}{2}\int_{-a}^{+a} P[(\acute{v})^2 + (\acute{w})^2]dx \quad (1)$$

where I_y, I_z and J are moment of inertia with respect to *y*-axis, moment of inertia with respect to *z*-axis and polar moments of inertia per unit length of shaft element respectively. Furthermore I_p is the second moment of inertia of the shaft element, $\acute{\theta}$ is the slope in rotational direction and $\acute{v}$ is the slope in vertical direction. After applying the Euler angle transformation, the total kinetic energy is derived as;

$$T = \frac{1}{2}\int_{-a}^{+a} \rho A((\dot{v})^2 + (\dot{w})^2)\,dx + \frac{1}{2}\int_{-a}^{+a} \rho I_y(\dot{\theta}_y)^2 + \rho I_z(\dot{\theta}_z)^2\,dx + \int_{-a}^{+a} \rho I_x \theta_z \dot{\theta}_y\,dx\,\Omega + \frac{1}{2}\int_{-a}^{+a} \rho I_x(\dot{\theta}_x)^2\,dx \quad (2)$$

Upon discretization of shaft and following standard derivation procedures, the Lagrange's equation gives

$$[M]\{\ddot{q}\} + \Omega[G]\{\dot{q}\} + ([K] - P(t)[K_g])\{q\} = 0 \quad (3)$$

where $[M]$, $[G]$, $[K]$ and $[K_g]$ are the mass, gyroscopic, stiffness and geometric stiffness matrices, respectively. Ω and $P(t)$ are the spin speed and the periodic axial force. The periodic axial force, $P(t)$ is

$$P(t) = P_s + P_t \cos\phi t = \alpha P^* + \beta P^* \cos\phi T \quad (4)$$

where P_s and P_t are the static and the dynamic of force component. P^* is the fundamental static buckling load while α and β are the static and dynamic load factors respectively. Inserting $P(t)$into (3), the Mathieu-Hill's equation can be obtained as

$$[M]\{\ddot{q}\} + \Omega[G]\{\dot{q}\} + ([K] - [K_g](\alpha P^* + \beta P^* \cos\phi T\,(t))\{q\} = 0 \quad (5)$$

Applying the Bolotin's method [9], an infinite eigenvalue problem can be derived such as

$$(\phi^2[M] + \phi[G] + K])q = 0 \quad (6)$$

where equation (6) actually represents the whirling frequency problem to be solved in this study.applied slenderness ratio, r_s = L/r where L is the length of the shaft.

3. RESULTS AND DISCUSSION

The effects of boundary condition and slenderness ratio on the whirling frequency of high-speed shaft considering the torsional effect are given in this section.

3.1. The Effect of Boundary Conditions

In this study, the effect of boundary condition towards whirling frequency of rotating shaft is determined. Fig. 2-3 show the Campbell diagram of shafts in the 1st and 2nd mode respectively corresponds to the 4DOF and 5DOF models for different types of boundary conditions.

In static condition, for both models of 4DOF and 5DOF, the C-P boundary condition gives the highest whirling frequencies, followed by the P-P and the C-F boundary conditions for both 1st and 2nd modes of frequencies. Table 2 and Table 3 show the whirling frequency of the static shaft system under C-P, C-F and P-P boundary conditions for the 1st and 2nd modes respectively. The small significance of the effect of the additional torsional DOF as in the 5DOF model towards whirling frequency can be observed at C-F boundary condition for the 1st mode compared to other boundary conditions, with 2.43 % difference in comparison between the 4DOF and 5DOF model.

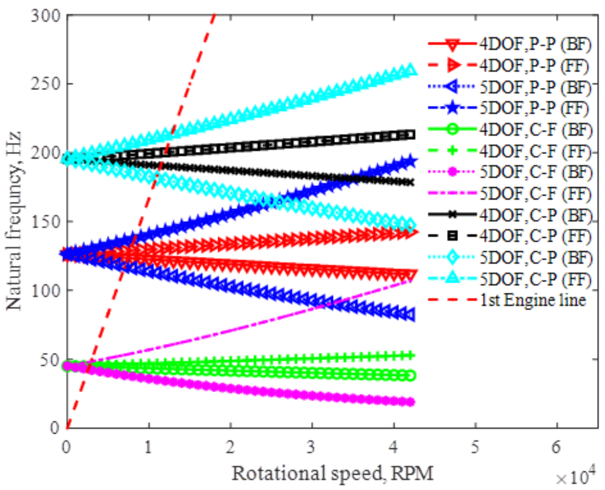

Fig. 2: The effect of boundary conditions on the 1st mode whirling frequency of shaft

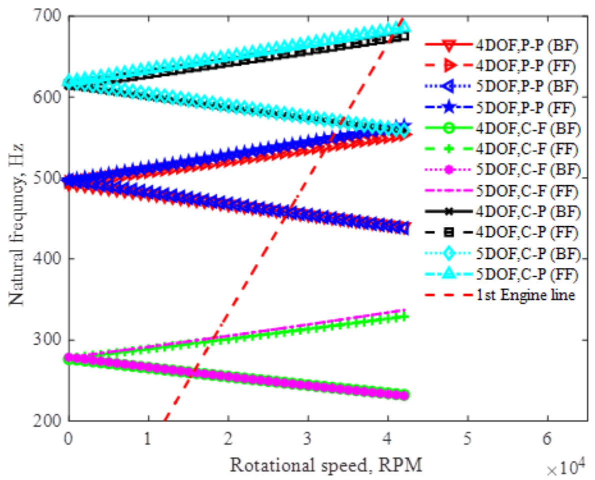

Fig. 3: The effect of boundary conditions on the 2nd mode whirling frequency of shaft

Table 2: The 1st mode whirling frequency of shaft for different boundary conditions

Boundary condition	Whirling frequency, Hz 4DOF	Whirling frequency, Hz 5DOF	Differences (%)
C-P	195.6	194.0	-0.82
P-P	126.4	125.3	-0.87
C-F	45.2	46.3	2.43

Table 3: The 2nd mode whirling frequency of shaft for different boundary conditions

Boundary condition	Whirling frequency, Hz 4DOF	Whirling frequency, Hz 5DOF	Differences (%)
C-P	616.3	619.1	0.45
P-P	495.8	497.5	0.34
C-F	276.3	278.7	0.86

As the spin speed is increased, Figures 2 and 3 shows that the FF and BF of the shaft behave as predicted where the FF increases with the spin speed while the BF decreases with the spin speed. The difference between the 4DOF model and the 5DOF model can be obviously seen as the spin speed is increased. Figure 4 and Figure 5 show the percentage differences of the 1st and 2nd modes of frequency, respectively given by the 4DOF and 5DOF model as rotating speed increases for all P-P, C-F and C-P boundary conditions. For the 1st mode, the difference increases as rotating speed increases for all boundary conditions with the C-F boundary condition shows the highest percentage difference of 45.3% and 31.0% at 20000 RPM for the FF and BF respectively. The second highest is the P-P boundary condition with the difference of 16.4% and 13.8% for the FF and BF respectively. C-P boundary condition shows the lowest but still having significant differences with 10.1% and 8.7% for the FF and BF respectively. This shows that the effect of torsional motion that comes as the additional DOF provided by the 5DOF model towards estimating the whirling frequency of shaft for rotating shaft is significant for all cases of boundary conditions. On the other hand, the effect of torsional DOF is not significant in estimating the whirling frequency of shaft as rotating speed increases for the 2nd mode since the percentage differences is below than 1.3%.

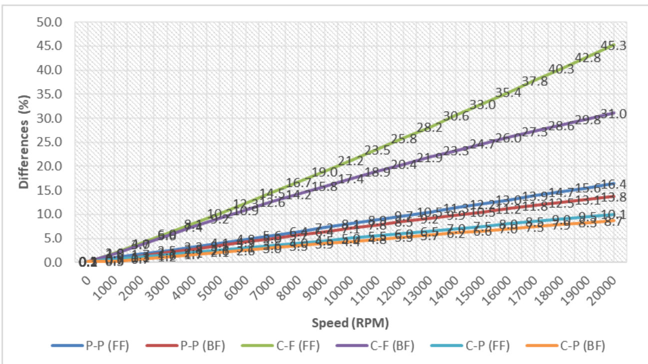

Fig. 4: The percentage differences in the 1st mode frequency

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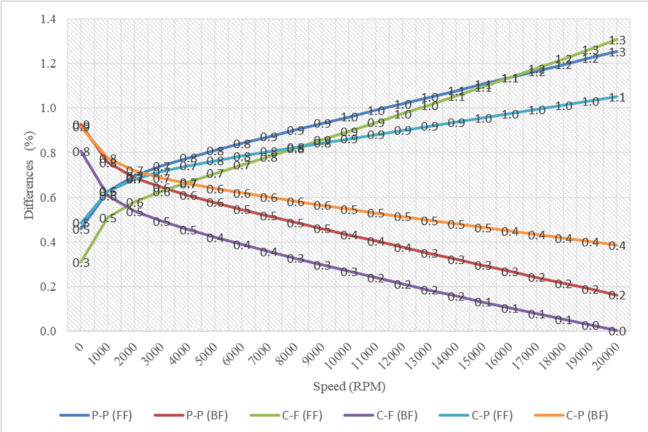

Fig. 5: The percentage differences in the 2nd mode frequency

3.2. The Effect of the Slenderness Ratio

The effect of slenderness ratio on the whirling frequency behaviour of shaft is given in this section. A rotating shaft with different cross-sectional areas but having constant length is used for this purpose. Table 4 shows the comparison on the effect of slenderness ratio towards whirling frequency for different boundary conditions of P-P, C-F and C-P in static condition of shaft. It can be said that as the slenderness ratio decreases, the whirling frequency of shafts corresponds to all boundary conditions and modes decreases. For both models of the 4DOF and 5DOF, reducing the slenderness ratio will decrease the whirling frequency of the shaft system which is in in line with the results produced by Chen and Peng [10].

Besides that, low slenderness ratio reduces the differences between the FF and BF whirling frequency for the 4DOF model. The differences between 4DOF and 5DOF are below than 1% for all boundary conditions and slenderness ratio. Thus, it shows that in static condition of shaft, the effect of additional torsional element is not significant in estimating whirling frequency for both shafts having high and low slenderness ratio at P-P, C-F and C-P boundary conditions.

Figures 6-7 show the effect of the slenderness ratio on the 1st mode whirling frequency of shaft for the P-P, C-F and C-P boundary conditions respectively having spin speed of up to 40000 RPM. They also show that at the 1st mode, the values for whirling frequency of FF and BF directions are almost similar at low slenderness ratio for all boundary conditions as rotating speed increases. In the case of the 5DOF model, the FF and BF show similar pattern at high slenderness ratio with slightly reducing in the gap between them.

Table 4: The effect of slenderness ratio on the whirling frequency of shaft in static condition

Boundary condition	Model	L/r	ω_1 (Hz)	ω_2 (Hz)	ω_3 (Hz)
P-P	4DOF	100	63.452	253.316	562.663
		200	126.108	492.797	1071.69
	5DOF	100	63.489	252.900	565.523
		200	126.420	497.362	1091.95
C-F	4DOF	100	22.629	141.011	391.495
		200	45.092	276.338	749.121
	5DOF	100	22.635	141.274	393.176
		200	45.176	278.566	761.694
C-P	4DOF	100	98.872	317.921	656.399
		200	195.045	613.33	1230.971
	5DOF	100	98.936	318.682	659.752
		25	195.564	618.951	1253.450

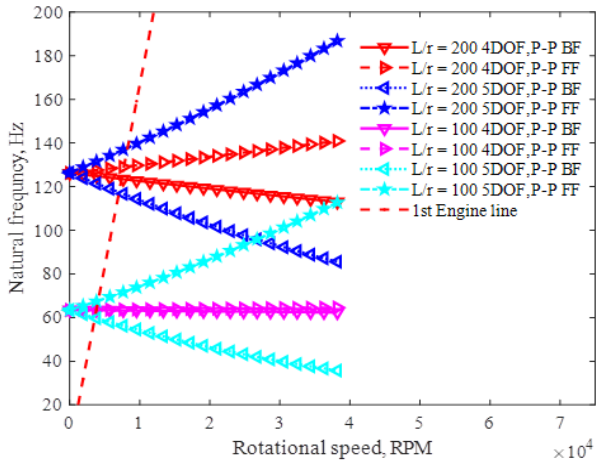

Fig. 6: The effect of slenderness ratio on the 1st mode of whirling speed of shaft for P-P boundary condition

Table 5 shows the comparison on critical speed of rotating shaft under P-P, C-F and C-P boundary conditions with different slenderness ratio. The table shows that higher slenderness ratio produces higher critical speed for all boundary conditions in both BF and FF directions. This show that at high slenderness ratio, the shaft can rotate at high speed compared to low slenderness ratio since the critical condition can occur only at higher speed. The differences between the 4DOF model and 5DOF model are in between 1% - 6% for all boundary conditions for both high and low slenderness ratio. This result shows the effect of torsional DOF on the critical speed is quite significant as slenderness ratio of the shaft changes in the system.

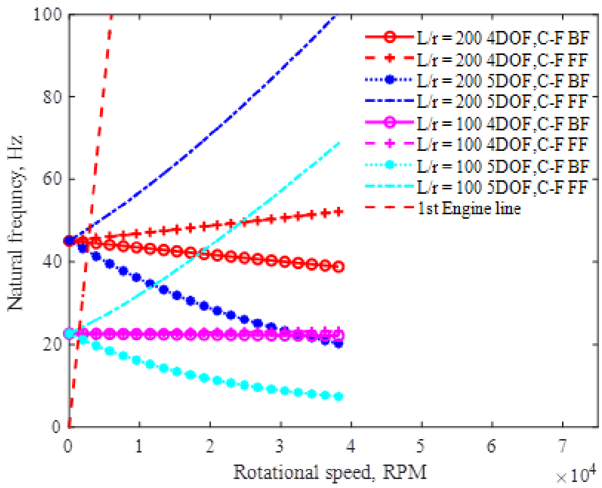

Fig. 7: The effect of slenderness ratio on the 1st mode whirling speed of shaft for C-F boundary condition

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Table 5: The comparison of the 1st mode of critical speed for different slenderness ratios and boundary conditions

Boundary conditions	L/r	Whirl direction	Critical speed, (RPM) 4DOF	Whirl speed, (RPM) 5DOF	Diff. (%)
P-P	100	BF	3650	3801	4.14
		FF	4068	3810	-6.34
	200	BF	7047	7420	5.29
		FF	8270	7744	-6.36
C-F	100	BF	1299	1356	4.39
		FF	1432	1365	-4.68
	200	BF	2559	2683	4.85
		FF	2903	2741	-5.58
C-P	100	BF	5605	5930	5.80
		FF	6331	5947	-6.07
	200	BF	10890	11430	4.96
		FF	12800	12610	-1.48

CONCLUSION

A study on the whirling frequency behavior of high speed shaft considering the torsional effect of the shaft is conducted. The focus of this paper is on the effect of the boundary condition and the slenderness ratio of the shaft. It was found that the C-P boundary condition gives the highest whirling frequency of shaft and the higher the slenderness ratio, the higher the whirling frequency and the critical speed are. The effect of adding torsional DOF in the formulation is less significant in the static condition of shaft but as the spin speed increases, the difference made by the torsional DOF can be as high as 45.3%.

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