

# Development of Shear Strength Expression for RC Corbels using Strut-and-Tie Model

Katari Durga Bhavani, J.D.Chaitanaya Kumar, M.L. Sai Ranga Rao

**Abstract:** Corbels are sections that project from the essences of columns and are used widely in precast concrete construction to support primary beams and girders. On account of the predominance of precast concrete, the design of corbels has become increasingly important. In this paper, a simple analytical expression is proposed for predicting the shear strength of reinforced concrete corbels using “Strut-and-Tie” Model. The proposed expression is compared with some of the existing expressions and ACI 318-14 code. The 178 corbels specimen’s experimental data has been collected from the literature and find out the unknown parameters. The proposed model accounted for compressive strength of concrete, shear span -to-depth ratio, breadth and effective depth. The results indicate that proposed expression as one the best fitting expression to predict the shear strength of RC corbels.

**Index terms:** Strut-and-Tie Model, Shear Span-to-depth ratio, Shear strength, Corbels.

## I. INTRODUCTION

Reinforced concrete (RC) corbel is a structural member, Corbel is a short-hunched cantilever used to support the reinforced concrete precast beams. Usually, shear span-to-depth ratio ( $a/d$ ) of corbel is lesser than or equal to one. It makes the corbel behave in a two-dimensional manner. Corbels are primarily designed to resist vertical loads and horizontal forces owing to restrained shrinkage, thermal deformation, and creep of the supported. Corbels cast monolithically with the column element (or) wall element.

Corbels are commonly termed as a discontinuity region (D-region), where “plane section remains plane” assumption is not valid. Different design procedure shall be adopted for regions with discontinuity in the structural member, such as areas, zone of load application (or) areas with a sudden change in the geometrical dimensions such as brackets and portal frames. Corbels are designed mainly to provide for the vertical reaction  $V_u$  at the end of the supported beam, and sometimes they must also resist a horizontal force  $N_{uc}$  transmitted from the reinforced beam due to restrained shrinkage, creep, or temperature change. In general, strut-and-tie model of a structure is an effective tool for the analysis and design of concrete members especially for D-regions.

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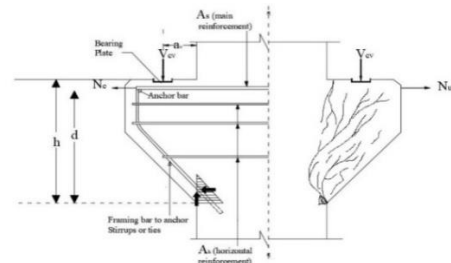


Figure 1: Typical shear failure in corbel

A strut-and-tie (STM) model visualizes a truss-like system in the structure and transmits forces from loading points to the supports. Strut resists compressive force applied on the member. They are primarily made of concrete or a combination of concrete and compression reinforcement. Ties are tensile members in the strut-and-tie model. They are made of a combination of reinforcement and concrete. However, the members like deep beams, corbels, beam-column joints and shear walls are designed with STM, Development of shear strength expression is needed for the following reasons: first, to limit the cracks under service loads and second, to limit the concrete contribution. Further, existing limiting expressions do not explicitly consider the effect of tension reinforcement ratio, shear span-to-depth ratio and size effect. Therefore, in this paper, an empirical expression is developed by considering the influencing phenomena and factors.

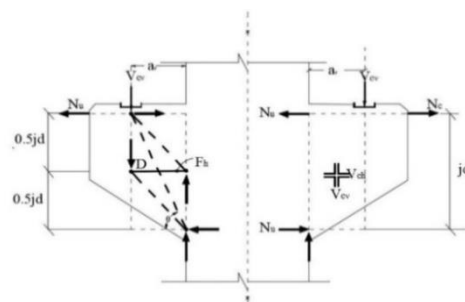


Figure 2: Strut and tie model for internal force

## II. RESEARCH SIGNIFICANCE

This paper reported a basic scientific expression proposed for predicting the shear strength of reinforced concrete corbels using “Strut-and-Tie” Method. The proposed expression is compared with some of the existing Expressions and ACI 318-14 code. The 178 corbels



specimen's experimental data has been collected from the literature and find out the unknown parameters. Experimental database of 178 samples is collected from the literature to validate the limiting expressions. Besides, the effect of tension reinforcement ratio, shear span-to-depth ratio and size effect was incorporated. The proposed model is safe, economic and consistent.

### III. EXPERIMENTAL DATA & RESULTS

The important parameters which are controlling the shear strength of corbels, based on previous research work are given below:

$a$  = shear span, i.e. distance from column face to resultant of vertical load, mm

$b$  = width of corbel, mm.

$d$  = effective depth of corbel measured at column face, mm

$f'_c$  = Compressive strength of concrete, MPa

$\rho$  = reinforcement ratio at column face,

$$\rho = \frac{A_v + A_s}{bd}$$

$v_u$  = nominal shear stress at ultimate strength, kN

$$v_u = \frac{V_u}{bd}$$

$V_u$  = vertical load at ultimate strength, i.e. shear at ultimate strength, kN

$\phi$  = capacity reduction factor

$A_s$  = area of tension reinforcement (mm<sup>2</sup>)

$A_v$  = total area of horizontal closed stirrups (mm<sup>2</sup>)

$a$  = shear span from concentrated load to face of column

$a_s$  = depth of diagonal strut

$D$  = compression force at diagonal strut (negative for compression)

$E_c$  = elastic modulus of concrete

$E_s$  = elastic modulus of steel bar

$F_h$  = tension force in horizontal tie (positive for tension)

$F_{yh}$  = yielding force of horizontal tie

$h$  = direction of horizontal hoop

$H$  = overall thickness of corbel

$j, k$  = coefficients

$N_c$  = tensile force applied at top of and corbel acting simultaneously with  $V_{cv}$  (positive for tension)

### IV. EXISTING EXPRESSIONS

#### A. ACI 318-2014

For critical members which is at maximum shear strength that is designed derived from Eq.1 given by ACI 318-14. The product of reduction factor and previous code eq which is designed results in obtaining actual equation more conservative. By using compressive strength of concrete and the cross-sectional area of corbels the maximum shear strength design can be calculated. A limit has been developed in order to decrease the cracking within service loads and to protect the corbels against diagonal compression failure.

$$V_n^{(ACI\ 318-11,14)} = \phi 0.83 \sqrt{f'_c} bd \quad (1)$$

Where  $\phi = 0.75$ .

#### B. RUSSO EXPRESSION

To predict the shear strength of corbels by means of single accurate expression to keep away time consuming computing procedures. The main commitment was derived from a limiting shear expression, while the second commitment was derived from the equilibrium of the strut-and-tie mechanism in presence of stirrups.

The ultimate shear strength of corbel Eq. (2)

$$v_u = 0.5(k \cdot \chi \cdot f'_c \cdot \cos\theta + 0.65 \cdot \rho_h \cdot f_{yh} \cdot \cos\theta) \quad (2)$$

Where  $k$  is acquired from the classical bending theory of reinforced concrete (RC) beams with tensile reinforcement only.

$$k = \sqrt{(n \cdot \rho_f)^2 + (2 \cdot n \cdot \rho_f) - (n \cdot \rho_f)}$$

$n$  = the ratio of the elastic moduli of steel and concrete

$$n = \frac{E_s}{E_c}$$

$\rho_f$  = flexural reinforcement ratio

$$\rho_f = \frac{A_s - A_n}{b \cdot d}$$

$$A_n = \frac{N_u}{f_{ys}}$$

$f_{ys}$  = yielding strength of the main reinforcement.

$f_{yh}$  = yielding strength of the stirrups.

$\rho_h$  = the stirrups ratio at column-corbel interface

$$\rho_f = \frac{A_h}{b \cdot d}$$

$\theta$  = is the angle between compressed concrete strut and the vertical direction.

$$\theta = 2 \cdot \arctan \left[ \frac{-1 + \sqrt{\left(\frac{a}{d}\right)^2 + 0.22(1 - \frac{k^2}{4})}}{\frac{a}{d} - \frac{k}{2}} \right]$$

$$\chi = \left[ 0.74 \left( \frac{f'_c}{105} \right)^3 - 1.28 \left( \frac{f'_c}{105} \right)^2 + 0.22 \left( \frac{f'_c}{105} \right) + 0.87 \right]$$

#### C. SIAO EXPRESSION

The ultimate shear capacity of corbel Eq. (3) may be calculated as follows:

$$v_u = 1.8 \cdot f_t \cdot b \cdot d \quad (3)$$

Where  $f_t$  is the allowable tensile strength of tension tie in refined compression strut.

$$f_t = 7 \sqrt{f'_c} (1 + n) \rho_h \sin^2 \theta + \rho_v \cos^2 \theta$$

$f_y$  = is the steel yield strength.

$n$  = is ratio of the elastic moduli of steel and concrete,

$$n = \frac{E_s}{E_c}$$

$\rho_h, \rho_v$  = steel reinforcement ratio of horizontal and vertical bars, respectively

$\theta$  = is the angle of compression strut to tension tie.



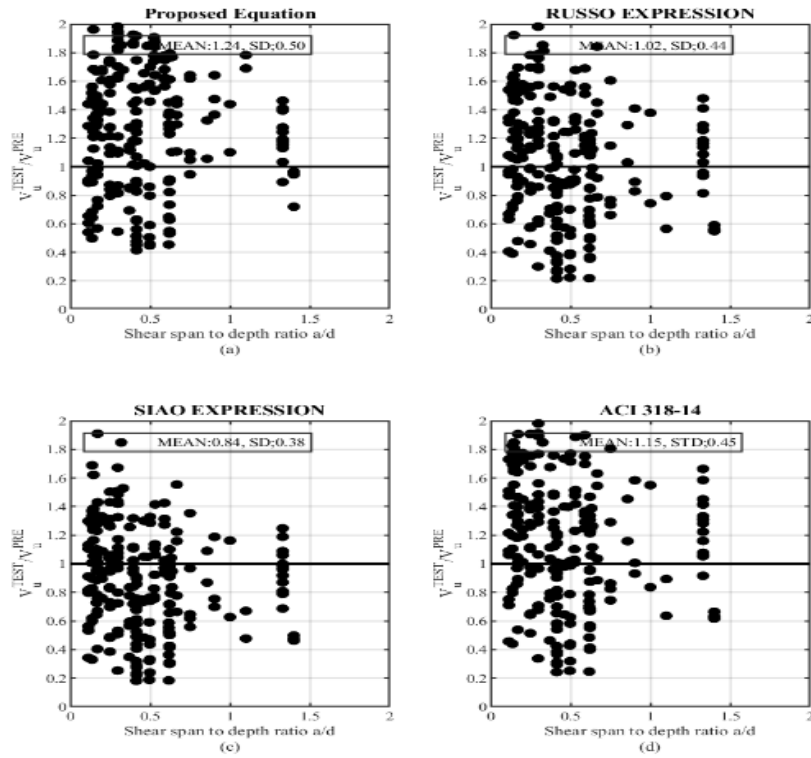


Figure 3. Experimental/Predicted Shear Strength ( $V_{u,TEST}/V_{u,PRED}$ ) vs. shear span-to-depth ratio ( $a/d$ )

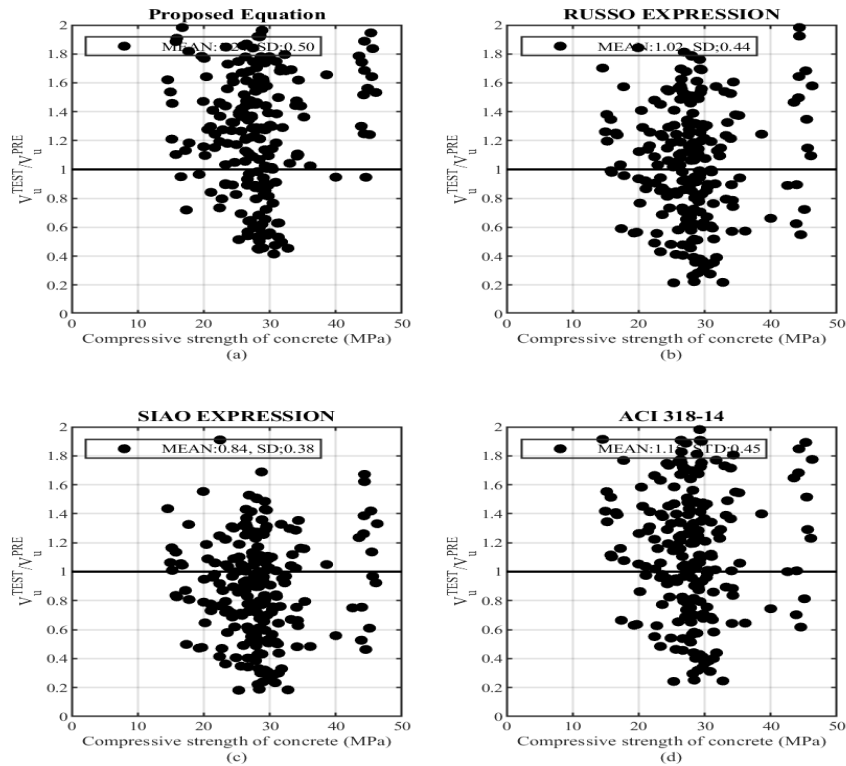


Figure 4. Experimental/Predicted Shear Strength vs. compressive strength of concrete

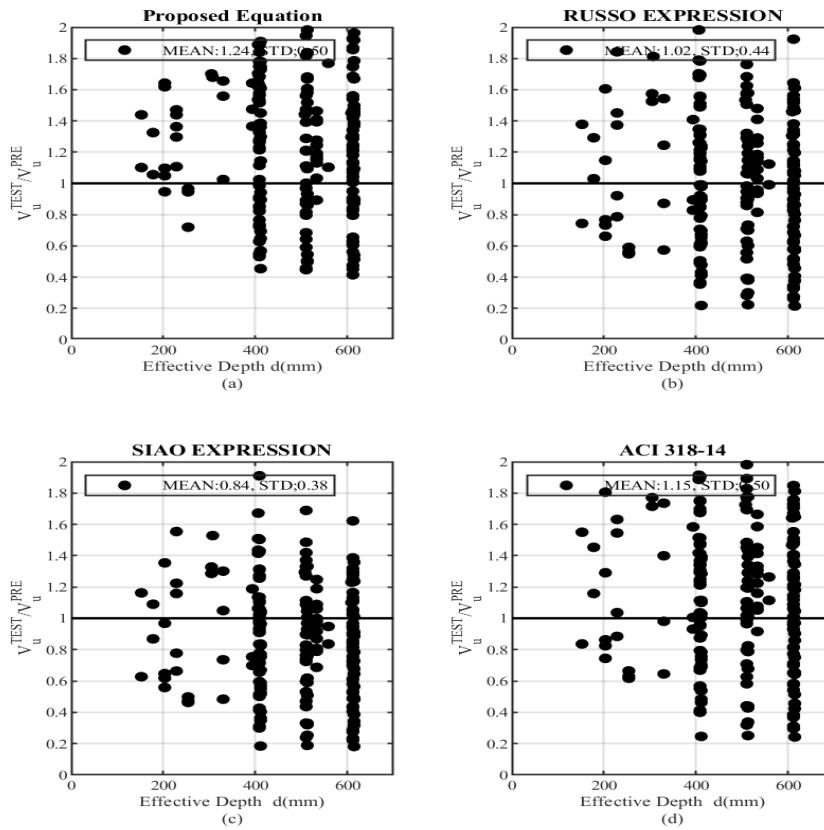


Figure 5. Experimental/Predicted Shear Strength vs. effective depth

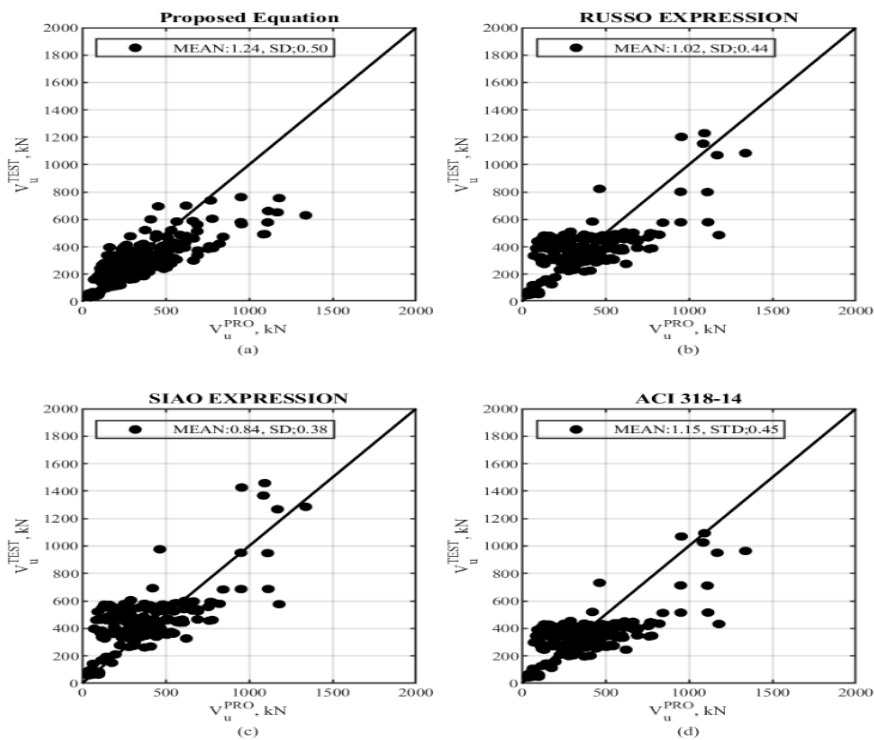


Figure 6. Experimental ( $V_u^{TEST}$ ) vs. predicted shear strength ( $V_u^{PRE}$ )

## V. MODIFIED MAXIMUM SHEAR STRENGTH

The influence of shear span-to-depth ratio and percentage tension reinforcement, the generic equation from ACI 318-14 is mentioned below.

$$V_n^{PRO} = \frac{f_c^{a1} \rho^{a2} b d^{a3}}{\left(\frac{a}{d}\right)^{a4}} \quad (4)$$

To determine the coefficients, namely,  $a_1$ ,  $a_2$ ,  $a_3$ , and  $a_4$ , ( $V_n / (b d f'_c)$ ) are plotted with ( $a/d$ ) ratio,  $f'_c$ ,  $b$  and  $d$ . From the literature the best-fit curve of ( $V_n / (b d f'_c)$ ) vs. ( $a/d$ ) is  $\left(\frac{a}{d}\right)^{-1.0}$ , and the trend decreases exponentially. Therefore,  $\left(\frac{a}{d}\right)^{a4}$  is replaced by term  $(\log_{10}[A + \frac{a}{d}])^B$ , where  $A$  and  $B$  are the empirical constants. Next, to consider the size effect, ( $V_n / (b d f'_c)$ ) is plotted with depth, and the best-fit curve is identified as  $d^{(-0.25)}$ . Based on trial and error method parameters like  $a_1$ ,  $a_2$ ,  $a_3$ , and  $a_4$  are determined. By substituting all the constants in equation (4) below equation is developed.

$$V_n^{PRO} = \frac{f_c^{a1} \rho^{a2} b d^{a3}}{(\log_{10}[1.0 + \frac{a}{d}])^2} \quad (5)$$

Where  $\left(\frac{a}{d}\right) < 1.0$

## VI. EFFECTS OF INFLUENCING PARAMETERS

### A. EFFECT OF SHEAR SPAN-TO-DEPTH RATIO

As per the provisions of RUSSO Expression, Siao Expression and ACI 318-14, the shear span-to-depth ratio decreases when there is an increase in the shear strength of corbel. In the case of proposed model, the effect of  $\frac{a}{d}$  ratio has been considered with a consistent term. The term  $\frac{a}{d}$  ratio is expressed in logarithmic form in order to minimize the errors.

### B. EFFECT OF COMPRESSIVE STRENGTH ON CORBEL

The compressive strength of corbel as per the proposed equation is greater when compared with RUSSO Expression, Siao Expression and ACI 318-14 Codes. The low strength of concrete corbels scatter was more when compared to the medium strength of concrete corbels. Irrespective of compressive strength of concrete, the proposed model predicts the capacity uniformly.

### C. EFFECT OF DEPTH ON CORBEL

The depth of corbel varies in between 200mm and 600mm. Size effect is accounted for in the proposed expression by adopting suitable power term 'd'. Consequently, prediction of the proposed model varies uniformly with depth of corbel.

## VII. COMPARISON OF PROPOSED MODEL WITH EXISTING MODELS

The proposed equation is relatively safe when compared to ACI 318-14, For the RUSSO Expression, Siao Expression and ACI 318-14, standard deviation values are 0.44, 0.38 and 0.45, respectively, the proposed equation standard deviation is 0.50, which is merely equal to ACI codes, whereas the mean of strength ratio is 1.24, while mean of ACI codes are

between 0.8 and 1.1. The coefficient of variation of proposed model is 40%, which can be reduced further by modifying the coefficients.

## VIII. CONCLUSION

The proposed equation is more conservative than that of the existing models suggested by ACI codes. Also, the effect of tension reinforcement ratio, shear span-to-depth ratio and size-effect were incorporated. The mean and coefficient of variation of strength ratio of proposed model are 1.24 and 0.40, respectively. From the comparison it has been concluded that the proposed model is safe, economical and consistent.

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