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Development of Shear Strength Expression for RC Deep Beams Using Strut-and-Tie Model

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Abstract: ***This paper reports, a new analytical expression is proposed for predicting the shear strength of RC deep beams using Strut-and-Tie model (STM). The proposed expression is compared with some of the existing expression and ACI 318-14 code expression. The 111 deep beams specimen's experimental data have been collected from the literature survey to find out the unknown parameters. The proposed expression encloses, concrete compressive strength, amount and arrangement of longitudinal and web reinforcement, shear span-to-depth ratio and effective depth. The results indicates that the proposed expression is one of the best fitting expression to predict the shear strength of variety of RC deep beams.***

Index Terms: Deep beams, Strut-and-tie Model, Shear Strength, Shear span-to-depth ratio.

I. INTRODUCTION

Beams with large depth in connection to span are called deep beams as shown in Fig. 1. Reinforced concrete deep beam have some useful applications in tall structures, seaward structures, pile cap as foundation etc. Deep beams are defined as non-flexural members, in which plane section do not remains plane even after bending [1]. As indicated shear span to depth (a/d) ratio, the strength of the deep beam is typically controlled by shear as opposed to flexure [2]. The possible mode of failure in deep beams are shear compression failure (crushing of concrete) or the shear tension failure [3].

Definition of deep beam as per different country codes as per IS-456 (2000) [4] Clause 29 when the ratio between effective span (L) to overall depth (D) is less than 2 for simply supported beams. When the ratio (L/D) is less than 2.5 for continuous span beams. The effective span is characterized as the centre-to-centre distance between the supports or 1.15 times the clear span whichever is less. As per New Zealand Code(NDS-3101-2006) [5] clear span ($L_{n)}$ equal to or less than 3.6 times the effective depth for simply supported deep beams and for continuous deep beams, where the clear span equal or less than 1.6 times the effective depth for cantilever deep beams. The Canadian code (CSA-A23.3-2004.) [6]Defines deep beam definition as when the ratio of the clear span (l_0) to the overall depth(h) is less than the limits for Simple span beams, l_0 / h < 1.25, and for Continuous span beams: l_0 / h < 2.5.

Further, the CIRIA Guide [7] defines as when the ratio between effective span/ depth (l/h) is less than 2 for single-span beams and less than 2.5 for continuous span beams. As per strut and tie provisions of ACI-318 code [8], deep beams are defined as when the ratio between effective span (L) to overall depth (D) is less than or equal to four.

Deep beams are also classified as disturbed regions, in which the strain distribution is non-linear[9]. Strut-and-tie models consists of struts and ties connected at nodes to form an idealized truss as shown in Fig II. In this idealized truss models, struts act as the compression member, ties act as the tension member, and nodes are the joints. Application of these models to structural elements by considering concrete in-between diagonal cracks are struts[10]. The strut-and-tie models are mostly used while designing the shear critical structural elements like deep beams, corbels, squat shear wall and pile caps [11].The validation of failure modes and evaluation of shear strength of the deep beams are very complex [12]. There are many numerous parameters influencing on the shear strength of RC deep beams, where the most critical of them are concrete compressive strength, shear span-depth ratio, the amount and arrangement of tension, horizontal and vertical web reinforcements [13].

The main aim of this paper is to develop the expression for predicting the shear strength of reinforced concrete deep beams using strut and tie model, which is applicable for a wide range of concrete compressive strength. To find the accuracy of the proposed model, one hundred and eleven single-span top-loaded deep beams experimental data is collected and compared, where the range is shown in table I. The comparison was done between the proposed model with existing simplified softened truss models of Russo et al[12]; Matamoros and Wong[13]; Arabzadeh et al [14]and strut-and-tie models of ACI 318-14 Code [8].

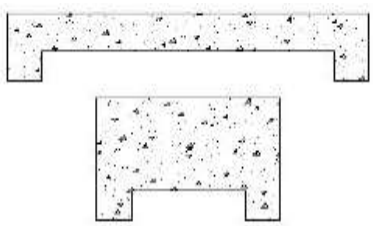

Fig. 1: Shallow beam and Deep beam

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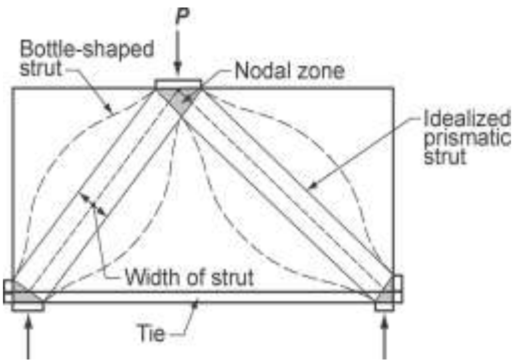

Fig. 2: Description of Strut-and-Tie Model (from ACI 318-14 Code)

II. RESEARCH SIGNIFICANCE

The main issue is of prediction of shear strength is accurate and uniform in the deep beams. Development of analytical expression for predicting the reinforced concrete deep beam shear strength is the main objective of this paper. When compared to existing expressions, the proposed expression showed the accurate and uniform prediction in the shear strength.

III. EXISTING EXPRESSIONS

A. Russo et al. Expression

The ultimate shear strength (V_n) is given by

$$v_n = c_1(k\chi f'_c \cos\theta + c_2\rho_h f_{yh} cot\theta + c_3\, {}^{a}/_{d}\, \rho_v f_{yv}) \quad (1)$$

Where c_1, c_2, c_3 are determined from the experimental results c_1=0.76 c_2=0.35 c_3=0.25. (1) can be written as

$$v_n = 0.76\left(k\chi f'c\, cos\theta + 0.35 p_h f_{yh} \cot\theta + 0.25\frac{a}{d} p_v f_{yv}\right) \quad (2)$$

In (2) the horizontal reinforcement ratio (p_h) is calculated as the total area given by the tension steel and the compression steel, and the horizontal web reinforcement and divided by *bd*. The vertical reinforcement ratio (p_v)is calculated as the area of one reinforcement vertical layer divided by *bd.*

"K" is obtained from the classical bending theory

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

Where *n* the ratio of steel to concrete modulus of elasticity $(= E_s/E_c)$and the ρ is the longitudinal reinforcement ratio($= {}^{A_s}/_{bd}$, with b width and d is the depth of the deep beam).

Assuming $E_s = 200000\ MPa$

χ is the non-dimensional interpolating function

$$\chi=[0.74\left(\frac{f'c}{105}\right)^3 - 1.28\left(\frac{f'c}{105}\right)^2 + 0.22\left(\frac{f'c}{105}\right) + 0.87]$$

θ is the inclined angle of compression strut.

$$\theta = 2arc\tan\left(\frac{\sqrt{\left(\frac{a}{d} - \frac{wl}{2d}\right)^2 + \left(1 - \frac{k^2}{4}\right) - 1}}{\frac{a}{d} - \frac{wl}{2d} - \frac{k}{2}}\right)$$

B. Matamoros and Wong Expression

The ultimate shear strength (V) is given by

$$v = \frac{0.3}{\frac{a}{d}} f'cbw_{st} + \rho_{wv} b\frac{a}{3} f_{yv} + 3(1 - \frac{a}{d})\rho_{wh} b\frac{d}{3} f_{yh} \quad (3)$$

Where in (3) $\Theta = \tan^{-1}\left(\frac{d}{a}\right)$, W_{st} is the width of strut $= l_b sin\theta + h_a cos\theta(l_b$ is thebase plate width and h_a is twice the distance between the centroid of the main reinforcement and the bottom of the beam), $\rho_{wh}\ and\ \rho_{wv}$are the horizontal and vertical reinforcement ratios, respectively and f_{yh}&$f_{yv}$are the yielding strength of the horizontal and vertical reinforcement, respectively. The following limits apply to Eq. (3) the term $\frac{0.3}{{}^{a}/_{d}}$ has an upper limit of$0.85\ sin\theta$, and the term $(1 - {}^{a}/_{d})$ has the lower limit of zero. Finally, the nominal shear strength $V_n$is given byV_n/bd.

C. Arabzadeh et al. Expression

The ultimate predicted shear strength expression is given by

$$v_c = \frac{f'c^{0.70}}{0.7+0.15\left(\frac{a}{d}\right)^2} A_{str} sin\theta + 0.065 p_p{}^{-0.35} A_{wp} cos\theta \quad (4)$$

Where in (4) 'θ' is the diagonal strut angle$=$ $(\tan^{-1}(\frac{jd}{a}))$'$a$' is the shear span measured from centre to centre distance between the load and support and 'jd' is the distance of lever arm. 'A_{str}' is the cross sectional area of the strut ($A_{str} = b\ a_s)b$ is the width of the beam $'a_s'$ is the uniform width of strut $= \min(l_b sin\theta + d_a cos\theta, l_p sin\theta + w_t cos\theta) l_p$ is depth of the bottom node, w_t is width of the support bearing plate, l_b is the depth of the top node and d_a is the width of loaded point bearing plate. 'A_{wp}' is the equivalent area of perpendicular web reinforcement $A_{wp} = (A_v cos\theta + A_h sin\theta = p_p A_{str}) A_{h,} A_v$ are the areas of horizontal and vertical reinforcement.

D. ACI 318-14 Code Expression

According to ACI 318-14 [8] the maximum shear strength of the deep beam is given by (5). To make the equation steady, the reduction factor (∅) is multipliedwith the design of previous code. Maximum shear strength depends on the concrete compressive strength and the cross-sectional area of deep beam and horizontal and vertical web reinforcement.

$$V_n = \emptyset\ 10\sqrt{f'c} bd; \quad \emptyset = 0.75 \quad (5)$$

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Table I: Details of selected deep beam specimens

Reference	No of tested beams	(a/d)	f'c(MPa)
Mau and Hsu	62	0.40-1.50	16.02-24.55
Subedi et al.	6	0.42-1.55	22.7-34.7
Kong et al.	19	0.23-1.71	37-75
Kong and Teng	14	0.26-2.15	41.06-58.84
Ramakrishnan	10	0.29-0.61	12.5-27.8

IV. PROPOSED EXPRESSION

The total shear strength (v_n) is given by the sum of compression strut (concrete) and web reinforcement

$$V_n = A\left(\frac{0.3}{a/d}\sqrt{f'c}\right) + B\left(\rho_v f_{yv} cos\theta + \rho_h f_{yh} sin\theta\right) \quad (6)$$

Regression analysis is carried out for formulation of proposed empirical expression for predicting shear strength of RC deep beams.

Multiple linear regression (MLR) is used to find out the linear relationship between the dependent variables and independent variables. This equation is also called as deterministic model. It given exact relationship between the variables.

By using this method, we get the unknown coefficients A and B in (6)

$$V_n = 0.44\left(\frac{0.3}{a/d}\sqrt{f'c}\right) + 0.85\left(\rho_v f_{yv} cos\theta + \rho_h f_{yh} sin\theta\right) \quad (7)$$

In (7)'θ' is the angle between diagonal strut and longitudinal tie $= \tan^{-1}\left(\frac{jd}{a}\right)$ as per the assumptions of linear bending theory 'jd' is the distance of lever arm$=\left(1-\frac{k}{3}\right)d$. The coefficient k can be defined as $k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$.'n' is the modular ratio of elasticity $\left(=\frac{E_s}{E_c}\right)$.'$\rho$' is the longitudinal reinforcement ratio (= A_s/(bd).

V. INFLUENCE OF PARAMETERS

A. Size Effect of Deep Beams

From experimental observation on different sizes of deep beam with web reinforcement and without web reinforcement and with different shear span to depth ratios, it is concluded that shear strength is decreases when depth is increases. This is called as size effect. when the beam depth was increased from 300mm to 1200mm there will be 40% reduction of the strength in the deep beams[16]. Furthermore, confirmed that when the beams depth is increased from 1200mm to 3000mm there will be the 25% reduction in shear strength[17].

B. Effect of Compressive Strength

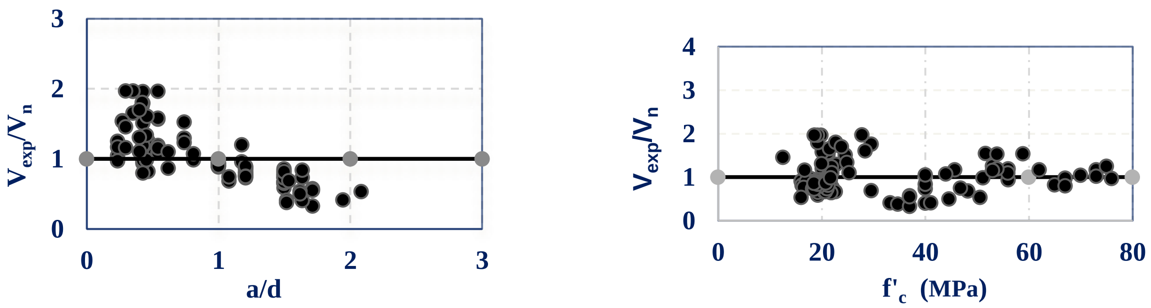
The concrete compressive strength plays an important role in the deep beam. Any change in the compressive strength of concrete leads to an obvious effect on the ultimate capacity[18]. As compressive strength increases, shear strength increases to certain limit.

C. Effect of a/d Ratio

According to the code provisions of various countries the shear span to depth ratio decreases there is an increase in the shear strength. When shear span to depth ratio increases the lever arm decreases.

VI. COMPARISON BETWEEN PROPOSED EXPRESSION AND EXISTING EXPRESSIONS

The proposed expression showed relatively good prediction when compared with existing expressions as shown in Fig. 3 and Fig. 4. The proposed expression has the least mean and coefficient of variation (COV) values of 1.035 and 0.25, respectively. On the other hand, for Russo et al mean and COV are 1.056 and 0.38, Matamoros and Wong mean and COV are 1.12 and 0.39, Arabzadeh et al mean and COV are 1.38 and 0.26 and for ACI 318-14 mean and COV values are 1.64 and 0.28.



(a) Russo et al

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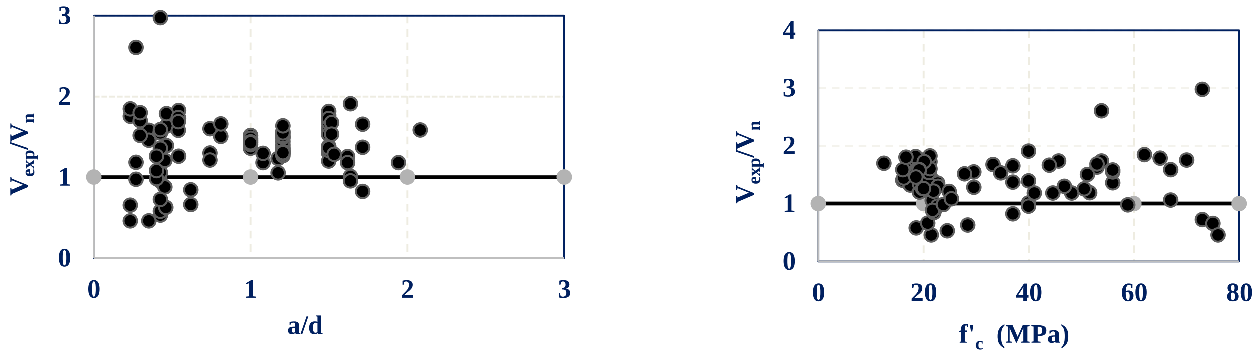
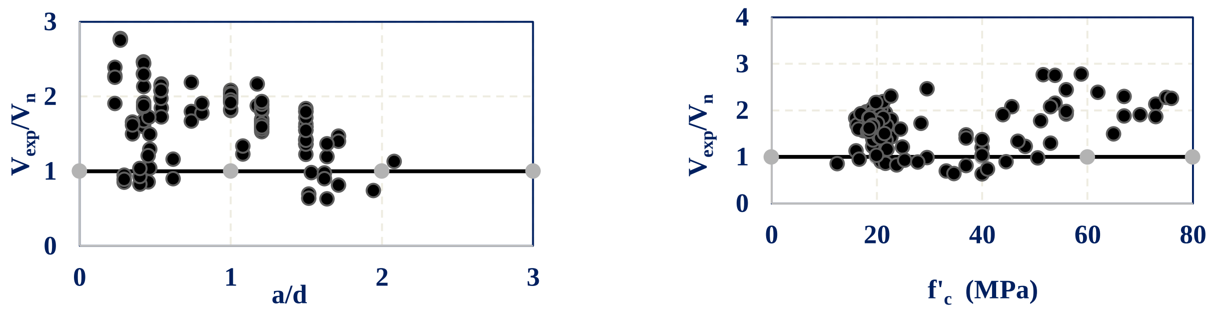
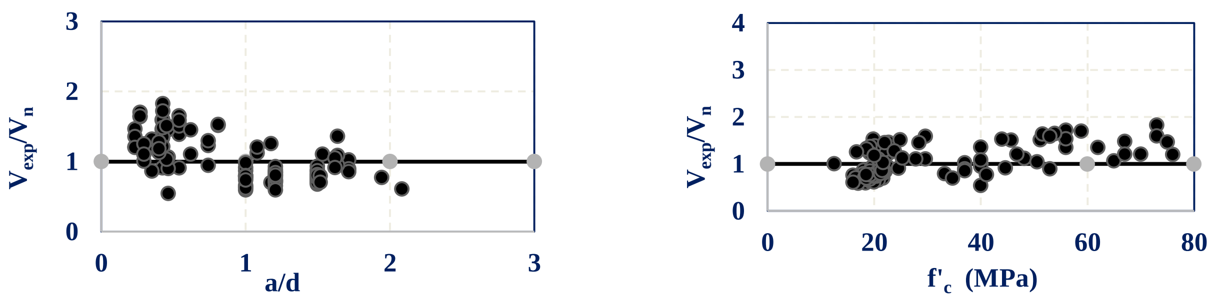
(b) Matamoros and Wong



(c) Arabzadeh et al.

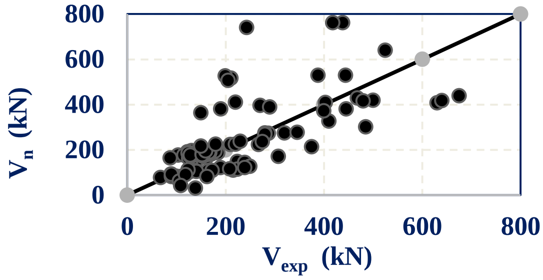


(d) ACI318-14 Code

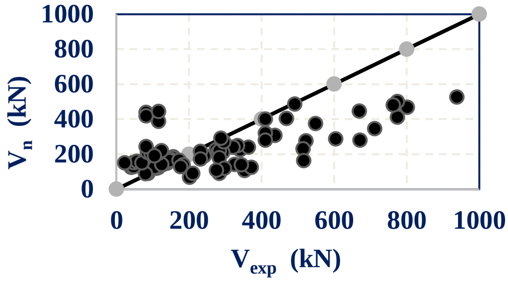


(e) Proposed Expression

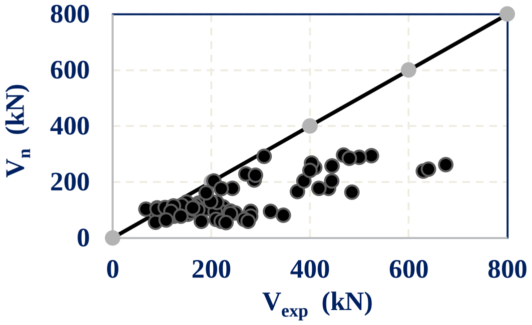
Fig. III: Effect of shear span to depth ratio (a/d) and compressive strength of concrete (f'_c) on the ratio of Predicted shear strength (V_{exp}/V_n).



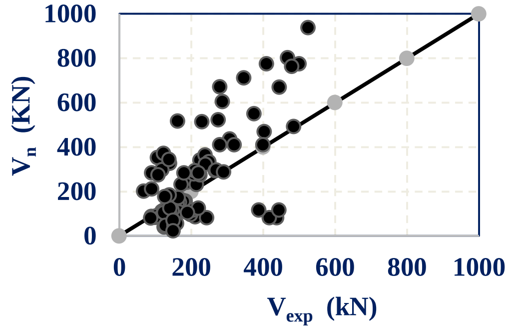
(a) Russo et al.



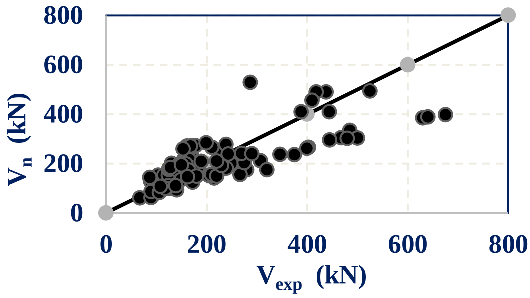
(b) Matamoros and Wong



(c) Arabzadeh et al



(d) ACI 318-14 Code



(e) Proposed Expression

Fig. 4: Effect of experimental shear strength (V_{exp}) Versus Provided shear strength (V_n)

VII. CONCLUSION

A modified model for determining the shear strength of reinforced concrete deep beam is proposed. The proposed expression is derived from the concept of strut and ties. Based on the experimental data, the comparisons were done between the proposed model and existing models (Russo et al, Matamoros and Wong, Arabzadeh et al, ACI 318-14 code), the following conclusions have been drawn:

1. For the proposed model mean of strength ratio is 1.035, and the standard deviation as well as coefficient of variation values are lesser compared to the existing models.
2. The proposed expression is applicable for design and capable for predicting the shear strength of reinforced concrete deep beam.
3. The horizontal web reinforcement shear strength contribution is relatively higher than the vertical web reinforcement in the deep beam.

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