Fixed Point Theorems of Soft Metric Space Using Altering Distance Function

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***Abstract:** In the present paper, some fixed point theorems are proved through rational expression in altering distance functions and property P for the contraction mappings.*

***Keywords:** - Soft metric space, Altering distance function, fixed point.*

Mathematics Subject Classification: - 47H10, 54H25.

I. INTRODUCTION & PRELIMINARIES

The fixed point theorems in metric spaces are playing a major role to solve many problems in a mathematical analysis. So the attraction of metric spaces to a large numbers of mathematicians is understandable.

Altering distance function for self-mapping on a metric space established by M. S. Khan in 1984 and it can be expanded by M. Swalesh and S. Seesa [7] that they introduced a control function which they called as altering distance function in the research of fixed point theory.

In the year 1999, Molodtsov [11] initiated a novel concept of soft sets theory as a new mathematical tool for dealing with uncertainties. A soft set is a collection of approximate descriptions of an object. Soft systems provide a very general framework with the involvement of parameters. Since soft set theory has a rich potential, applications of soft set theory in other disciplines and real life problems are progressing rapidly.

Maji et al. [8, 9] worked on soft set theory and presented an application of soft sets in decision making problems. Chen [2] introduced a new definition of soft set parameterization reduction and a comparison of it with attribute reduction in rough set theory. Many researchers contributed towards many structure on soft set theory. [1,2].

M. Shabir and M. Naz [12] presented soft topological spaces and they investigated some properties of soft topological spaces. Later, many researches about soft topological spaces were studied in [6,10,12,13]. In these studies, the concept of soft point is expressed by different approaches. In the study we use the concept of soft point which was given in [4, 13].

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Definition 2.1: Let X be an initial universe set and E be a set of parameters. A pair (F,E) is called a soft set over X if and only if X is a mapping from E into the set of all subsets of the set $X, i.e. F: E \to P(X)$, where $P(X)$ is the power set of X.

Definition 2.2: The intersection of two soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.This is denoted by $(F,A) \cap (G,B) = (H,C)$.

Definition 2.3: The union of two soft sets (F,A) and (G,B) over X is the soft set, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if\ \varepsilon \in A - B \\ G(\varepsilon), & if\ \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \varepsilon \in A \cap B \end{cases}$$

This relationship is denoted by $(F,A) \cup (G,B) = (H,C)$.

Definition 2.4: The soft set (F,A) over X is said to be a null soft set denoted by Φ if for all $\varepsilon \in A, F(\varepsilon) = \phi\ (null\ set)$.

Definition 2.5: A soft set (F,A) over X is said to be an absolute soft set, if for all $\varepsilon \in A, F(\varepsilon) = X$.

Definition 2.6: The difference (H,E) of two soft sets (H,E) and (H,E) over X denoted by $(H,E)\backslash(H,E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.

Definition 2.7: The complement of a soft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$ where $F^c: A \to P(X)$ is mapping given by $F^c(\alpha) = X - F(\alpha), \forall \alpha \in A$.

Definition 2.8: Let $\mathfrak{R}$ be the set of real numbers and $B(\mathfrak{R})$ be the collection of all nonempty bounded subsets of $\mathfrak{R}$ and E taken as a set of parameters. Then a mapping $F: E \to B(\mathfrak{R})$ is called a soft real set. It is denoted by (F,E). If specifically (F,E) is a singleton soft set, then identifying (F,E) with the corresponding soft element, it will be called a soft real number and denoted $\tilde{r}, \tilde{s}, \tilde{t}$ etc.

$\overline{0}, \overline{1}$ are the soft real numbers where $\overline{0}(e) = 0, \overline{1}(e) = 1$ for all $e \in E$, respectively.

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Definition 2.9: For two soft real numbers

(i) $\tilde{r} \leq \tilde{s}$, if $\tilde{r}(e) \leq \tilde{s}(e)$, for all $e \in E$.

(ii) $\tilde{r} \geq \tilde{s}$, if $\tilde{r}(e) \geq \tilde{s}(e)$, for all $e \in E$.

(iii) $\tilde{r} < \tilde{s}$, if $\tilde{r}(e) < \tilde{s}(e)$, for all $e \in E$.

(iv) $\tilde{r} > \tilde{s}$, if $\tilde{r}(e) > \tilde{s}(e)$, for all $e \in E$.

Definition 2.10: A soft set over X is said to be a soft point if there is exactly one $e \in E$, such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \phi, \forall e' \in E\backslash\{e\}$. It will be denoted by $\tilde{x}_e$.

Definition 2.11: Two soft points $\tilde{x}_e, \tilde{y}_e$ are said to be equal if $e = e'$ and $P(e) = P(e')$ i.e. $x = y$. Thus $\tilde{x}_e \neq \tilde{y}_e \Leftrightarrow x \neq y$ or $e \neq e'$.

Definition 2.12: A mapping $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$, is said to be a soft metric on the soft set $\tilde{X}$ if d satisfies the following conditions:

(M1) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \geqq \overline{0}$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \widetilde{\in} \tilde{X}$,

(M2) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \overline{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$,

(M3) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \cong \tilde{d}(\tilde{y}_{e_2}, \tilde{x}_{e_1})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \widetilde{\in} \tilde{X}$,

(M4) $\tilde{d}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) \leqq \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{d}(\tilde{y}_{e_2}, \tilde{z}_{e_3})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \widetilde{\in} \tilde{X}$.

The soft set $\tilde{X}$ with a soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Definition 2.13 (Cauchy Sequence): A sequence $\{\tilde{x}_{\lambda,n}\}_n$ of soft points in $(\tilde{X}, \tilde{d}, E)$ is considered as a Cauchy sequence in $\tilde{X}$ if corresponding to every $\tilde{\varepsilon} \geqq \overline{0}, \exists m \in N$ such that $d(\tilde{x}_{\lambda,i}, \tilde{x}_{\lambda,j}) \leqq \tilde{\varepsilon}, \forall\, i,j \geq m$, i.e. $d(\tilde{x}_{\lambda,i}, \tilde{x}_{\lambda,j}) \to \overline{0}$, as $i,j \to \infty$.

Definition 2.14 (Soft Complete Metric Space): A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called complete, if every Cauchy Sequence in $\tilde{X}$ converges to some point of $\tilde{X}$.

Definition 2.15: A function $\psi: \mathbb{R}_+ \to \mathbb{R}_+$ is called an altering distance function if the following properties are satisfied.

i. $\psi(t) = 0 \Leftrightarrow t = 0$,
ii. ψ is monotonically non decreasing,
iii. ψ is continuous,

By ψ denotes the set of all altering distance function.

Definition 2.16: Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space for a self-mapping (f, φ) with a nonempty set $E_{(f,\varphi)}$. Then (f, φ) is said to satisfy the property P if $E_{(f,\varphi)} = E_{(f,\varphi)^n}$ for each $n \in N$.

Theorem 2.1 Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space, let $\psi \in \Psi$ and Let mapping $(f, \varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ satisfies the following inequality:

$$\psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda), (f,\varphi)(\tilde{y}_\mu)\right)\right] \leq \tilde{a}\, \psi[\tilde{d}(\tilde{x}_\lambda, \tilde{y}_\mu)] \qquad \textbf{... [2.1.1]}$$

For all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$, $0 < \tilde{a} < 1$. Then (f, φ) has a unique fixed point in $\tilde{y}^*_\lambda \in \tilde{X}$ and moreover for each $\tilde{y}_\mu \in \tilde{X}, \lim_{n\to\infty}(f,\varphi)^n \tilde{y}_\mu = \tilde{y}^*_\lambda$.

Theorem 2.2 Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space, and Let $(f, \varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ be a mapping such that:

$$\psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda), (f,\varphi)(\tilde{y}_\mu)\right)\right] \leq \tilde{a}\, \psi[\tilde{d}(\tilde{x}_\lambda, \tilde{y}_\mu)]$$

$$+\tilde{b}\left[\frac{\tilde{d}^2(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)) + \tilde{d}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)).\tilde{d}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right) + \tilde{d}^2\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}{1 + \tilde{d}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)) + \tilde{d}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}\right] \qquad \textbf{... [2.2.1]}$$

For all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$, $\tilde{a}, \tilde{b} > 0, 0 < \tilde{a} + 2\tilde{b} < 1$. Then (f, φ) has a unique fixed point in $\tilde{y}^*_\lambda \in \tilde{X}$ and moreover for each $\tilde{y}_\mu \in \tilde{X}, \lim_{n\to\infty}(f,\varphi)^n \tilde{y}_\mu = \tilde{y}^*_\lambda$.

II. MAIN RESULTS

Theorem 3.1: Let a complete soft metric space $(\tilde{X}, \tilde{d}, E)$, we have $\psi \in \Psi$. Let mapping $(f, \varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ satisfies the following condition:

$$\psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda), (f,\varphi)(\tilde{y}_\mu)\right)\right] \leq \tilde{a}\, \psi[\tilde{d}(\tilde{x}_\lambda, \tilde{y}_\mu)]$$

$$+\tilde{b}\psi\left[\frac{\tilde{d}^2(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)) + \tilde{d}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)).\tilde{d}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right) + \tilde{d}^2\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}{1 + \tilde{d}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda)) + \tilde{d}\left(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu)\right)}\right]$$

... **(3.1.1)**

For all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$, $\tilde{a} > 0$, $\tilde{b} > 0$, $\tilde{a} + 2\tilde{b} < 1$. Then (f, φ) has a unique fixed point in $\tilde{y}^*_\lambda \in \tilde{X}$ and moreover for each $\tilde{y}_\mu \in \tilde{X}, \lim_{n\to\infty}(f,\varphi)^n \tilde{y}_\mu = \tilde{y}^*_\lambda$.

Proof: Let $\tilde{x}^0_\lambda$ be any soft point in $SP(X)$.

Set $\tilde{x}^1_{\lambda_1} = (f,\varphi)(\tilde{x}^0_\lambda) = \left(f(\tilde{x}^0_\lambda)\right)_{\varphi(\lambda)}$

$\tilde{x}^2_{\lambda_2} = (f,\varphi)(\tilde{x}^1_{\lambda_1}) = \left(f^2(\tilde{x}^0_\lambda)\right)_{\varphi^2(\lambda)}$

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$\tilde{x}^{n+1}_{\lambda_{n+1}} = (f,\varphi)(\tilde{x}^n_{\lambda_n}) = \left(f^{n+1}(\tilde{x}^0_\lambda)\right)_{\varphi^{n+1}(\lambda)}$, – –

Now consider

$$\psi[\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})] = \psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}^{n-1}_{\lambda_{n-1}}), (f,\varphi)(\tilde{x}^n_{\lambda_n})\right)\right]$$

$$\leq \tilde{a}\,\psi[\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n})]$$

$$+\tilde{b}\psi\left[\frac{\tilde{d}^2\left(\tilde{x}^{n-1}_{\lambda_{n-1}},(f,\varphi)\left(\tilde{x}^{n-1}_{\lambda_{n-1}}\right)\right)+\tilde{d}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},(f,\varphi)\left(\tilde{x}^{n-1}_{\lambda_{n-1}}\right)\right).\tilde{d}\left(\tilde{x}^n_{\lambda_n},(f,\varphi)\left(\tilde{x}^n_{\lambda_n}\right)\right)+\tilde{d}^2\left(\tilde{x}^n_{\lambda_n},(f,\varphi)\left(\tilde{x}^n_{\lambda_n}\right)\right)}{1+\tilde{d}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},(f,\varphi)\left(\tilde{x}^{n-1}_{\lambda_{n-1}}\right)\right)+\left(\tilde{x}^n_{\lambda_n},(f,\varphi)\left(\tilde{x}^n_{\lambda_n}\right)\right)}\right]$$

$$\leq \tilde{a}\,\psi[\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n})]$$

$$+\tilde{b}\psi\left[\frac{\tilde{d}^2\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}\right)+\tilde{d}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}\right).\tilde{d}\left(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}}\right)+\tilde{d}^2\left(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}}\right)}{1+\tilde{d}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}\right)+\tilde{d}\left(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}}\right)}\right]$$

$$\leq \tilde{a}\,\psi[\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n})] + \tilde{b}\psi\left[\frac{\left\{\tilde{d}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}\right)+\tilde{d}\left(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}}\right)\right\}^2-\tilde{d}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}\right).\tilde{d}\left(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}}\right)}{1+\tilde{d}\left(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}\right)+\tilde{d}\left(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}}\right)}\right]$$

$\leq \tilde{a}\,\psi[\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n})] + \tilde{b}\psi[\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n}) +$ *dxλnn,xλn+1n+1*

$$\psi[\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})] \leq (\tilde{a} + \tilde{b})\psi[\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n})] + \tilde{b}\psi\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})$$

$$\psi[\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})] \leq \frac{(\tilde{a} + \tilde{b})}{(1-\tilde{b})}\psi[\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n})]$$

$$\psi[\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})] \leq \left[\frac{(\tilde{a} + \tilde{b})}{(1-\tilde{b})}\right]^2 \psi[\tilde{d}(\tilde{x}^{n-2}_{\lambda_{n-2}}, \tilde{x}^{n-1}_{\lambda_{n-1}})] \leq - - -$$

$$\psi[\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})] \leq \left[\frac{(\tilde{a} + \tilde{b})}{(1-\tilde{b})}\right]^n \psi[\tilde{d}(\tilde{x}^0_{\lambda_0}, \tilde{x}^1_{\lambda_1})] \quad \dots \textbf{(3.1.2)}$$

Since $\frac{(\tilde{a} + \tilde{b})}{(1-\tilde{b})} < 1$ from **(3.1.2).** We have

$$\lim_{n\to\infty} \psi[\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})] = 0$$

From the result given that $\psi \in \Psi$, we have

$$\lim_{n\to\infty} \tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}}) = 0 \quad \dots \textbf{(3.1.3)}$$

Now we will show that $\{\tilde{x}^n_{\lambda_n}\}$ is Cauchy sequence in $\tilde{X}$. Suppose that $\{\tilde{x}^n_{\lambda_n}\}$ is not a Cauchy sequence, which means that there is a constant $\epsilon_0 > 0$ such that for each positive integer k, there exist a positive integer $\lambda_{m(k)}$ and $\lambda_{n(k)}$ with $\lambda_{m(k)} > \lambda_{n(k)} > k$ such that

$$\tilde{d}\left(\tilde{x}^{m(k)}_{\lambda_{m(k)}}, \tilde{x}^{n(k)}_{\lambda_{n(k)}}\right) \geq \epsilon_0, \tilde{d}\left(\tilde{x}^{m(k)-1}_{\lambda_{m(k)-1}}, \tilde{x}^{n(k)}_{\lambda_{n(k)}}\right) < \epsilon_0$$

By triangle inequality

$$\epsilon_0 \leq \tilde{d}\left(\tilde{x}^{m(k)}_{\lambda_{m(k)}}, \tilde{x}^{n(k)}_{\lambda_{n(k)}}\right) \leq \tilde{d}\left(\tilde{x}^{m(k)}_{\lambda_{m(k)}}, \tilde{x}^{m(k)-1}_{\lambda_{m(k)-1}}\right) + \tilde{d}\left(\tilde{x}^{m(k)-1}_{\lambda_{m(k)-1}}, \tilde{x}^{n(k)}_{\lambda_{n(k)}}\right)$$

$$< \tilde{d}\left(\tilde{x}^{m(k)}_{\lambda_{m(k)}}, \tilde{x}^{m(k)-1}_{\lambda_{m(k)-1}}\right) + \epsilon_0$$

Letting $k \to \infty$ and using (3.1.3), we have

$$\lim_{k\to\infty} \tilde{d}\left(\tilde{x}^{m(k)}_{\lambda_{m(k)}}, \tilde{x}^{n(k)}_{\lambda_{n(k)}}\right) = \epsilon_0 \quad \textbf{…(3.1.4)}$$

Similarly, we have

$$\lim_{n\to\infty} \tilde{d}\left(\tilde{x}^{m(k)+1}_{\lambda_{m(k)+1}}, \tilde{x}^{n(k)+1}_{\lambda_{n(k)+1}}\right) = \epsilon_0 \quad \textbf{…(3.1.5)}$$

Putting $\tilde{x}_\lambda = \tilde{x}^{m(k)}_{\lambda_{m(k)}}$ and $\tilde{y}_\mu = \tilde{x}^{n(k)}_{\lambda_{n(k)}}$ in **(3.1.1)** we have

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$$\psi\left[\tilde d\left(\tilde x^{m(k)+1}_{\lambda_{m(k)+1}}, \tilde x^{n(k)+1}_{\lambda_{n(k)+1}}\right)\right] =$$
$$\psi\left[\tilde d\left((f,\varphi)\left(\tilde x^{m(k)}_{\lambda_{m(k)}}\right), (f,\varphi)\left(\tilde x^{n(k)}_{\lambda_{n(k)}}\right)\right)\right]$$
$$\leq \tilde a\ \psi\left[\tilde d\left(\tilde x^{m(k)}_{\lambda_{m(k)}}, \tilde x^{n(k)}_{\lambda_{n(k)}}\right)\right]$$
$$+\ \tilde b\psi\left[\frac{\tilde d^2\left(\tilde x^{m(k)}_{\lambda_{m(k)}}, \tilde x^{m(k)+1}_{\lambda_{m(k)+1}}\right) + \tilde d\left(\tilde x^{m(k)}_{\lambda_{m(k)}}, \tilde x^{m(k)+1}_{\lambda_{m(k)+1}}\right).\tilde d\left(\tilde x^{n(k)}_{\lambda_{n(k)}}, \tilde x^{n(k)+1}_{\lambda_{n(k)+1}}\right) + \tilde d^2\left(\tilde x^{n(k)}_{\lambda_{n(k)}}, \tilde x^{n(k)+1}_{\lambda_{n(k)+1}}\right)}{1 + \tilde d\left(\tilde x^{m(k)}_{\lambda_{m(k)}}, \tilde x^{m(k)+1}_{\lambda_{m(k)+1}}\right) + \tilde d\left(\tilde x^{n(k)}_{\lambda_{n(k)}}, \tilde x^{n(k)+1}_{\lambda_{n(k)+1}}\right)}\right]$$

Using **(3.1.3), (3.1.4)** and **(3.1.5)** we have

$$\psi[\epsilon_0] = \lim_{n\to\infty}\psi\left[\tilde d\left(\tilde x^{m(k)+1}_{\lambda_{m(k)+1}}, \tilde x^{n(k)+1}_{\lambda_{n(k)+1}}\right)\right]$$

$$\leq \lim_{n\to\infty}\tilde a\ \psi\left[\tilde d\left(\tilde x^{m(k)}_{\lambda_{m(k)}}, \tilde x^{n(k)}_{\lambda_{n(k)}}\right)\right] \leq \tilde a\psi[\epsilon_0]$$

Since $\tilde a \in (0,1)$, we get a contradiction. Thus $\{\tilde x^n_{\lambda_n}\}$ is a Cauchy sequence in the complete soft metric space $\tilde X$. Thus there exist $\tilde x^*_\lambda \in \tilde X$ such that $\tilde x^n_{\lambda_n} \to \tilde x^*_\lambda,\ n \to \infty$.

Putting $\tilde x_\lambda = \tilde x^n_{\lambda_n}$ and $\tilde y_\mu = \tilde x^*_\lambda$ in **(3.1.1)** we have

$$\psi\left[\tilde d\left(\tilde x^{n+1}_{\lambda_{n+1}}, (f,\varphi)(\tilde x^*_\lambda)\right)\right] =$$
$$\psi\left[\tilde d\left((f,\varphi)(\tilde x^n_{\lambda_n}), (f,\varphi)(\tilde x^*_\lambda)\right)\right]$$
$$\leq \tilde a\ \psi[\tilde d(\tilde x^n_{\lambda_n}, \tilde x^*_\lambda)]$$

+ ??ψ $\frac{?^2\ ??,\ ?,?\ ??\ +?\ ??,\ ?,?\ ??\ .?\ ??,\ ?,?\ ??\ +?^2\ ??,\ ?,?\ ??}{1+?\ ??,\ ?,?\ ??\ +?\ ??,\ ?,?\ ??}$

Therefore $\psi[\tilde d(\tilde x^*_\lambda, (f,\varphi)(\tilde x^*_\lambda))] \leq \tilde b\psi[\tilde d(\tilde x^*_\lambda, (f,\varphi)(\tilde x^*_\lambda))]$

Since $\tilde b \in (0,1)$, then $\psi[\tilde d(\tilde x^*_\lambda, (f,\varphi)(\tilde x^*_\lambda))] = 0$, which implies that $\tilde d(\tilde x^*_\lambda, (f,\varphi)(\tilde x^*_\lambda)) = 0$.

Thus $\tilde x^*_\lambda = (f,\varphi)(\tilde x^*_\lambda)$.

Now we are going to establish the uniqueness of the fixed point. Let $\tilde x^*_\lambda, \tilde y^*_\lambda$ be two fixed point of (f,φ) such that $\tilde x^*_\lambda \neq \tilde y^*_\lambda$,

Putting $\tilde x_\lambda = \tilde x^*_\lambda$ and $\tilde y_\mu = \tilde y^*_\lambda$ in **(3.1.1)** we have

$$\psi[\tilde d(\tilde x^*_\lambda, \tilde y^*_\lambda)] = \psi[\tilde d((f,\varphi)(\tilde x^*_\lambda), (f,\varphi)(\tilde y^*_\lambda))]$$
$$\leq \tilde a\ \psi[\tilde d(\tilde x^*_\lambda, \tilde y^*_\lambda)]$$
$$+\ \tilde b\psi\left[\frac{\tilde d^2(\tilde x^*_\lambda, (f,\varphi)(\tilde x^*_\lambda)) + \tilde d(\tilde x^*_\lambda, (f,\varphi)(\tilde x^*_\lambda)).\tilde d(\tilde y^*_\lambda, (f,\varphi)(\tilde y^*_\lambda)) + \tilde d^2(\tilde y^*_\lambda, (f,\varphi)(\tilde y^*_\lambda))}{1 + \tilde d(\tilde x^*_\lambda, (f,\varphi)(\tilde x^*_\lambda)) + \tilde d(\tilde y^*_\lambda, (f,\varphi)(\tilde y^*_\lambda))}\right]$$

$$\psi[\tilde d(\tilde x^*_\lambda, \tilde y^*_\lambda)] \leq \tilde a\,\psi[\tilde d(\tilde x^*_\lambda, \tilde y^*_\lambda)]$$

Which implies that $\psi[\tilde d(\tilde x^*_\lambda, \tilde y^*_\lambda)] = 0$, so $\tilde d(\tilde x^*_\lambda, \tilde y^*_\lambda) = 0$

Thus $\tilde x^*_\lambda = \tilde y^*_\lambda$.

Corollary 3.2: Let a complete soft metric space $(\tilde X, \tilde d, E)$ and let $(f,\varphi) : (\tilde X, \tilde d, E) \to (\tilde X, \tilde d, E)$ be a mapping. We assume that for each $\tilde x_\lambda, \tilde y_\mu \in \tilde X$,

$$\int_0^{\psi[\tilde d((f,\varphi)(\tilde x_\lambda),(f,\varphi)(\tilde y_\mu))]}\varphi(t)dt \leq$$
$$\tilde a\int_0^{\psi[\tilde d(\tilde x_\lambda, \tilde y_\mu)]}\varphi(t)dt$$

+? $\int_0^{\psi \frac{?^2\ ??,\ ?,?\ ??\ +?\ ??,\ ?,?\ ??\ .?\ ??,\ ?,?\ ??\ +?^2\ ??,\ ?,?\ ??}{1+?\ ??,\ ?,?\ ??\ +?\ ??,\ ?,?\ ??}}$?? ? ???

…(3.2.1)

Where $0 < \tilde a + 2\tilde b < 1$ and $\psi: \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a Lebesrue intergable mapping which is Summable on each compact subset of $[0,\infty)$, non negative and such that

$$\int_0^\epsilon \varphi(t)dt > 0,\ \text{for all}\ \epsilon > 0.$$

Then (f,φ) has a unique fixed point in $\tilde x^*_\lambda \in \tilde X$ such that $\lim_{n\to\infty}(f,\varphi)^n\,\tilde x_\lambda = \tilde x^*_\lambda$, for each $\tilde x^*_\lambda \in \tilde X$.

Proof: Let $\psi: \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ be a mapping as we define $\psi(t) = \int_0^t \varphi(t)dt$, $t \in R_+$. It is clear that $\psi(t) = 0$ and ψ is monotonically non decreasing and by hypothesis Ψ is absolutely continuous. Hence ψ is continuous. Therefore, $\psi \in \Psi$, so by (3.1.1) becomes

$$\psi\left[\tilde d\left((f,\varphi)(\tilde x_\lambda), (f,\varphi)(\tilde y_\mu)\right)\right] \leq \tilde a\psi[\tilde d(\tilde x_\lambda, \tilde y_\mu)]$$

+??ψ $\frac{?^2\ ??,\ ?,?\ ??\ +?\ ??,\ ?,?\ ??\ +?\ ??,\ ?,?\ ??\ +?^2\ ??,\ ?,?\ ??}{1+?\ ??,\ ?,?\ ??\ .?\ ??,\ ?,?\ ??}$

Hence from **Theorem 3.1** there exist a unique fixed point $\tilde{y}_\lambda^* \in \tilde{X}$ such that for each $\tilde{y}_\mu \in \tilde{X}, \lim_{n\to\infty}(f,\varphi)^n \tilde{y}_\mu = \tilde{y}_\lambda^*$.

In this section we are going to prove that the mappings satisfying the contractive condition [2.1.1],[2.2.1] and [3.1.1] fulfil the property P.

Theorem3.3: Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space, we have $\psi \in \Psi$. Let mapping $(f,\varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ satisfies the following condition:

$$\psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda), (f,\varphi)(\tilde{y}_\mu)\right)\right] \leq \tilde{a}\, \psi[\tilde{d}(\tilde{x}_\lambda, \tilde{y}_\mu)]$$

For all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$, and for some $0 < \tilde{a} < 1$. Then $E_{(f,\varphi)} \neq \phi$ and (f,φ) has a property P.

Proof: From **Theorem [2.1],** (f,φ) has fixed point therefore $E_{(f,\varphi)^n} \neq \phi$ for every $n \in N, n > 1$ and we assume that $\tilde{y}_\lambda^* \in E_{(f,\varphi)^n}$ we have to prove that $\tilde{y}_\lambda^* \in E_{(f,\varphi)}$,

Assume that $\tilde{y}_\lambda^* \neq (f,\varphi)\tilde{y}_\lambda^*$, from [2.1.1] we have

$$[\tilde{d}(\tilde{y}_\lambda^*, (f,\varphi)(\tilde{y}_\lambda^*))] = \psi[\tilde{d}((f,\varphi)^n(\tilde{y}_\lambda^*), (f,\varphi)^{n+1}(\tilde{y}_\lambda^*))]$$

$$\leq \tilde{a}\, \psi[\tilde{d}((f,\varphi)^{n-1}(\tilde{y}_\lambda^*), (f,\varphi)^n(\tilde{y}_\lambda^*))] \leq \cdots \leq \tilde{a}^n \psi[\tilde{d}(\tilde{y}_\lambda^*, (f,\varphi)(\tilde{y}_\lambda^*))]$$

Since $\tilde{a} \in (0,1), \lim_{n\to\infty} \psi[\tilde{d}(\tilde{y}_\lambda^*, (f,\varphi)(\tilde{y}_\lambda^*))] = 0$.

From the face that, $\psi \in \Psi$ we get $\tilde{y}_\lambda^* = (f,\varphi)(\tilde{y}_\lambda^*)$ which is a contradiction.

Therefore $\tilde{y}_\lambda^* \in E_{(f,\varphi)}$ i.e. (f,φ) has a property P.

Theorem3.4: Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space, and let mapping $(f,\varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ satisfies the contractive condition:

$$\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda), (f,\varphi)(\tilde{y}_\mu)\right) \leq \tilde{a}\, [\tilde{d}(\tilde{x}_\lambda, \tilde{y}_\mu)]$$

$$+?? \frac{??^2\ ??,\ ?,??\ ??\ +??\ ??,\ ?,??\ ??\ .??\ ??,\ ?,??\ ??\ +??^2\ ??,\ ?,??\ ??}{1+??\ ??,\ ?,??\ ??\ +??\ ??,\ ?,??\ ??}$$

For all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$, and $\tilde{a} > 0, \tilde{b} > 0, \tilde{a} + 2\tilde{b} < 1$. Then $E_{(f,\varphi)} \neq \phi$ and (f,φ) has a property P.

Proof: From Theorem [2.2], $E_{(f,\varphi)} \neq \phi$, therefore $E_{(f,\varphi)^n} \neq \phi$ for every $n \in N, n > 1$ and

we assume that $\tilde{y}_\lambda^* \in E_{(f,\varphi)^n}$ we have to prove that $\tilde{y}_\lambda^* \in E_{(f,\varphi)}$

Assume that $\tilde{y}_\lambda^* \neq (f,\varphi)\tilde{y}_\lambda^*$, from [2.2.1] we have

$$\tilde{d}(\tilde{y}_\lambda^*, (f,\varphi)(\tilde{y}_\lambda^*)) = \tilde{d}((f,\varphi)^n(\tilde{y}_\lambda^*), (f,\varphi)^{n+1}(\tilde{y}_\lambda^*))$$

$$\leq \tilde{a}\, \tilde{d}((f,\varphi)^{n-1}(\tilde{y}_\lambda^*), (f,\varphi)^n(\tilde{y}_\lambda^*))$$

$$+?? \frac{??^2\ ?,??^{?-1}\ ??^*,\ ?,??^{?}\ ??^*\ +??\ ?,??^{?-1}\ ??^*,\ ?,??^{?}\ ??^*\ .??\ ?,??^{?}\ ??^*,\ ?,??^{?+1}\ ??^*\ +??^2\ ?,??^{?}\ ??^*,\ ?,??^{?+1}\ ??^*}{1+??\ ?,??^{?-1}\ ??^*,\ ?,??^{?}\ ??^*\ +??\ ?,??^{?}\ ??^*,\ ?,??^{?+1}\ ??^*}$$

$$\leq \tilde{a}\, \tilde{d}((f,\varphi)^{n-1}(\tilde{y}_\lambda^*), (f,\varphi)^n(\tilde{y}_\lambda^*))$$

$+\tilde{b}[\tilde{d}((f,\varphi)^{n-1}(\tilde{y}_\lambda^*), (f,\varphi)^n(\tilde{y}_\lambda^*)) +$ *df,φnyλ∗,f,φn+1yλ∗*

$$\begin{aligned} &\tilde{d}(\tilde{y}_\lambda^*, (f,\varphi)(\tilde{y}_\lambda^*)) \\ &= \tilde{d}((f,\varphi)^n(\tilde{y}_\lambda^*), (f,\varphi)^{n+1}(\tilde{y}_\lambda^*)) \\ &\leq \frac{\tilde{a}}{(1-\tilde{b})} \tilde{d}((f,\varphi)^{n-1}(\tilde{y}_\lambda^*), (f,\varphi)^n(\tilde{y}_\lambda^*)) \end{aligned}$$

$$\leq \cdots \ldots \ldots \leq \left[\frac{\tilde{a}}{(1-\tilde{b})}\right]^n \tilde{d}(\tilde{y}_\lambda^*, (f,\varphi)(\tilde{y}_\lambda^*))$$

Which is the contradiction. Consequently $\tilde{y}_\lambda^* \in E_{(f,\varphi)}$ and (f,φ) has a property P.

Theorem3.5: Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space and let $\psi \in \Psi$. Let $(f,\varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ be a mapping satisfies the contractive condition:

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$$\psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right)\right] \leq \tilde{a}\,\psi[\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu)]$$

$$+??\psi\ \frac{??^2\ ??_{??},\ ??,??\ \ ??_{??}\ \ +??\ ??_{??},\ ??,??\ \ ??_{??}\ \ .??\ ??_{??},\ ??,??\ \ ??_{??}\ \ +??^2\ ??_{??},\ ??,??\ \ ??_{??}}{1+??\ ??_{??},\ ??,??\ \ ??_{??}\ \ +??\ ??_{??},\ ??,??\ \ ??_{??}}$$

For all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$, and $\tilde{a} > 0, \tilde{b} > 0, \tilde{a} + 2\tilde{b} < 1$. Then $E_{(f,\varphi)} \neq \phi$ and (f,φ) has a property P.

Proof: From Theorem [2.1], (f,φ) has a fixed point.

Therefore $E_{(f,\varphi)^n} \neq \phi$ for every $n \in N, n > 1$ and

We assume that $\tilde{y}^*_\lambda \in E_{(f,\varphi)^n}$ we have to prove that $\tilde{y}^*_\lambda \in E_{(f,\varphi)}$

Assume that $\tilde{y}^*_\lambda \neq (f,\varphi)\tilde{y}^*_\lambda$,

$$\psi[\tilde{d}(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))] = \psi[\tilde{d}((f,\varphi)^n(\tilde{y}^*_\lambda),(f,\varphi)^{n+1}(\tilde{y}^*_\lambda))]$$

$$\leq \tilde{a}\,\psi[\tilde{d}((f,\varphi)^{n-1}(\tilde{y}^*_\lambda),(f,\varphi)^n(\tilde{y}^*_\lambda))]$$

$$+??\psi\ \frac{??^2\ \ ??,??^{??-1}\ ??^*_{??},\ ??,??^{??}\ ??^*_{??}\ \ +??\ \ ??,??^{??-1}\ ??^*_{??},\ ??,??^{??}\ ??^*_{??}\ \ .??\ \ ??,??^{??}\ ??^*_{??},\ ??,??^{??+1}\ ??^*_{??}\ \ +??^2\ \ ??,??^{??}\ ??^*_{??},\ ??,??}{1+??\ \ ??,??^{??-1}\ ??^*_{??},\ ??,??^{??}\ ??^*_{??}\ \ +??\ \ ??,??^{??}\ ??^*_{??},\ ??,??^{??+1}\ ??^*_{??}}$$

$$\leq \tilde{a}\psi[\tilde{d}((f,\varphi)^{n-1}(\tilde{y}^*_\lambda),(f,\varphi)^n(\tilde{y}^*_\lambda))]$$

$+\tilde{b}\psi[\tilde{d}((f,\varphi)^{n-1}(\tilde{y}^*_\lambda),(f,\varphi)^n(\tilde{y}^*_\lambda)) +$ *df,φnyλ∗,f,φn+1yλ∗*

Hence

$$\psi[\tilde{d}(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))] = \psi[\tilde{d}((f,\varphi)^n(\tilde{y}^*_\lambda),(f,\varphi)^{n+1}(\tilde{y}^*_\lambda))]$$

$$\leq \left(\frac{\tilde{a}+\tilde{b}}{1-\tilde{b}}\right)\psi[\tilde{d}((f,\varphi)^{n-1}(\tilde{y}^*_\lambda),(f,\varphi)^n(\tilde{y}^*_\lambda))]$$

$$\leq \cdots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots .. \leq$$

$$\leq \left(\frac{\tilde{a}+\tilde{b}}{1-\tilde{b}}\right)^n \psi[\tilde{d}(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))]$$

$\psi[\tilde{d}(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))] \leq$ *a+b1−bnψdyλ∗,f,φyλ∗*

This is a contradiction. Therefore $\psi[\tilde{d}(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))] = 0$, since $\psi \in \Psi$

This conclude that $\tilde{d}(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda)) = 0$, thus $\tilde{y}^*_\lambda \in E_{(f,\varphi)}$ and (f,φ) has the property P.

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