Inventory Optimization Model for Quadratic Increasing Holding Cost and Linearly Increasing Deterministic Demand

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***Abstract:** This article proposes an inventory optimization model for quadratic increasing holding cost and with linearly increasing deterministic demand. Demand function changes with time up to the shortage occurrence. During the period of shortages, demand is constant. Partial backlogging type shortage is considered by assuming constant deterioration. As an economic order quantity (EOQ) problem, an equation for the total cost function is formulated as an optimization problem by applying Maclaurin series approximation. For the optimization of this problem, the second order derivative method is applied. The Cost function convexity is demonstrated with the help of graph in three dimensions. Numerical experimentation is carried out with the support of two numerical examples. The experimented optimal results are included in tabular form for more clarity. Some graphical representations are drawn to show the variations in various parameters of the model. An analysis of sensitivity of the model is performed to detect the most as well as the least sensitive parameters in the proposed optimization problem.*

***Keywords:** Maclaurin series, Shortages, Inventory Optimization.*

I. INTRODUCTION

Basically, demand of products cannot be fixed over time. Rather, it depends on inventory level and selling price, and sometimes on time. The seasonal items, for example warm cloths, dairy-products, green vegetables, etc. are termed as deteriorating products. On account of deterioration, the shortages are occurred in the inventory system, which affects the total inventory cost as well as total profit. Therefore, one of the factors affecting the inventory control is deterioration. U. Dave, L. Patel **[1]** studied a (T, si)-policy for deteriorating products considering variable demand. A. Goswami, and K. S. Chauhduri **[2]** developed an economic order quantity problem with shortages, and finite replenishment rate. They considered the demand as a linear function.

K. J. Chung, and P. S. Ting **[3]** presented a heuristic method to the replenishment of inventory with a variable demand. P. Abad **[4]** developed an inventory optimization model for the optimality of lot-size and price, following the restrictions on partial backordering, perish-ability and deterioration. H. Chang, and C. Dye **[5]** proposed an economic order quantity problem for partial backlog type shortages, by assuming variable demand as a function of time.

Some researchers and scientist considered the concepts of fuzzy number and interval number due to uncertainty in inventory parameters.

J. S. Yao and J. S. Su **[6]** studied a fuzzy inventory problem related with interval-valued fuzzy set. They considered the backorder policy for total demand under fuzzy uncertainty. P. Abad **[7]** suggested an inventory problem for optimal selling price and for order-size of a reseller under shortages. Wu, O. and Cheng **[8]** presented an inventory control problem with deterioration and shortages. They considered the demand function which is exponentially decreasing with time.

C. Dye **[9]** studied an inventory optimization model, where the optimal lot-size and selling price are two decision variables. He considered a variable deterioration rate and exponential time-dependent partial backlog. J. Teng et. al. **[10]** proposed two models for selling price and lot-size. They considered the shortages as a partial backlog, and studied a comparison between these models. A. Alamri, and Z. Balkhi **[11]** proposed an analytical model for optimization of production lot-size. They considered deterioration and time varying demand, and studied the deviations of learning as well as forgetting.

During the management of an inventory, the cost of holding the items in stock is a crucial factor which largely affects the total cost function in many ways. Therefore, it is managerial suggestion to maintain the reasonably low cost of holding the items. In all practical applications, it is seen that the holding cost may be constant at some time, and may be variable at some other time. Therefore, holding cost depends on time and lot-size, and some other factors. A. Roy **[12]** presented an optimization model for inventory problem with time varying holding cost and price dependent demand. He considered the deterioration, and constant holding cost. K. Skouri et. al. **[13]** developed a mathematical model for inventory problem by considering demand as a ramp type function and holding cost as a constant. They considered the shortages with partial backlog, and deterioration as a weibull distribution function. Y. He **[14]** studied a model for optimal production-inventory problem for constant holding cost and the demand as multiple-market. V. K. Mishra, and L. S. Singh **[15]**

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presented a mathematical formulation for inventory problem where the demand depends on time. They considered constant holding cost, and constant deteriorating seasonal products. D. Dutta, and Pavan Kumar **[16]** proposed a model for the inventory. They did not consider the shortages. They considered fuzzy type uncertainty in inventory parameters. For the purpose of fuzzification of the crisp model, they implemented the trapezoidal fuzzy number.

Some researchers have studied the inventory problems where unit holding cost is a function of stock level. Valentín Pando et. al. **[17]** studied the profit maximization in an inventory problem while the demand as well as per unit time holding cost depends on the current stock level. Recently, Valentín Pando et.al. **[18]** presented a crisp model for the economic lot-size problem. They considered non-linear holding cost depending on time as well as quantity. Inventory parameters may vary in the form of intervals also. S. Chakrabortty et. al. **[19]** proposed an inventory problem and solved this to obtain the optimum total cost by introducing a computer algorithm. They considered the interval type uncertainty to develop the formulation. D. Dutta and Pavan Kumar **[20]** formulated an inventory problem as a fractional programming problem with many objective functions, in fuzzy type uncertainty. They applied goal programming method to solve the model.

D. Dutta and Pavan Kumar **[21]** presented a model for an inventory problem. They considered the time-variable demand. They applied an interval number based method to optimize the cost function. D. Dutta and Pavan Kumar **[22]** studied a model developed for an inventory problem. They treated both the demand and holding cost as time depending. Very recently, Pavan Kumar and D. Dutta **[23]** proposed a method to solve an inventory problem for multiple products, which is formulated as fractional programming problem with many objective functions. They considered the demand as the price-dependant with fuzziness. Bhunia and Shaikh **[24]** investigated the two-warehouse inventory problems. They considered the interval environment and inflation. They applied the PSO techniques. U. Mishra **[25]** presented a model for inventory problem with shortages, where the deterioration is a two parameter weibull function. They considered the demand as decreasing function. Proceeding in this direction, Pavan Kumar and P. S. Keerthika **[26]** proposed an inventory formulation for variable holding cost and partial backlog. They considered the interval type uncertainty, and applied the global criteria method to solve the model.

In this article, we consider an inventory problem with quadratic increasing holding cost. We formulate the total cost function as an optimization problem. Our objective is to calculate the optimal value of the total inventory cost. In brief, the remainder of this article is planned to organize as mentioned bellow.

We describe notations and assumption in the section II. While in the section III, we formulate the mathematical optimization model of the proposed inventory problem. In the section IV, we give some numerical illustration, following the section V with sensitivity analysis and observations. Finally, we conclude in the section VI.

II. NOTATIONS & ASSUMPTIONS

A. NOTATIONS

C_1(t)= holding cost for per unit per time unit	W = maximum level of inventory
C_2 = purchasing cost per unit	Q = quantity of order to be placed
C_3 = order cost per order	C_H = holding cost per cycle
C_4 = shortage cost	C_D = cost of deterioration per cycle
C_5 = lost sales cost per unit	C_S = shortage cost per cycle
θ = deterioration rate	C_L = cost due to lost sales per cycle
T = cycle length, t_1 = shortage starting time	Z = average total cost
$T-t_1$ = waiting time	$I(t)$ = level of inventory during [0, T]
D_B = maximum quantity of demand backlogged	$I_1(t)$ = level of inventory during [0, t_1]
δ = backlog parameter	$I_2(t)$ = level of inventory during [t_1, T]

B. ASSUMPTIONS

(1) For the positive inventory, demand is time-dependent linear, while for the negative inventory, it becomes constant. For the arbitrary constants $\beta > 0, and\ D > 0$, the demand is given by

$$R(t) = \begin{cases} \beta t, & \text{when } I(t) > 0 \\ D, & \text{when } I(t) \le 0, \end{cases}.$$

(2) Only one item is considered inventory system.

(3) Infinite planning horizon, with zero lead time.

(4) Deteriorating rate, θ ($0 < \theta < 1$), is fixed. Deterioration starts immediately just after receiving the items. No repair or replacement of deteriorated items.

(5) Partially backlog type shortage is permitted along with backordered rate: $B(t) = \frac{1}{1+\delta\,(T-t)}$, where $t_1 \le t \le T$. If $\delta = 0$, then B(t) equal to 1, which is completely backlogged situation. We consider $\delta < 1$ for the applicability of Maclaurin's series for approximation.

(6) Decision variables are T and t_1, where $0 \le t_1 \le T$.

(7) Holding cost is an increasing quadratic function as follows (for h>0):

$$C_1(t) = ht^2$$

C. JUSTIFICATION FOR CONSIDERING QUADRATIC HOLDING COST

Holding cost is linked with the storage of items until the usages or stored of inventory. It consists of various costs, such as, insurance, space of the ware house, cost of capital tied up, protection, etc.

Moreover, it can be viewed as function of multiple factors such as optimal quantity stored during a period and can be accessed either on a periodic manner or continuously, all these factors may occur simultaneously. As the holding cost depends on various factors, it may be represented as quadratic function of time.

III. MATHEMATICAL MODEL

Replenishment of inventory is done when t = 0. On account of demand as well as deterioration, the stock level starts to decline for the period $0 \leq t \leq t_1$. After that the stock level comes down to zero, when $t = t_1$.
Moreover, during the period $t_1 \leq t \leq T$, the shortage is permitted, and the demand rate is treated as partial backlog. Therefore, the shortage period is $0 < t \leq t_1$, and the no-shortage period is $t_1 < t \leq T$. For the status of inventory system, the controlling equations are:

$$\frac{dI_1(t)}{dt} + \theta . I_1(t) = -\beta t, \quad 0 \leq t \leq t_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt} = \frac{-D}{1+\delta(T-t)}, \quad t_1 \leq t \leq T \qquad (2)$$

with boundary conditions:

$$\left. \begin{array}{l} I_1(t) = I_2(t) = 0 \ at \ t = t_1, \\ I_1(t) = W \ at \ t = 0 \end{array} \right\} \quad (3)$$

Now there arise two cases:

CASE – I (For I(t) > 0): Applying (3), to solve (1), we obtain

$$I_1(t) = \frac{-\beta}{\theta}[(t-\tfrac{1}{\theta}) - (t_1 - \tfrac{1}{\theta})e^{\theta(t_1-t)}], \quad 0 \leq t \leq t_1 \qquad (4)$$

For each cycle, the maximum inventory level:

$$W = I_1(0) = \frac{-\beta}{\theta}\left(\frac{1}{\theta} + (t_1 - \tfrac{1}{\theta})e^{\theta t_1}\right) \qquad (5)$$

CASE –II (For I(t) ≤ 0): Applying (3), the solution of (2) will be

$$I_2(t) = \frac{D}{\delta}[\log\{1+(T-t)\delta\} - \log\{1+(T-t_1)\delta\}],$$

$$\text{for } t_1 \leq t \leq T \qquad (6)$$

Implying that

$$D_B = -I_2(T) = \frac{D}{\delta}[\log\{1+(T-t_1)\delta\}] \qquad (7)$$

And

$$Q = W + D_B$$

$$= \frac{\beta}{\theta}[\frac{1}{\theta} + (t_1 - \tfrac{1}{\theta})e^{\theta t_1}] + \frac{D}{\delta}[\log\{1+(T-t_1)\delta\}]$$

Applying Maclaurin series, and taking terms upto power two, as θ<1, we get

$$Q = \left(\frac{t_1^2}{2}\right)\beta + D[T - t_1 - \frac{\delta(T-t_1)^2}{2}] \qquad (8)$$

We derive the expressions for all associated costs as follows:

Holding cost per cycle:

$$C_H = \int_0^{t_1} C_1(t)\, I_1(t) dt = \int_0^{t_1} -\frac{\beta h t^2}{\theta}[(t-\tfrac{1}{\theta}) - (t_1 - 1\theta)e\theta(t1-t)]dt$$

$$= -\frac{\beta h}{\theta}\int_0^{t_1}[(t^{2+1} - \frac{t^2}{\theta}) - (t_1 - \tfrac{1}{\theta})t^2\{1 + \theta(t_1 - t) + \theta 2t1 - t22\}]dt$$

$$= -\frac{\beta h}{\theta}[\frac{t_1^4}{4} - \frac{t_1^3}{3\theta} - t_1^3(t_1 - \tfrac{1}{\theta})(\tfrac{1}{3} + \frac{\theta t_1}{12} + \frac{\theta^2 t_1^2}{60})] \qquad (9)$$

where $e^{\theta(t_1-t)}$ is expanding into ascending powers of $\theta(t_1-t)$, taking the terms up to 2nd power in θ.

Deterioration cost per cycle:

$$C_D = c_2\left[W - \int_0^{t_1} R(t)dt\right]$$

$$= c_2[\frac{\beta}{\theta}\{\frac{1}{\theta} + \left(t_1 - \frac{1}{\theta}\right)e^{\theta t_1}\} - \int_0^{t_1}\beta t\, dt]$$

$$= c_2\beta[\frac{1}{\theta^2} + \frac{e^{\theta t_1}}{\theta}\left(t_1 - \frac{1}{\theta}\right) - \frac{t_1^2}{2}]$$

$$= c_2\beta[\frac{1}{\theta^2} + \left(1 + \theta t_1 + \frac{\theta^2 t_1^2}{2}\right)\left(\frac{t_1}{\theta} - \frac{1}{\theta^2}\right) - \frac{t_1^2}{2}]$$

$$= \frac{c_2\beta}{2}\theta t_1^3 \qquad (10)$$

Shortage cost:

$$C_S = c_4[-\int_{t_1}^{T} I_2(t)dt]$$

$$= -c_4\frac{D}{\delta}\int_{t_1}^{T}[\log\{1+\delta(T-t)\} - \log\{1+\delta(T-t_1)\}]dt$$

$$= c_4 D[\frac{T-t_1}{\delta} - \frac{1}{\delta^2}\log\{1+\delta(T-t_1)\}] \qquad (11)$$

Lost sale cost:

$$C_L = c_5\int_{t_1}^{T}[1 - \frac{1}{1+\delta(T-t)}]D\, dt$$

$$= c_5 D[(T-t_1) - \frac{1}{\delta}\log\{1+\delta(T-t_1)\}] \qquad (12)$$

Average of total cost per unit time per cycle:

$$Z = \frac{\text{Total cost}}{\text{cycle length}}$$

$$= \frac{C_H + C_D + c_3 + C_S + C_L}{T}$$

$$= \frac{1}{T}\{-\frac{\beta h}{\theta}\left[\frac{t_1^4}{4} - \frac{t_1^3}{3\theta} - t_1^3(t_1 - \tfrac{1}{\theta})(\tfrac{1}{3} + \frac{\theta t_1}{12} + \frac{\theta^2 t_1^2}{60})\right] + \frac{c_2\beta}{2}\theta t_1^3 + c_3 + D(\frac{c_4+\delta c_5}{\delta})[T - t_1 - \frac{\log[1+\delta(T-t_1)]}{\delta}]\}$$

Since,

$$[T - t_1 - \frac{\log[1+\delta(T-t_1)]}{\delta}]$$

$$= (T - t_1) - \frac{1}{\delta}[\delta(T-t_1) - \frac{\delta^2(T-t_1)^2}{2}]$$

$$= \frac{\delta}{2}(T-t_1)^2,$$

By using Macraurin series, and ignoring higher power terms, as δ<1, we obtain

$$Z = \frac{1}{T}\{-\frac{\beta h}{\theta}\left[\frac{t_1^4}{4} - \frac{t_1^3}{3\theta} - t_1^3(t_1 - \tfrac{1}{\theta})(\tfrac{1}{3} + \frac{\theta t_1}{12} + \right.$$

$\frac{\theta^2 t_1^2}{60})\Big] + \frac{c_2\beta}{2}\,\theta t_1^3 + c_3 + D(\frac{c_4+\delta c_5}{\delta})[\frac{\delta}{2}(T-t_1)^2]\}$

$$Z= \frac{1}{T}\{-\frac{\beta h}{\theta}\Big[\frac{t_1{}^4}{4} - \frac{t_1{}^3}{3\theta} - t_1{}^3(t_1 - \frac{1}{\theta})(\frac{\theta^2 t_1^2+5\theta t_1+20}{60})\Big] + \frac{c_2\beta\theta}{2}t_1^3 + c_3 + \frac{D}{2}(c_4+\delta c_5)(T-t_1)^2\} \quad (13)$$

The optimization problem can be written as:

Minimize

$$Z=\frac{1}{T}\{-\frac{\beta h}{\theta}\Big[\frac{t_1{}^4}{4} - \frac{t_1{}^3}{3\theta} - t_1{}^3(t_1 - \frac{1}{\theta})(\frac{\theta^2 t_1^2+5\theta t_1+20}{60})\Big]+\frac{c_2\beta\theta}{2}t_1^3 + c_3 + \frac{D}{2}(c_4+\delta c_5)(T-t_1)^2\}$$

Subject to the constraints

$$T - t_1 > 0$$

$$t_1 > 0,\ and\ T > 0 \quad (14)$$

The optimum values of decision variables T and t_1, (say T^* *and* t_1^*) may be computed after solving following equations:

$$\frac{\partial C(t_1,T)}{\partial t_1} = 0, \ \text{and} \ \frac{\partial C(t_1,T)}{\partial T} = 0, \quad (15)$$

provided that

$$[\frac{\partial^2 C(t_1,T)}{\partial^2 t_1}]_{at\ (t_1^*,\ T^*)} > 0,\ [\frac{\partial^2 C(t_1,T)}{\partial^2 T}]_{at\ (t_1^*,\ T^*)} > 0,$$

And

$$[\Big[\frac{\partial^2 C(t_1,T)}{\partial^2 t_1}\Big]\Big[\frac{\partial^2 C(t_1,T)}{\partial^2 T}\Big] - [[\frac{\partial^2 C(t_1,T)}{\partial t_1 \partial T}]^2]_{at\ (t_1^*,\ T^*)} > 0 \quad (16)$$

IV. NUMERICAL EXPERIMENTATION

Example I: Consider the following input data, in proper units:

$$R(t)=\begin{cases}\beta t, & \text{when } I(t) > 0\\ D, & \text{when } I(t) \le 0,\end{cases}, \text{ where } \beta = 24, and\ D=16.$$

Also let h=4 > 0, m=2, θ = 0.06, δ = 0.4, c_2 = 4, c_3 = 60, c_4= 8, c_5 = 12. Then, solving the optimization problem (14) by computer packages MATHEMATICA 8.0, we obtain the optimal results as: t_1= 1.2122, T = 2.2971, and Z= 83.3151.

Example II: Consider the following input data, in proper units:

$$R(t) =\begin{cases}\beta t, & \text{when } I(t) > 0\\ D, & \text{when } I(t) \le 0,\end{cases}, \text{ where } \beta = 150, D = 80.$$

Also, h = 12 > 0, m=4, θ = 0.09, δ = 0.7, c_2 = 15, c_3= 300, c_4= 32, c_5= 44. Then, solving the optimization problem (14) by computer packages MATHEMATICA 8.0, we obtain the optimal results as: t_1= 1.1272, T= 1.6889, and Z=1384.1100.

Table 1: Experimented Results in Tabular Form

NUMERICAL EXPERIMENT	OPTIMAL VALUE OF		
	t_1	T	Z
Example I	1.2122	2.2971	83.3151
Example II	1.1272	1.6889	1384.11

V. SENSITIVITY ANALYSIS AND OBSERVATIONS

To analyze the flexibility of the proposed model, we attempt to study the impact of variations in different parameters against average total cost function Z. That is accomplished by the value change of each parameter by 20%, 10%, −10% and −20%, considering only one parameter at a time, while maintaining the remaining parameters unaltered. The obtained results are displayed by the following Table 2:

Table 2: Sensitivity Analysis (Example-I)

INPUT DATA			OUTPUT DATA		
Parameter	**Change in parameter (%)**	**Parameter Value**	t_1	T	Z
c_2 = 4	− 20	3.20	1.2205	2.2994	82.8640
	− 10	3.60	1.2163	2.2982	83.0906
	+ 10	4.40	1.2081	2.2959	83.5374
	+ 20	4.80	1.2041	2.2947	83.7575
c_3= 60	− 20	48	1.1908	2.2.62	77.9860
	− 10	54	1.2018	2.2522	80.6774
	+ 10	66	1.2222	2.3408	85.9024
	+ 20	72	1.2319	2.3834	88.4425
c_4=8	− 20	6.4	1.1893	2.2000	77.6229
	−10	7.2	1.2010	2.2492	80.4997
	+ 10	8.8	1.2229	2.3437	86.0732
	+ 20	9.6	1.2331	2.3891	88.7777
c_5=12	− 20	9.6	1.1943	2.4775	78.8371
	− 10	10.8	1.2040	2.3790	81.2215
	+ 10	13.2	1.2195	2.2277	85.1715
	+ 20	14.4	1.2258	2.1680	86.8311
D=16	− 20	12.6	1.1695	2.3752	72.9262
	− 10	14.4	1.1930	2.3287	78.5026
	+ 10	17.6	1.2303	2.2722	88.0160
	+ 20	19.2	1.2473	2.2523	92.6223
β=24	− 20	19.2	1.2780	2.3348	81.1570
	− 10	21.6	1.2429	2.3145	82.2966
	+ 10	26.4	1.1850	2.2818	84.2352
	+ 20	28.8	1.1605	2.2682	85.0741
θ=0.06	− 20	0.048	1.2216	2.3002	82.8364
	− 10	0.054	1.2170	2.2987	83.0771
	+ 10	0.066	1.2076	2.2955	83.5504

	+ 20	0.072	1.2030	2.2940	83.783
δ=0.4	− 20	0.32	1.1943	2.4775	78.8371
	− 10	0.36	1.2040	2.3790	81.2215
	+ 10	0.44	1.2195	2.2277	85.1715
	+ 20	0.48	1.2258	2.1680	86.8311
h=4	− 20	3.20	1.2684	2.3317	81.6664
	− 10	3.60	1.2385	2.3132	82.5344
	+ 10	4.40	1.1888	2.2829	84.0242
	+ 20	4.80	1.1677	2.2702	84.6737

Since the total cost function Z is highly nonlinear, it is better to show the convexity of Z graphically rather than analytically. We plot the graph for the total cost function Z to demonstrate its convexity, which results in a $3D$ convex graph (Fig.1 and Fig. 2).

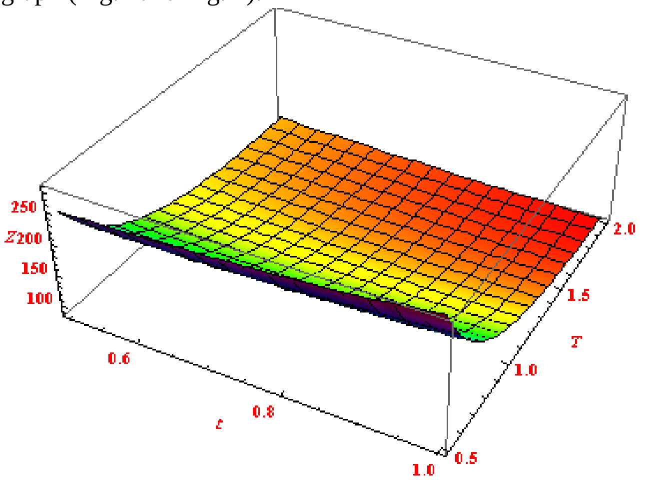

Fig. 1: Convexity of Cost Function (Example-I)

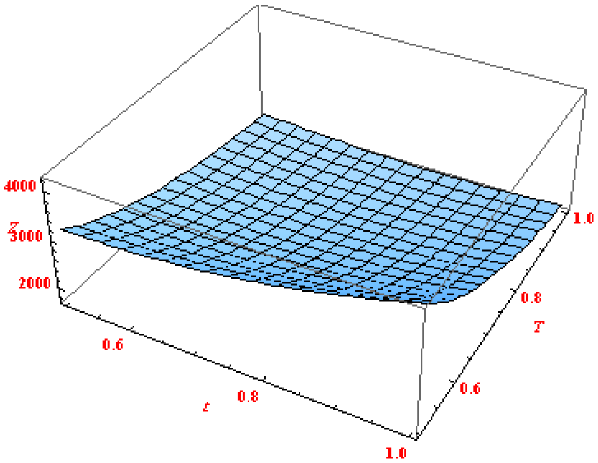

Fig. 2: Convexity of Cost Function (Example-II)

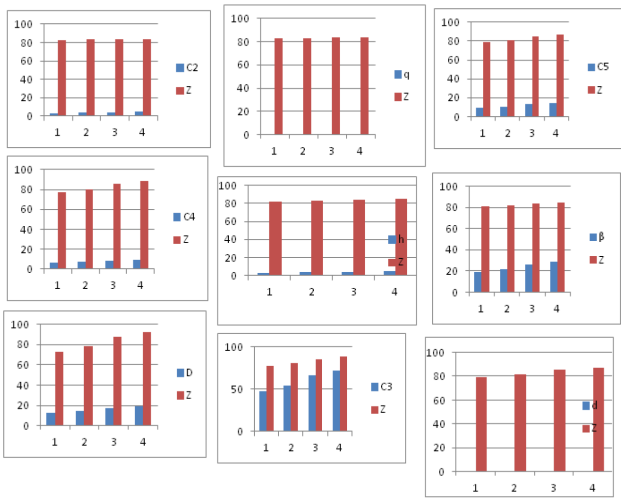

Fig. 3: Graphical Representations For Variations in Various Parameters

OBSERVATIONS: By sensitivity analysis as given in Table 2, some points are recorded as the below observations:

(i) Average of the total cost Z starts to increase as any one of the parameters c_2, c_3, c_4 , c_5 , D, β, and δ increases.

(ii) Z slightly increase with the increase in deterioration θ.

(iii) Z shows high sensitivity to the variation in demand component D in comparison of others.

(iv) Z shows comparatively less sensitivity to the variations in purchase cost c_2 in comparison of the other remaining parameters.

(v) If backlogging parameter δ increases/decreases, Z also increases/decreases. Therefore, for optimal value of Z, the backlogging δ must be minimum.

Therefore, to obtain the optimal value of the total cost function, the optimal strategy must be to keep the stock high so as to decrease the cost due to lost sales. Moreover, for quitting large holding cost, the stock level must be kept as low as possible.

VI. CONCLUSIONS

In this work, a deterministic inventory model for time-dependant demand rate and quadratic holding cost considering shortages is proposed. The second derivative method was used to derive the equation for the total inventory cost. Additionally, for its practical applications, we solved some numerical examples following the analysis of sensitivity. All computational works in both the numerical examples are carried out using the computer packages MATLAB and LINGO, and the computational results are discussed. From the sensitivity analysis, the results indicate that the effect of variation of rate of changes of the ordering cost and demand on the system behavior is more significant. Also, the deterioration rate and the backordering rate have some impact on the total cost of the system.

For the optimization of cost function, we should design the most economical replenishment strategy to reduce the component of ordering cost. The developed model can be very helpful to the industries and retailers in accurately determining the optimal inventory cost. There are some products in market, like warm cloths, etc., for which during the season period, the demand declines over time, and when the season is off, the demand immediately decreases and eventually becomes fixed. On account of this tendency of demand rate, the proposed model might be applied in inventory management problem of seasonal products.

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