

Prediction of Shear Strength of Steel Fibre Based Concrete Beams without Shear Stirrups

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Abstract: Seventy Seven tests were conducted on reinforced concrete beams with four steel fibre volume fractions (0, 0.5, 0.75, 1.0 %), two aspect ratios of fibres (66.6 and 80.0), five shear span-depth ratios (1, 1.3, 1.6, 2.6 and 4.0) and three concrete strengths (23.30, 29.72 and 34.23). The results demonstrate that ultimate shear strength increases with increasing fibre volume, decreasing shear span to depth ratio, and increasing concrete compressive strength. It has been seen that as the fibre content increased, the failure mode changed from shear to flexure mode for the beams with moderate amount of longitudinal tension steel. The test results were used to evaluate existing and proposed empirical equations for estimating shear strength. It was found that the equations proposed herein and the equations developed by Narayanan and Darwish provide a good accuracy for the estimation of shear strength

I. INTRODUCTION

The shear strength of concrete is indeed a complex phenomenon involving many variables. Even the modified compression field theory is supported on certain assumptions (plain sections before and after loading remain plain which is not particularly true in case of deep beams) and empirical relations (stress-strain relationships of concrete). Due to these shortcomings, the design procedures for the evolution of shear strength of concrete members are still based primarily on experimental results rather than purely theoretical findings.

Early design specifications considered cylinder strength of concrete as the only principal variable in the prediction of shear strength (Rebeiz -12). In 1909, Talbot [14] (University of Illinois Experiment Station) demonstrated that "percentage of reinforcement and the length-to-depth ratio also play an important role in the shear and diagonal tension strength of concrete beams without web reinforcement." Subsequent to these findings, other variables such as the maximum aggregate size, spacing of the flexural cracks, and diameter of tensile reinforcing bars have also been found to impact the shear strength of concrete members (Kim and Park [4]). Nevertheless, it is now widely accepted that the three main variables affecting the shear strength of concrete members without web reinforcement are the concrete compressive strength (f_c), the shear span-to-depth ratio (a/d), and the tensile reinforcement ratio (ρ) (Kim and Park [4]).

There have been a number of studies in the past, which confirm the effectiveness of steel fibres as shear reinforcement for RC beams [1, 2, 3, 5-9, 11 and 13] and hence adding to the variables which govern the shear strength of fibre based concrete. The important additional factor is the post cracking tensile strength of fibre based concrete. Over the years, several semi-empirical relations have been proposed to determine the ultimate shear strength of Steel Fibre Based Concrete (SFBC, hereafter) beams. A number of formulas proposed by researchers, namely, Ashour et al., Li et al., Mansur et al., Narayanan and Darwish, Sharma, Shin et al and Swamy et al. have been listed by Khuntia et al. [3]. However in most of the design expressions, the segregation of experimental results in terms of shear span to depth (a/d) ratio have not been taken into consideration appropriately and more important the parameters which constitute the post cracking tensile strength of SFBC have not been incorporated in toto. This has resulted in a significant deviation in the predicted results for shear strength of SFBC, when computed using these formulas.

Research Significance.

The sole inspiration to conduct this study is to incorporate in an appropriate way the important variables governing the shear strength of SFBC, so as to develop a more reliable expression for shear strength of SFBC in a rational manner. Large numbers of experiments have been conducted to produce sufficient data in order to cover the variables to a large range for the best fit of empirical expressions.

Factors effecting Shear Strength of SFBC

With reference to shear strength, the major difference in RC beams with and without fibres, lies in the significant post cracking tensile strength of SFBC. Hence the parameters influencing the ultimate shear strength of SFBC beams are those that affect the shear strength of conventional RC beams plus the post cracking tensile strength or the split cylinder strength of SFBC. Therefore it becomes imperative to expedite sensibly the post cracking tensile strength of SFBC (σ_{pc}) for the prediction of shear strength and is discussed in detail ahead, here.

II. Experimental Program.

77 sets of beams with two of each kind (154 total) were tested to failure to evaluate the affect of different variables, discussed in earlier section, on the shear performance of SFBC beams.

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The shear span ($a = 400 \text{ mm}$) was maintained constant while as the depth of beams, longitudinal reinforcement ratio, grade of concrete, fibre content and aspect ratio of fibres were changed for each set of beams. The longitudinal bars were hooked upwards behind the supports and enclosed by one stirrup at each end. This detail precluded the possibility of anchorage failure, which is of great importance in practice. No stirrups were included within the span.

The beams were tested under four point loading system (Fig. 1) over a span of 1000 mm, under 50 ton capacity loading frame, attached with loading jack, proving ring and dial gauges to note the readings. At the beginning of each test, deflections were imposed by increasing the load in small increments, but as the beam approached its capacity, the test was controlled by gradually increasing the beam deflection.

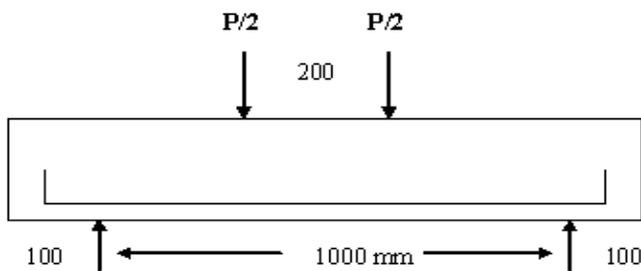


Fig. 1: Testing setup

Table 1 : Concrete Mix Design

Mix.	Cement kg/m ³	Fine agg. kg/m ³	Coarse agg. kg/m ³	Plasticizer By wt. Of cement %	w / c ratio	Slump mm	f'_c MPa	f_t MPa
M1	400	720	1320	0.65	0.40	70	34.23	4.09
M2	360	720	1320	0.50	0.45	71	29.72	3.81
M3	320	720	1320	0.50	0.50	65	23.30	3.33

III. TEST RESULTS

The ultimate shear strengths of 77 beams (mean of two specimens for each beam) are reported in table 2 through table 4 in terms of the average stress at failure, which is defined as the maximum shear force divided by the beam width and effective depth ($v_e = V_u / bd$). Fig-2a, b, shows that the average shear stress at failure consistently decreased with increasing a/d . The difference in capacity of the beams with a/d of 2.6 is significantly large in comparison to those with a/d of 4.0 which can be attributed to the fact that arching action becomes less effective with increasing a/d , and is found insignificant here beyond a/d value of 3.0. It is also evident from the tabular results that there is not appreciable increase in the strength with the increase in longitudinal steel content beyond 2% when a/d is on lower side (less than 1.3 here), as such in predicting shear capacity of very deep beams, the contribution of ρ is not considered of vital importance, Warwick et al [16] and is attributed to the direct compression failure of deep beams.

Material Properties.

Three basic plain concrete mixture series were prepared to cover the range of concrete from normal to medium strength. The proportions of ingredients and properties for each basic mix are shown in table 1. Each mix was embedded with different combinations of steel fibre volume fractions. The coarse aggregates were crushed gravel with a maximum size of 16 mm and the fine aggregates were natural river sand with a fineness modulus of 2.17. A high range water reducing admixture was used to improve the workability of SFBC. The steel fibres were crimped, 30 and 36 mm long, 0.45 mm diameter, which corresponds to aspect ratio of 66.6 and 80 respectively and had nominal yield strength of 840 MPa.

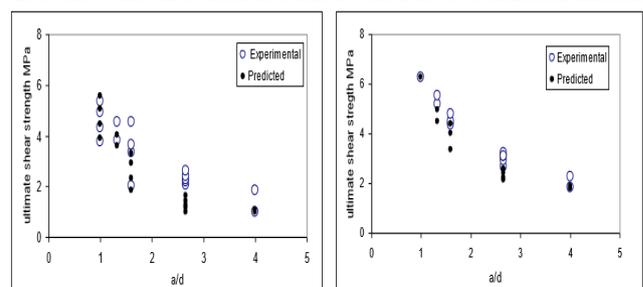


Fig. 2-a: influence of a/d on shear strength ($V_f = 0 \%$) Fig. 2-b: influence of a/d on shear strength ($V_f = 1 \%$)

Table 2: Experimental and Predicted Shear Strength, MPa (V_f = 0%)

S. No.	V _f	ρ	d	a/d	f _c	f _t	v _e (Expt.)	Predicted Strength		
								v _u (Proposed)	v _a (Narayanan)	v _e / v _u
1	0	0.39	400	1.00	34.23	4.09	2.01*	-	-	-
2	0	0.78	400	1.00	34.23	4.09	3.77	3.90	4.11	0.96
3	0	1.17	400	1.00	34.23	4.09	4.32	4.46	4.87	0.96
4	0	1.70	400	1.00	34.23	4.09	4.92	5.05	5.91	0.97
5	0	2.27	400	1.00	34.23	4.09	5.35	5.56	7.04	0.96
6	0	2.09	300	1.33	34.23	4.09	3.81	3.58	4.09	1.06
7	0	3.01	300	1.33	34.23	4.09	4.52	4.04	5.07	1.11
8	0	0.62	250	1.60	34.23	4.09	2.03	1.82	1.98	1.11
9	0	1.25	250	1.60	34.23	4.09	3.34	2.30	2.43	1.45
10	0	2.51	250	1.60	34.23	4.09	3.64	2.90	3.32	1.25
11	0	3.61	250	1.60	34.23	4.09	4.52	3.28	4.10	1.37
12	0	1.04	150	2.66	34.23	4.09	2.05	0.99	1.08	2.07
13	0	1.72	150	2.66	34.23	4.09	2.19	1.17	1.23	1.87
14	0	2.08	150	2.66	34.23	4.09	2.25	1.24	1.31	1.81
15	0	3.12	150	2.66	34.23	4.09	2.40	1.43	1.54	1.67
16	0	4.50	150	2.66	34.23	4.09	2.60	1.61	1.84	1.61
17	0	1.04	150	2.66	29.72	3.81	1.71	0.94	1.02	1.82
18	0	1.72	150	2.66	29.72	3.81	1.92	1.11	1.17	1.73
19	0	2.08	150	2.66	29.72	3.81	2.01	1.19	1.25	1.68
20	0	3.12	150	2.66	29.72	3.81	2.18	1.36	1.48	1.60
21	0	1.04	150	2.66	23.30	3.33	1.36	0.87	0.92	1.56
22	0	1.72	150	2.66	23.30	3.33	1.70	1.03	1.07	1.65
23	0	2.08	150	2.66	23.30	3.33	1.98	1.09	1.15	1.81
24	0	3.12	150	2.66	23.30	3.33	2.25	1.25	1.38	1.80
25	0	4.50	150	2.66	23.30	3.33	2.25	1.42	1.68	1.58
26	0	0.60	100	4.00	34.23	4.09	0.34*	-	-	-
27	0	1.00	100	4.00	34.23	4.09	0.60*	-	-	-
28	0	1.40	100	4.00	34.23	4.09	0.98	0.98	1.19	1.00
29	0	1.88	100	4.00	34.23	4.09	1.83	1.08	1.26	1.69

*: indicates moment failure and has not been considered for prediction of shear strength

Table 3: Experimental and Predicted Shear Strength, MPa (V_f = 0.5 & 0.75%)

S. No.	V _f	l _f /d _f	ρ	d	a/d	f _c	f _t	v _e (Expt.)	Predicted Strength		
									v _u (Proposed)	v _a (Narayanan)	v _e / v _u
1	0.5	66.6	0.39	400	1.00	36.25	4.51	2.62*	-	-	-
2	0.5	66.6	0.78	400	1.00	36.25	4.51	4.14	4.31	4.98	0.96
3	0.5	66.6	1.17	400	1.00	36.25	4.51	4.66	4.55	5.75	1.02
4	0.5	66.6	1.70	400	1.00	36.25	4.51	5.13	5.49	6.79	0.93
5	0.5	66.6	2.27	400	1.00	36.25	4.51	5.55	6.01	7.92	0.92
6	0.5	66.6	2.09	300	1.33	36.25	4.51	4.41	3.99	4.90	1.10
7	0.5	66.6	3.01	300	1.33	36.25	4.51	5.23	4.45	5.88	1.17
8	0.5	66.6	0.62	250	1.60	36.25	4.51	2.32	2.20	2.75	1.05
9	0.5	66.6	2.51	250	1.60	36.25	4.51	3.78	3.30	4.08	1.14
10	0.5	66.6	3.61	250	1.60	36.25	4.51	4.44	3.69	4.86	1.20

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11	0.5	80.0	1.04	150	2.66	37.55	4.65	1.76*	-	-	-
12	0.5	80.0	1.72	150	2.66	37.55	4.65	2.32	1.62	2.10	1.43
13	0.5	80.0	2.08	150	2.66	37.55	4.65	2.53	1.70	2.18	1.48
14	0.5	80.0	3.12	150	2.66	37.55	4.65	2.74	1.89	2.41	1.45
15	0.5	80.0	4.50	150	2.66	37.55	4.65	2.88	2.08	2.71	1.38
16	0.75	80.0	1.04	150	2.66	38.29	4.76	1.80*	-	-	-
17	0.75	80.0	1.72	150	2.66	38.29	4.76	2.4	1.84	1.84	1.30
18	0.75	80.0	2.08	150	2.66	38.29	4.76	2.60	1.92	2.59	1.35
19	0.75	80.0	3.12	150	2.66	38.29	4.76	2.98	2.11	2.82	1.41
20	0.75	80.0	4.50	150	2.66	38.29	4.76	3.0	2.30	3.12	1.30
21	0.5	66.6	0.60	100	4.00	36.25	4.51	0.39*	-	-	-
22	0.5	66.6	1.00	100	4.00	36.25	4.51	0.56*	-	-	-
23	0.5	66.6	1.40	100	4.00	36.25	4.51	1.41	1.34	1.90	1.05
24	0.5	66.6	1.88	100	4.00	36.25	4.51	2.10	1.45	1.98	1.44

Table 4: Experimental and Predicted Shear Strength, MPa ($V_f = 1.0\%$)

S. No.	V_f	l_f/d_f	ρ	d	a/d	f_c	f_t	v_e (Expt.)	Predicted Strength		
									v_u (Proposed)	v_a (Narayanan)	v_e / v_u
1	1.0	80	0.39	400	1.00	41.50	5.15	2.68*	-	-	-
2	1.0	80	1.70	400	1.00	41.50	5.15	6.28	6.26	7.75	1.00
3	1.0	66.6	2.09	300	1.33	39.80	4.98	5.19	4.47	5.80	1.16
4	1.0	66.6	3.01	300	1.33	39.80	4.98	5.52	4.95	6.77	1.11
5	1.0	80	0.62	250	1.60	41.50	5.15	2.32	2.82	3.95	0.82
6	1.0	80	1.25	250	1.60	41.50	5.15	4.35	3.33	4.39	1.30
7	1.0	80	2.51	250	1.60	41.50	5.15	4.50	3.98	5.28	1.13
8	1.0	80	3.61	250	1.60	41.50	5.15	4.78	4.38	6.06	1.09
9	1.0	80.0	1.04	150	2.66	41.50	5.15	1.94*	-	-	-
10	1.0	80.0	1.72	150	2.66	41.50	5.15	2.64	2.12	3.03	1.24
11	1.0	80.0	2.08	150	2.66	41.50	5.15	2.93	2.20	3.11	1.33
12	1.0	80.0	3.12	150	2.66	41.50	5.15	3.23	2.40	3.34	1.34
13	1.0	80.0	4.50	150	2.66	41.50	5.15	3.08	2.56	3.64	1.2
14	1.0	80	1.04	150	2.66	30.11	4.38	1.72*	-	-	-
15	1.0	80	1.72	150	2.66	30.11	4.38	2.47	1.99	2.64	1.24
16	1.0	80	2.08	150	2.66	30.11	4.38	2.54	2.07	2.71	1.22
17	1.0	80	3.12	150	2.66	30.11	4.38	2.54	2.24	2.94	1.13
18	1.0	80	1.04	150	2.66	23.50	3.78	1.46	1.75	2.24	0.83
19	1.0	80	1.72	150	2.66	23.50	3.78	1.80	1.91	2.36	0.94
20	1.0	80	2.08	150	2.66	23.50	3.78	2.06	1.97	2.44	1.04
21	1.0	80	3.12	150	2.66	23.50	3.78	2.31	2.13	2.66	1.08
22	1.0	80	4.50	150	2.66	23.50	3.78	2.34	2.30	2.97	0.84
23	1.0	66.6	1.40	100	4.00	39.80	4.98	1.82	1.78	2.69	1.08
24	1.0	66.6	1.88	100	4.00	39.80	4.98	2.27	1.89	2.76	1.20

The affect of fibre content on the strength of fibre concrete beams is shown if Fig. 3. The strength of SFBC beams ranged from 120 to 136% of the strength of beams without fibres.

The strength increase was particularly large for the beams with lower a/d value, which failed in shear or shear flexure mode.

For the beams with large a/d value, which also failed in shear or shear - flexure mode, the increase in strength was found to be less than 20% and the beams which failed in flexure mode only (marked with * in table 2), the increase in strength was even lesser. The results of these beams were deleted for the prediction of shear strength for the reason that applied load at failure is not equal to the shear strength load; instead, this load only provides a lower bound on the shear strength. However it was noted that the beams with very low longitudinal reinforcement ratio (0.39 %, table 2), the failure mode was mainly flexure, irrespective of a/d value.

Figure 4 & 5, depicts the modes of failure of typical beam elements with and without fibres. Fig-4 shows that as the depth increases for same shear span, the failure inclines towards instantaneous brittle state, but the addition of fibres causes gradual failure in the beam elements. It is clear from fig-5 that the increased fibre content changed the mode of failure from brittle shear to ductile manner with large deformation and dissipation of energy prior to ultimate failure. Hence the addition of fibres results in the production of sufficient ductility in shear dominant structural members, thus avoiding sudden collapse which is highly treacherous.

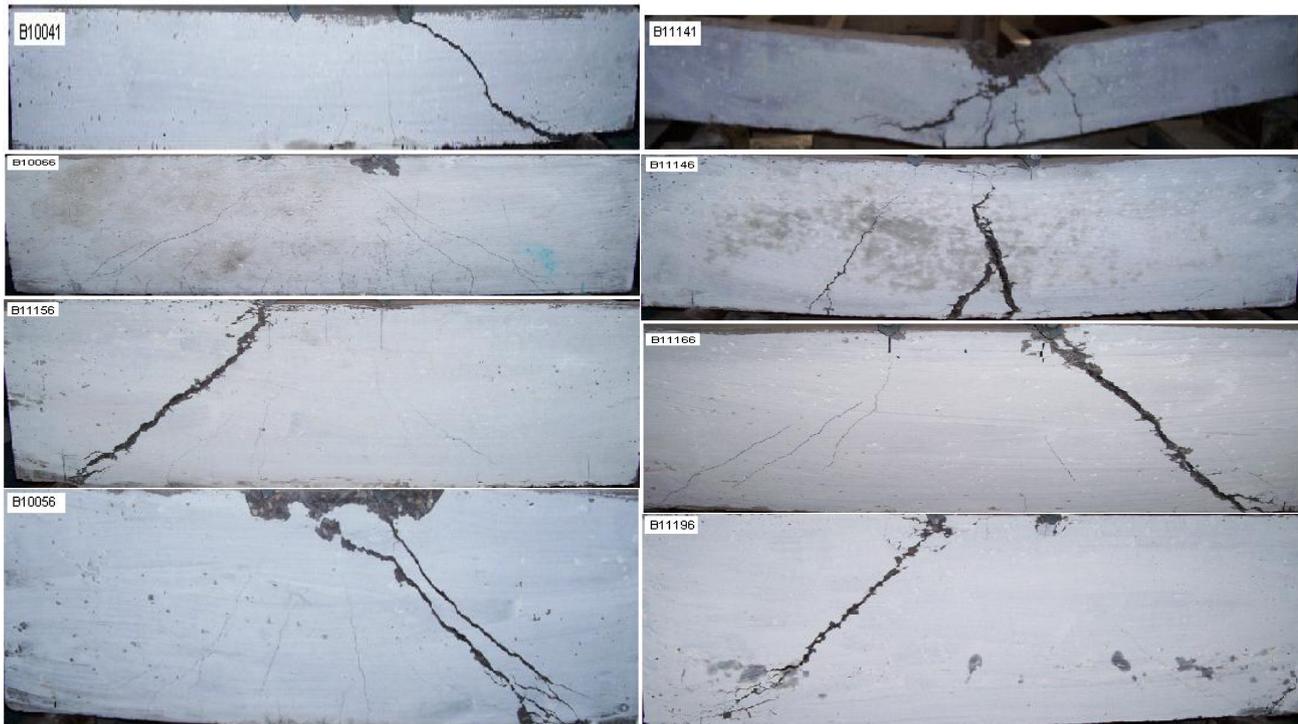


Fig. 4, Comparison of typical beam elements of different depths with and without fibres

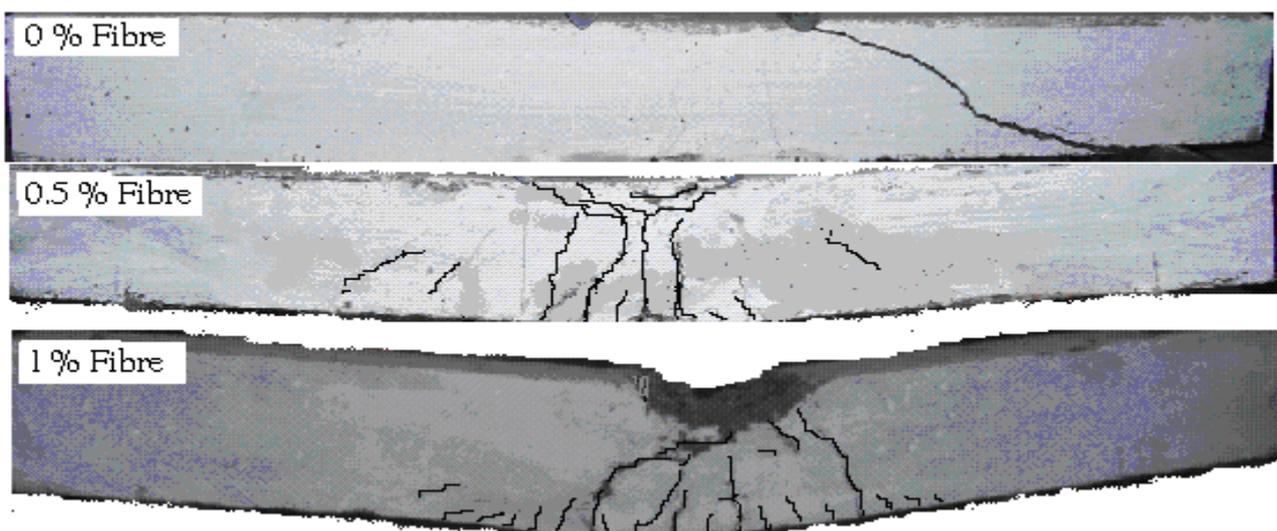


Fig. 5: Typical Crack Patterns, $\rho = 1 \%$, $d = 150 \text{ mm}$

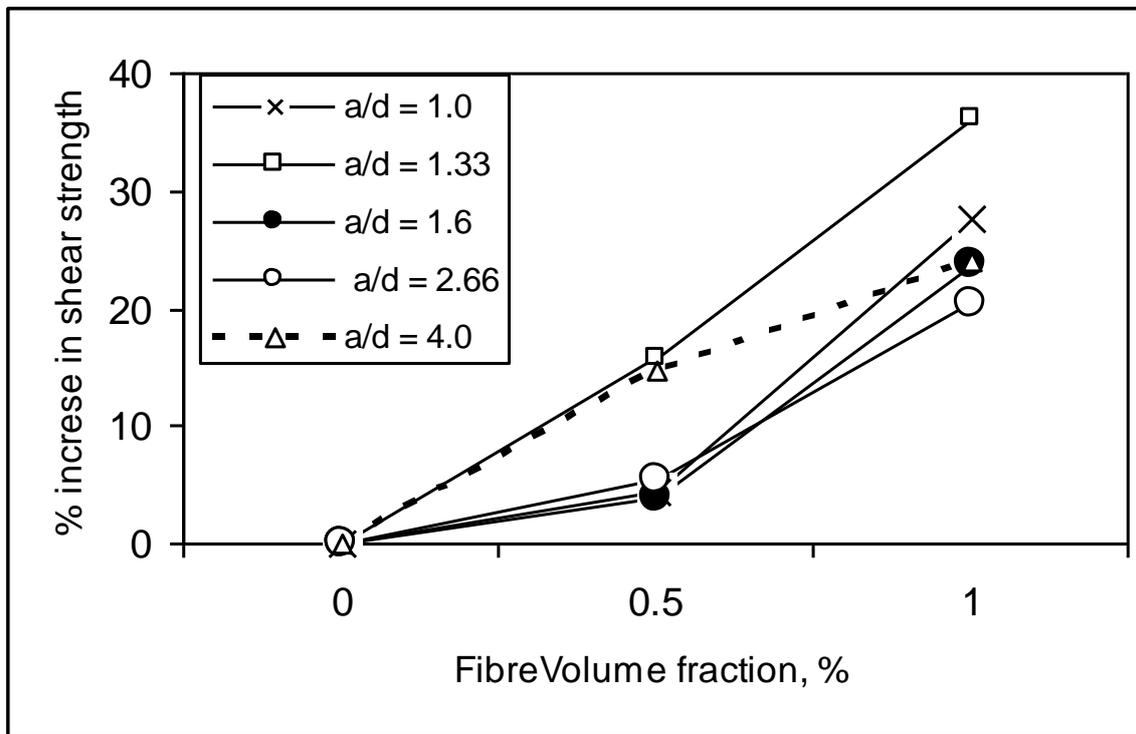


Fig. 3: Influence of Fibre volume on Shear Strength

The effect of concrete strength can be evaluated by comparing the test results of beam elements which are identical in all respects except for the strength of concrete. Comparing the results of Beam No. 11 and 20 (table 4). Both the beams have $V_f = 1\%$, $\rho = 2.08\%$, $a/d = 2.66$ and differ only in concrete strength i.e. 41.5 and 23.5 MPa respectively. It is seen that as the concrete strength was approximately doubled, the strength increased for the given fibre content by 28 %.

Existing Shear Strength Equations.

Several empirical equations are available in the literature to predict the shear capacity of normal to medium strength fibre based concrete beams without stirrups, one of which is explained here.

Narayanan and Darwish equation.

Using steel fibres as shear reinforcement, Narayanan and Darwish [8] have presented the ultimate shear strength (v_a) of SFBC beams as:

$$v_a = e \left[0.24 f_t + 80 \rho \frac{d}{a} \right] + v_b$$

Where

$$e = 1.0 \quad \text{when } a/d > 2.8$$

$$e = 2.8(d/a) \quad \text{when } a/d \leq 2.8$$

$$v_b = 0.41 \tau F$$

τ = fibre matrix interfacial bond stress, MPa

$$= 0.75 \sqrt{f'_c}$$

f'_c = cylinder compressive strength of FBC, MPa

f_t = split cylinder strength of FBC, MPa

ρ = percentage of longitudinal tensile reinforcement

d/a = shear span to effective depth ratio

Post cracking tensile strength of FBC

There is considerable reserve strength in FBC beams failing in shear after the appearance of first diagonal crack and is attributed to significant post cracking tensile strength of FBC

compared to plain concrete. This residual stress is referred as post-cracking-tensile strength (σ_{pc}). Extensive experimental investigations and analytical studies have shown that this post cracking tensile strength can be expressed in the following form.

$$\sigma_{pc} = \alpha \beta \tau V_f \left(\frac{l_f}{d_f} \right) = \alpha \beta F \quad \text{where:}$$

$$F = \beta V_f \left(\frac{l_f}{d_f} \right) \quad \text{is the fibre factor}$$

β = factor for fibre shape and concrete type, taken as 1 for hooked or crimped steel fibres, 2/3 for plain or round steel fibres with normal concrete

α = orientation factor taken as 0.4 on conservative side, Lim. Et al. [6]; Swamy et al. [13]

τ = Fibre-matrix interfacial bond stress

A comprehensive study by Naaman and Najm[10] on bond slip mechanisms of steel fibres in concrete demonstrated that there is a definite increase in, τ , with the increase in compressive strength of concrete (f'_c). The table 3 below shows the variation of bond stress, τ , with cylindrical compressive strength of concrete.

It is seen from the table that interfacial bond stress is influenced by the concrete strength and the type of fibres (crimped, plain or hooked) with a minimum ratio $\tau / \sqrt{f'_c}$ equal to 0.75 for both normal and high strength FBC. But Lim et al.

in their study obtained that, τ , varied from 5.3 MPa to 8.0 MPa for same strength concrete (34 MPa) and same type fibres (hooked steel). This can be attributed to the relative diameter / Eq. diameter of fibre w.r.t size of fine aggregate in the matrix.

When the size of fibres is equal to or less than the size of higher fraction of fine aggregates, then there will be sufficient space left in between particles of fine aggregate in most parts of the matrix to accommodate a fibre loosely, and hence

Investigator	Fibre type	Con. Strength, (f'_c)	Fib-mat. Bond stress, τ	Ratio($\tau / \sqrt{f'_c}$)
Naaman et al.	Hooked steel	33.4	4.31	0.75
		51	5.87	0.82
		60	7.5	0.97
Swamy et al.	Crimped steel	47	5.12	0.75
Lim et al.	Hooked steel	34	5.3 to 8.0	0.91 to 1.38

would result in the decrease in interfacial bond strength. As such a factor, γ , has been introduced in this study to account for relative size of fibres w.r.t size of fine aggregates. Based on intuitions and comparison of experimental results in this study with other's results, the values of, γ , for different fibre sizes in normal sands (fineness modulus = 2 to 2.5) are proposed as under.

- $\gamma = 1.3$ for $d_f \leq 0.5$ mm
- $= 1.2$ for $0.5 \leq d_f \leq 0.75$
- $= 1.1$ for $0.75 \leq d_f \leq 1.0$
- $= 1.0$ for $d_f > 1.0$

Where, d_f denotes the diameter or equivalent diameter of fibre
Considering the minimum ratio $\tau / \sqrt{f'_c}$ of 0.75, the interfacial bond stress can be expressed as.

$$\tau / \sqrt{f'_c} = 0.75 / \gamma$$

$$\text{or } \tau = (0.75 / \gamma) \sqrt{f'_c}$$

$$\text{therefore } \sigma_{pc} = \alpha \cdot \tau \cdot F$$

$$= 0.4 [(0.75 / \gamma) \sqrt{f'_c}] F$$

$$= (0.29 / \gamma) F \sqrt{f'_c}$$

Proposed equation for shear strength of SFBC

There exists a wide divergence of opinions, design approaches and code equations related to shear strength of reinforced concrete members. The differences between the shear strength equations are the result of considerable scatter of the experimentally observed shear strengths. A consequence of this lack of fit to experimental data is the acceptance of conservative lower limits for code equations. It can be shown that total shear resistance of FBC beams is the sum of shear contribution from concrete and shear contribution from the fibres. Consider the forces acting at a diagonal crack in FBC beam without shear reinforcement (Fig. 4).

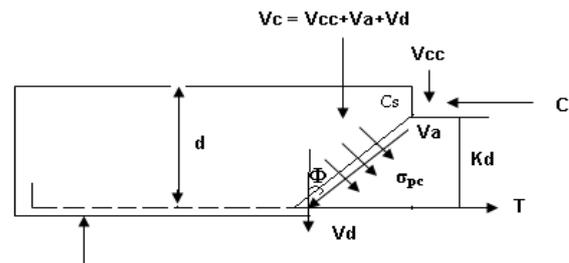


Fig. 4: Shear resistance of FRC beams without stirrups

The total shear resistance, V_u is given by:

$$V_u = V_c + V_f$$

Where V_c and V_f is the contribution in shear from concrete and fibres respectively.

From the figure:

$$V_f = (\sigma_{pc} \sin \Phi) b_w \cdot (kd / \cos \Phi)$$

$$= \sigma_{pc} b_w kd \tan \Phi$$

$$= (0.29 k / \gamma) F \sqrt{f'_c} b_w d \tan \Phi$$

where $k = (d - c_s)/d$; c_s is the depth of compression zone above critical diagonal crack. The value of c_s has been considered here as obtained by Zariras [17] for reinforced concrete beams failing in shear compression mode; which in simplified form can be expressed as:

$$\frac{c_s}{d} = \frac{1 + 0.27(a/d)^2}{2 + 2(a/d)^2}$$

The concrete contribution (V_c), which includes resistance of compressed concrete, aggregate interlock and dowel action of reinforcement can be taken equal to that given by Zsutty [18] with segregation value of 3.0 instead of 2.5 to differentiate short and long beams and is given as under.

$$V_c = 2.2 \left(f'_c \rho \frac{d}{a} \right)^{1/3} b_w d \quad \text{for } a/d \geq 3.0 \text{ and}$$

$$V_c = 6.6 \frac{d}{a} \left(f'_c \rho \frac{d}{a} \right)^{1/3} b_w d \quad \text{for } a/d < 3.0$$

Prediction of Shear Strength of Steel Fibre Based Concrete Beams without Shear Stirrups

Thus the ultimate shear strength of SFBC beams can be expressed as

$$V_u = 2.2 \left(f'_c \rho \frac{d}{a} \right)^{1/3} b_w d + (0.29 k / \gamma) F \sqrt{f'_c} b_w d \tan \Phi \quad (\text{SI units}) \quad a/d \geq 3.0$$

$$V_u = 6.6 \frac{d}{a} \left(f'_c \rho \frac{d}{a} \right)^{1/3} b_w d + (0.29 k / \gamma) F \sqrt{f'_c} b_w d \tan \Phi \quad (\text{SI units}) \quad a/d < 3.0$$

Simplified expressions

From the ease of design considerations, assume the value of constants on conservative side as under
 $\Phi = 45^\circ$, $k = 0.8$, $\gamma = 1.3$ we have

$$v_u = 2.2 \left(f'_c \rho \frac{d}{a} \right)^{1/3} + 0.17 F \sqrt{f'_c} \quad (\text{SI units}) \quad a/d \geq 3.0$$

$$v_u = 6.6 \frac{d}{a} \left(f'_c \rho \frac{d}{a} \right)^{1/3} b_w d + 0.17 F \sqrt{f'_c} \quad (\text{SI units})$$

for $a/d < 3.0$

The results of tested beam elements were compared with those obtained from proposed expressions and those given by previous author (Narayanan et al) as indicated in table 1. Good agreement was found between the experimental and predicted results. For the beams with very low shear span to depth ratio ($a/d \approx 1.0$), the results from proposed expressions preserve the conservativeness for extra margin of safety to brittle failure for such beams as confirmed on comparison with the results of Narayanan and Darwish.

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Notation

a = Shear Span, mm
 b_w = beam width, mm
 c_s = depth of compression zone above diagonal crack, mm
 d = effective depth, mm
 d_f = fibre diameter, mm
 e = arch action factor
 F = fibre factor
 f'_c = cylinder, concrete compressive strength, MPa
 f'_t = split cylinder strength of concrete, MPa
 l_f = length of fibre, mm
 v_e = experimental shear strength, MPa
 v_a = evaluated shear strength, MPa
 v_u = predicted shear strength, MPa
 V_f = volume fraction of fibres, percent
 ρ = percentage of longitudinal tensile reinforcement
 τ = interfacial fibre matrix bond stress, MPa
 γ = factor accounting for relative fibre size in the matrix
 Φ = diagonal cracking angle with vertical, degrees

IV. CONCLUSIONS

Based on the experimental results, the following conclusions have been drawn.

1. the presence of steel fibres transformed the mode of failure of the tested beams into a more ductile one, especially for large values of a/d
2. Shear strength of beams increase with an increase in fibre content and a decrease in a/d ratio. For beams with $\rho \approx 2\%$, increasing the fibre content from 0 to 1.0 % causes an increase in shear strength of 22% and 36% percent for $a/d = 4$ and 1.33, respectively.
3. Beams with very low flexural steel content exhibit flexural failure irrespective of a/d value.
4. Based on experimental results two modified equations are proposed to predict the shear strength of Fibre Based Concrete Beams without stirrups. These equations gave good predictions for the shear strength of the tested beams.

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