

A Multi-Objective Transportation Problem Solve By Lexicographic Goal Programming

Kirti Kumar Jain, Ramakant Bhardwaj, Sanjay Choudhary

ABSTRACT In this research paper, we assume a Multi-Objective Transportation Problem, which minimizes the total transportation transfer time and transportation cost. Here the transportation problem is solved by Lexicographic goal programming method. The solution resultant to the minimum distances gives the best settlement solution. This is a process used to solve a numerical example.

Keywords: Lexicographic goal programming, Multi-objective transportation problem, allocation resources.

I. INTRODUCTION :

Transportation problem is one of the most established issues of linear programming problem. The simple type of transport issue was first created by Hitchcock (1941) with innovative techniques for the system. A linear programming problem may be characterized as the issue of expanding or limiting the direct capacity under direct requirements and it may also be in essential balance or inequality. Transport models consider an important task to reduce the expenditure and increase the administration. Lee and Moore (1973) have examined the multi-target development of transportation issues. Iserman (1979) introduced a calculation for tackling direct multi-target transportation issues by which the arrangement of every single proficient arrangement was counted. Ringuest and Rinks (1987) proposed two methodologies for getting the arrangement of direct multi-target transportation issues. Das et al. (1999) utilized a fluffy programming approach for taking care of multi-target interim transportation issue.

In the transportation issue the item the source should be delivered to the target. For achieving this goal, goal programming (GP) has been linked to many correct issues in many areas and is used, for example,

bookkeeping, building, transport, advertising, financial matters, further. In this research paper a Multi-Objective Transportation Problem solved by Lexicographic goal programming method. which minimizes the total transportation transfer time and transportation cost. By the lingo software solved a numerical problem.

Multi-objective Transportation Problem

In reality circumstances, the transportation issue typically incorporates various, incommensurable and in conflict target capacities. Such issues are called multi-purpose transport problem. Relating to a typical transportation issue is to be transported item to n goals from m sources and their ability are b_1, b_2, \dots, b_n and a_1, a_2, \dots, a_m and separately. In added substance, C_{ij} is penalty connect with transportation a an item to goal j from source i. The penalty might be cost or conveyance time or wellbeing of conveyance or and so forth. A variable x_{ij} speaks to the indefinite amount to be sent from source i to goal j. The Multi-Objective transportation model are following:

$$\text{Minimize } f_k(y) = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^k y_{ij}$$

Subject to

$$\sum_{j=1}^n y_{ij} = a_i \quad , \quad \text{for } i = 1, 2, 3, \dots, m$$

$$\sum_{i=1}^m y_{ij} = b_j \quad , \quad \text{for } j = 1, 2, 3, \dots, n$$

$$\sum_{j=1}^n b_j = \sum_{i=1}^m a_i$$

$$y_{ij} \geq 0 \quad \text{for } i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n$$

Where $F_k(y) = \{ F_1(y), F_2(y), F_3(y), \dots, F_k(y) \}$ is a vector. $k = 1, 2, 3, \dots, K$

The MOTP obtained by (1) for $k = 1, 2, 3, \dots, K$

$$f_1(y) = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^1 y_{ij}$$

for time minimize

$$f_2(y) = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 y_{ij}$$

for time minimize

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Subject to

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$$\sum_{i=1}^m y_{ij} = b_j, \quad \text{for } i = 1, 2, 3, \dots, m$$

$$\sum_{j=1}^n b_j = \sum_{i=1}^m a_i$$

$$y_{ij} \geq 0 \quad i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n$$

LEXICOGRAPHIC GOAL PROGRAMMING

First let total cost more than the total time

$$\text{Minimize } f_2(y) = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 y_{ij} + \mu_1$$

$$\text{Subject to } \sum_{i=1}^m \sum_{j=1}^n c_{ij}^1 y_{ij} - \mu_1 \leq F_1(y)$$

$$\sum_{j=1}^n y_{ij} = a_i, \quad \text{for } j = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^m y_{ij} = b_j, \quad \text{for } i = 1, 2, 3, \dots, m$$

$$\sum_{j=1}^n b_j = \sum_{i=1}^m a_i$$

$$y_{ij} \geq 0 \quad i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n$$

$$\mu_1 \geq 0 \quad \mu_2 \geq 0$$

μ_1 is deviation variable. Solving these equation and find $F_2(y)$

$$\text{Now Minimize} = \mu_1 + \mu_2$$

Subject to

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij}^1 y_{ij} - \mu_1 \leq F_1(y)$$

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 y_{ij} - \mu_2 \leq F_2(x)$$

$$\sum_{j=1}^n y_{ij} = a_i, \quad \text{for } j = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^m y_{ij} = b_j, \quad \text{for } i = 1, 2, 3, \dots, m$$

$$\sum_{j=1}^n b_j = \sum_{i=1}^m a_i$$

$$y_{ij} \geq 0 \quad i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n$$

$$\mu_1 \geq 0 \quad \mu_2 \geq 0$$

find from this problem $y_{11}, y_{12}, \dots, y_{44}$
 next we consider the total transferable time is more than the cost then we solve this problem minimize:

$$f_1(y) = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^1 y_{ij} + \mu_1$$

Subject to

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 y_{ij} - \mu_1 \leq F_2(y)$$

$$\sum_{j=1}^n y_{ij} = a_i, \quad \text{for } j = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^m y_{ij} = b_j, \quad \text{for } i = 1, 2, 3, \dots, m$$

$$\sum_{j=1}^n b_j = \sum_{i=1}^m a_i$$

$$y_{ij} \geq 0 \quad i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n$$

$$\mu_1 \geq 0 \quad \mu_2 \geq 0$$

Now,

Ware house →	A	B	C	D	Capacity
Factories ↓					
X	6	10	9	7	35
Y	5	7	5	6	30
Z	8	10	8	7	25
Requirement	30	26	19	15	90

$$\text{Minimize} = \mu_1 + \mu_2$$

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 y_{ij} - \mu_1 \leq F_2(y)$$

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij}^1 y_{ij} - \mu_2 \leq F_1(y)$$

$$\sum_{j=1}^n y_{ij} = a_i, \quad \text{for } j = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^m y_{ij} = b_j, \quad \text{for } i = 1, 2, 3, \dots, m$$

$$\sum_{j=1}^n b_j = \sum_{i=1}^m a_i$$

$$y_{ij} \geq 0 \quad i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n$$

$$\mu_1 \geq 0, \quad \mu_2 \geq 0$$

Solve the above all equation we find a set of solutions. Out of these solutions, we obtain an Ideal solution which gives a minimum distance. This can be the closest to the ideal solution.

Numerical Examples:

For a numerical problem we have cement company and the company have four warehouse A ,B ,C , D , and three factories X, Y, Z .The monthly capacity of factories are 35, 30, and 25 mini loading van and monthly requirement of warehouse from each factories are 30, 26, 19, 15, . The shipping cost are

$$f_1(y) = 18y_{11} + 22y_{12} + 19y_{13} + 16y_{14} + 16y_{21} + 14y_{22} + 11y_{23} + 12y_{24} + 24y_{31} + 22y_{32} + 18y_{33} + 15y_{34}$$

$$f_2(y) = 6y_{11} + 10y_{12} + 9y_{13} + 7y_{14} + 5y_{21} + 7y_{22} + 5y_{23} + 6y_{24} + 8y_{31} + 10y_{32} + 8y_{33} + 7y_{34}$$

Subject to

$$y_{ij} \geq 0$$

$$y_{11} + y_{12} + y_{13} + y_{14} \leq 35$$

$$y_{23} + y_{22} + y_{23} + y_{24} \leq 30$$

$$y_{31} + y_{32} + y_{33} + y_{34} \leq 25$$

$$y_{11} + y_{21} + y_{31} \geq 30$$

$$y_{12} + y_{22} + y_{32} \geq 26$$

$$y_{13} + y_{23} + y_{33} \geq 19$$

$$y_{14} + y_{24} + y_{34} \geq 15$$

$$18y_{11} + 22y_{12} + 19y_{13} + 16y_{14} + 16y_{21} + 14y_{22} + 11y_{23} + 12y_{24} + 24y_{31} + 22y_{32} + 18y_{33} + 15y_{34} - \mu_1 \leq 1448$$

$$6y_{11} + 10y_{12} + 9y_{13} + 7y_{14} + 5y_{21} + 7y_{22} + 5y_{23} + 6y_{24} + 8y_{31} + 10y_{32} + 8y_{33} + 7y_{34} - \mu_2 \leq 607$$

$$y_{11} + y_{12} + y_{13} + y_{14} \leq 35$$

$$y_{21} + y_{22} + y_{23} + y_{24} \leq 30$$

$$y_{31} + y_{32} + y_{33} + y_{34} \leq 25$$

$$y_{11} + y_{21} + y_{31} \geq 30$$

$$y_{12} + y_{22} + y_{32} \geq 26$$

$$y_{13} + y_{23} + y_{33} \geq 19$$

$$f_2(y) = 6y_{11} + 10y_{12} + 9y_{13} + 7y_{14} + 5y_{21} + 7y_{22} + 5y_{23} + 6y_{24} + 8y_{31} + 10y_{32} + 8y_{33} + 7y_{34} + \mu_2$$

The shipping time
Minimize:

Ware house→ Factories ↓	A	B	C	D	capacity
X	18	22	19	16	35
Y	16	14	11	12	30
Z	24	22	18	15	25
Requirement	30	26	19	15	90

Solving by Lexicographic goal programming using LINGO SOFTWARE and find the solutions are

$$y_{11} = 30, \quad y_{14} = 4, \quad y_{22} = 26, \quad y_{23} = 4, \\ y_{33} = 15, \quad y_{34} = 10,$$

$$\text{SO } f_1(y) = 1448, \quad f_2(y) = 607,$$

Now solve the MOTP by goal programming approach

$$\text{Minimize} = \mu_1 + \mu_2$$

Subject to

$$y_{14} + y_{24} + y_{34} \geq 15$$

$$y_{ij} \geq 0$$

solve by LINGO SOFTWARE and find

$$y_{11} = 30, \quad y_{12} = 5, \quad y_{22} = 21, \quad y_{23} = 9, \\ y_{33} = 10, \quad y_{34} = 15,$$

In a same way again solve for $f_2(y)$ and find the solution

Minimize

Subject to



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$$18y_{11} + 22y_{12} + 19y_{13} + 16y_{14} + 16y_{21} + 14y_{22} + 11y_{23} + 12y_{24} + 24y_{31} + 22y_{32} + 18y_{33} + 15y_{34} - \mu_1 \leq 1448$$

$$y_{11} + y_{12} + y_{13} + y_{14} \leq 35$$

$$y_{21} + y_{22} + y_{23} + y_{24} \leq 30$$

$$y_{31} + y_{32} + y_{33} + y_{34} \leq 25$$

$$y_{11} + y_{21} + y_{31} \geq 30$$

$$y_{12} + y_{22} + y_{32} \geq 26$$

$$y_{13} + y_{23} + y_{33} \geq 19$$

$$y_{14} + y_{24} + y_{34} \geq 15$$

$$y_{ij} \geq 0$$

solve by LINGO SOFTWARE and find

$$y_{11} = 30, \quad y_{12} = 5, \quad y_{22} = 11, \quad y_{23} = 19,$$

$$y_{33} = 10, \quad y_{34} = 15$$

and obtained $f_1(y) = 1458$, and $f_2(y) = 607$

Minimize = $\mu_1 + \mu_2$

Subject to

$$18y_{11} + 22y_{12} + 19y_{13} + 16y_{14} + 16y_{21} + 14y_{22} + 11y_{23} + 12y_{24} + 24y_{31} + 22y_{32} + 18y_{33} + 15y_{34} - \mu_1 \leq 1458$$

$$6y_{11} + 10y_{12} + 9y_{13} + 7y_{14} + 5y_{21} + 7y_{22} + 5y_{23} + 6y_{24} + 8y_{31} + 10y_{32} + 8y_{33} + 7y_{34} - \mu_2 \leq 607$$

$$y_{11} + y_{12} + y_{13} + y_{14} \leq 35$$

$$y_{21} + y_{22} + y_{23} + y_{24} \leq 30$$

$$y_{31} + y_{32} + y_{33} + y_{34} \leq 25$$

$$y_{11} + y_{21} + y_{31} \geq 30$$

$$y_{12} + y_{22} + y_{32} \geq 26$$

$$y_{13} + y_{23} + y_{33} \geq 19$$

$$y_{14} + y_{24} + y_{34} \geq 15$$

$$y_{ij} \geq 0$$

solve by LINGO SOFTWARE and obtained the value

$$y_{11} = 30, \quad y_{12} = 5, \quad y_{22} = 11, \quad y_{23} = 19,$$

$$y_{33} = 10, \quad y_{34} = 15,$$

which satisfies $f_1(y)$ and $f_2(y)$

The solution table :

Priorities	$f_1(y)$, $f_2(y)$	$f_1(y)$, $f_2(y)$	Perfect result D	Transfer cost	Transfer time
y ₁₁	30	30	30	0	0
y ₁₂	5	5	5	0	0
y ₁₃	0	0	0	0	0
y ₁₄	0	0	0	0	0
y ₂₁	0	0	0	0	0
y ₂₂	21	11	11	10	0
y ₂₃	9	19	9	0	10
y ₂₄	0	0	0	0	0
y ₃₁	0	0	0	0	0
y ₃₂	0	10	0	0	10
y ₃₃	10	0	0	10	0
y ₃₄	15	15	15	0	0
				20	20

II. RESULT:

In above table we obtained distance calculate solution from all ideal solution. It is clear that the minimum distance and minimum time of two priorities are equal and 20. We choose any one of them.

III. CONCLUSION:

The Lexicographic goal programming is a great method to find the minimum distance for solving MOTP, the problem allocation in transportation is considered as a multi-objective problem. Using this procedure we can find the solution of others transportation problem. This is very simple way to find out the solution of other transport problem.

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