The Performance of Control Charts in the Presence of Assignable Causes

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Abstract*: *The main objective of control charts is to evaluate the quality of a production process. A process is stable or in-control when the variability of the production process is only produced by common causes. However, it is said that the process is out-of-control if its variability is produced by assignable causes. An advantage of control charts is the detection and identification of assignable causes within the production process. The aim of this article is to analyse the performance of control charts under simulated (therefore known) changing conditions. Monte Carlo simulation studies are carried out to analyze the empirical performance of control charts under different scenarios. In particular, we first consider that the processes have a correct operation. Second, we consider processes that operate with quality characteristics that do not satisfy the required assumptions, and this issue may have an impact on the proportion of non-conforming articles. Third, we consider processes that suffer from a change on the performance of the production volume. The proposed studies allow to estimate an impact of the analyzed scenarios on the performance of control charts.

Index Terms*: *Control limits, simulation study, statistical process control, variance.

I. INTRODUCTION

Control charts are one of the main techniques of statistical process control, and they are used by many companies due to the fact that they may provide relevant benefits [1], [2]. The main objective of control charts is to control the parameters associated with the quality characteristic of arcticles or items under inspection. In general, companies are interested in controlling the variability of the process. In line with this issue, control charts monitor the production process to detect causes of assignable variation, which allows to apply corrective actions as soon as possible, thus minimizing the losses generated by the manufacture of non-conforming items. Therefore, the control charts allow to identify the status of the process. A control chart is a graphic representation that includes a chronological series of the statistic corresponding to the process parameter to be controlled, a central line and the control limits. The chronological series of the statistic is determined from random samples of articles that have been selected in a step prior to the elaboration of the control chart.

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The value of the statistic for each sample will be represented by a point, which will be joined with the points of the following samples by straight lines between each two consecutive points. The central line (CL) of the control chart is a horizontal line that corresponds to the expected value of the estimator of the population parameter under study. The upper limit of control (ULC) and the lower control limit (LCL) are two horizontal lines at a predetermined distance from the central line that delimit the values that the statistic can present when the process is under control [3]-[7]. The process is in-control if: (i) all the points represented in the control chart are located in the area delimited by the control limits, and (ii) the points are distributed randomly, without ascending or descending streaks of points. When the process is in this situation, the process is stable and predictable, that is, the variability within the process is due exclusively to fortuitous causes, and the uniformity between articles is maximum. Besterfield [3] lists some of the benefits that arise when the process is in this state. For example, the possibility of minimizing the cost per inspection, since fewer samples are needed to evaluate the quality. Conversely, if any statistic value is above the UCL, or below the LCL, the process will be in an out-of-control state. In addition, although all the statistic's values are located in the area delimited by both limits, if these are not distributed randomly, the process is also out-of-control. Some useful rules can be applied to determine if the process is out-of-control (see, for example, [3], [6], [8]). Some very common rules are: (i) the presence of ascending or descending streaks of points; (ii) four of five consecutive points located at a distance greater than the standard deviation of the quality characteristic of the CL; (iii) a series of eight consecutive points on one side of the CL; (iv) fourteen points alternating up and down or an unusual pattern in the data; etc. The main objective of this work is to analyze the empirical impact of the presence of asignable causes on the performance of different control charts. For this purpose, Monte Carlo simulation studies will be used. The contribution of this paper consists on using different scenarios, which correspond to situations that may arise in practice. Some examples of situations that we discuss in this paper are described as follows:

a) We suppose a company that operates with a process in which a certain machine is composed of a series of gears. A particular gear contains a tooth of unsuitable dimension, resulting from excessive wear. Some articles are influenced by the defective tooth of this machine. We assume that this fact directly influences the quality characteristic of the product, resulting in a higher value than is usual.

In the control chart we expect the higher value of statistic corresponding to samples that include articles produced with the intervention of the defective tooth. This point could be located above UCL or will represent a pattern of behaviour in the control chart. Both cases can indicate that the process is not under control and can be qualified as a state out of control. That is, a part of the variability of the process is produced by an assignable cause: the tooth of unsuitable dimensions.

b) A company involves a machine that, after producing a number of items, overheats. While this machine is overheated, the process produces articles with a certain alteration in their quality characteristic, such as a slightly longer length. The control chart corresponding to this process can indicate a pattern or a streak on more or less long series of points located at a certain distance from the central line of the control chart in the chronological series of the sample statistic. Points outside the range the control limits are considered as out-of-control and the assignable cause is the defective machine. This paper is organized as follows. In section 2, we describe the simulation studies carried out to analyze the performance of control charts in the presence of assignable causes. Production processes with different characteristics have been considered. Note that the production processes may suffer assignable causes, and this issue may have an impact on the quality of the articles. Control chart is an appropriate tool to detect the presence of such assignable causes, and which may arise in practice. In addition, note that simulation studies have been carried out using the R software [9], and the R codes will be available from the authors upon request. In section 3, the most relevant results are presented and discussed. The main conclusions derived from the various simulation studies are summarized in section 4.

II. METHODOLOGY

The analysis of the performance of control charts in the presence of asignable causes is carried out using Monte Carlo simulation studies, which consists in drawing the corresponding control charts for each run of the simulation study. The total number of results are merged to provide a single final result. For each situation in this study, we simulate that the production is stored in lots, from which a sample of items is extracted. In each item, the quality characteristic is measured, and the statistic of the sample is calculated. Afterwards, the control chart is elaborated. We assume that the parameters associated to the process are known. In the case of unknown parameters, they should be estimated using the information contained in the samples of articles. It is important to select the samples when it is certain that the process is in an in-control state [10]-[12]. In order to analyze the importance of this assumption, we consider the case of control charts with control limits based on estimated parameters, but the corresponding processes are in a state of out-of-control. Some references that discuss the construction of control charts with known and unknown parameters are [13]-[15]. Let $x \to N(\mu,\sigma)$ be a quality characteristic associated with a process, where μ is the true process mean and σ is true process standard deviation. We assume m random samples with size n, where x_{ij} denotes the observed value of the quality characteristic for the j th article, with $j=1,\dots,n$, in the i th sample, with $i=1,\dots,m$. The process can be monitored by plotting the sample means and the sample standard deviations, and which are defined as follows:

$$\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij},$$

$$s_i = \left(\frac{1}{n-1}\sum_{j=1}^{n}\left(x_{ij}-\bar{x}_i\right)^2\right)^{1/2}, \qquad i=1,\dots,m.$$

In the case of known parameters, the control limits for the $\bar{x}$ control chart are defined as:

$$LCL_\mu = \mu - 3\frac{\sigma}{\sqrt{n}};$$

$$CL_\mu = \mu;$$

$$UCL_\mu = \mu + 3\frac{\sigma}{\sqrt{n}}$$

On the other hand, the control limits for the s control chart are defined as:

$$LCL_\sigma = c_4\sigma - 3\sigma\sqrt{1-c_4^2};$$

$$CL_\sigma = c_4\sigma;$$

$$UCL_\sigma = c_4\sigma + 3\sigma\sqrt{1-c_4^2}$$

where c_4 is a function that depends on the sample size [12].

If we consider the case of unknown parameters, the control limits for the $\bar{x}$ and s control charts must be estimated, and they are defined as:

$$LCL_\mu = \bar{\bar{x}} - 3\frac{\bar{s}}{\sqrt{n}};$$

$$CL_\mu = \bar{\bar{x}};$$

$$UCL_\mu = \bar{\bar{x}} + 3\frac{\bar{s}}{\sqrt{n}}$$

and

$$LCL_\sigma = \bar{s} - 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2};$$

$$CL_\sigma = \bar{s};$$

$$UCL_\sigma = \bar{s} + 3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2}$$

respectively, where $\bar{\bar{x}} = m^{-1}\sum_{i=1}^{m}\bar{x}_i$ is the grand sample mean, and $\bar{s} = m^{-1}\sum_{i=1}^{m}s_i$ is the average of the m standard deviations.

Three different production processes have been considered in this study. The first scenario corresponds to a process that operates correctly. In the second scenario, the process operates with a certain alteration in the quality characteristic, and which only affects a proportion $p*$ of the articles produced. In this second scenario, we will assume that each lot contains that proportion $p*$ of items. Finally, the third scenario corresponds to a process that, from a certain volume of production, changes its performance. Since storage of lots is chronological, we assume that only a certain number of lots contain items produced by the process when the incidence is present. In addition, we will assume that the proportion of lots with item with this type of incidences is also given $p*$. For example, if we assume $p* = 0.2$ and lots of size 100, in the second scenario each lot stores 20 items with incidence. Under the third scenario and in a cyclical way, 20% of the lots will contain items with incidents, i.e., the production will be stored in four consecutive lots with articles without incidence, while the next lots will contain articles with incidence. Note that the case studies presented in the previous section can be identified with the second and third scenarios, respectively. Therefore, two different types of production are presented. On the one hand, the production corresponding to a process when it operates without any incidence and, on the other hand, the production result of a process when it is affected by some incidence. We assume that the variable that represents the quality characteristic of the articles produced by a process without incidences has a normal distribution with mean μ and standard deviation σ. Consequently, the quality characteristic corresponding to production with incidences follows a normal distribution with mean $\mu + \varepsilon_{\mu}$ and standard deviation $\sigma + \varepsilon_{\sigma}$. In the simulation carried out in this work, control charts have been drawn up in which 50 samples of size 5 are represented and the control limits are defined using the USA method [6]. To simulate the three previously defined scenarios, lots of size 100 and an incidences proportion of 20% $(p* = 0.2)$ for the second and third scenarios have been assumed. We consider $\mu = 10$ and $\sigma = 1$ for the parameters associated with the production without incidents. In addition, in order to represent the changes in production discussed, the values $\varepsilon_{\mu} = \{0,1,2,3\}$ and $\varepsilon_{\sigma} = \{0,0.5,1,2\}$ and all its possible combinations are considered. Note that the objective of this work is to evaluate the impact of assignable causes in the process with different effects, both in intensity and in form.

III. CASE STUDY

The most relevant results derived from the simulation studies are shown in Fig. 1-3. Each Fig. contains the control charts corresponding to the three scenarios. Note that the first scenario is represented in all the figures as a that the first scenario is represented in all the figures as a reference, since it is characterized as a process under control that does not suffer Variation.

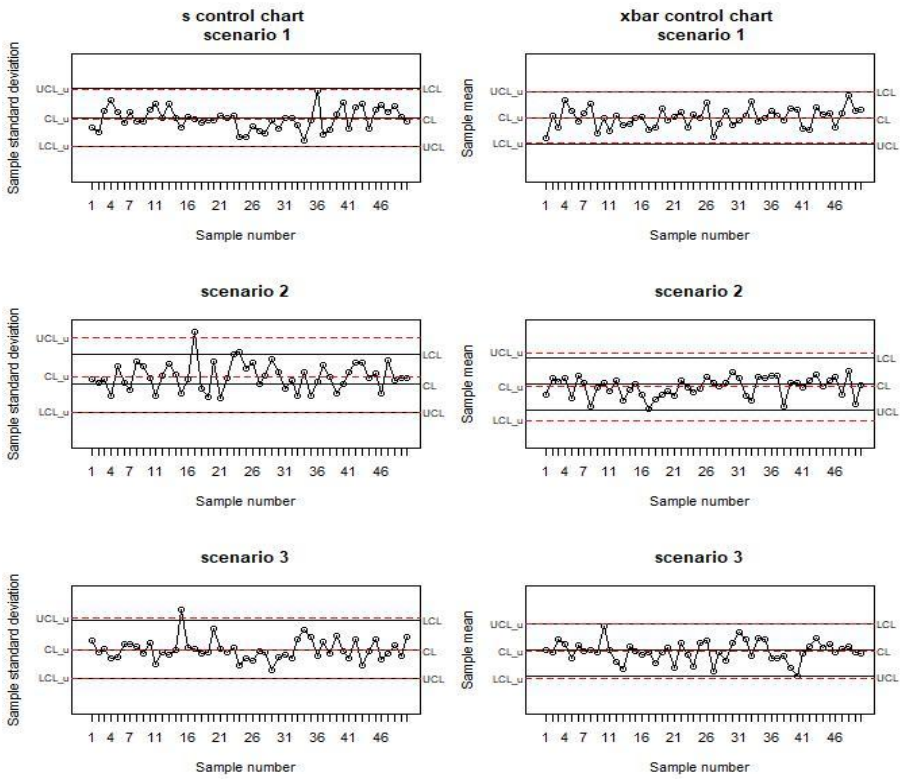

Figure 1 $\bar{x}$ and s control charts of a process. Samples are selected from a Normal distribution $\mu = 10$ and $\sigma = 1$. The production with incidence of 0.2 and its quality characteristic is affected $\varepsilon_{\mu} = 0$ and $\varepsilon_{\sigma} = 0.5$

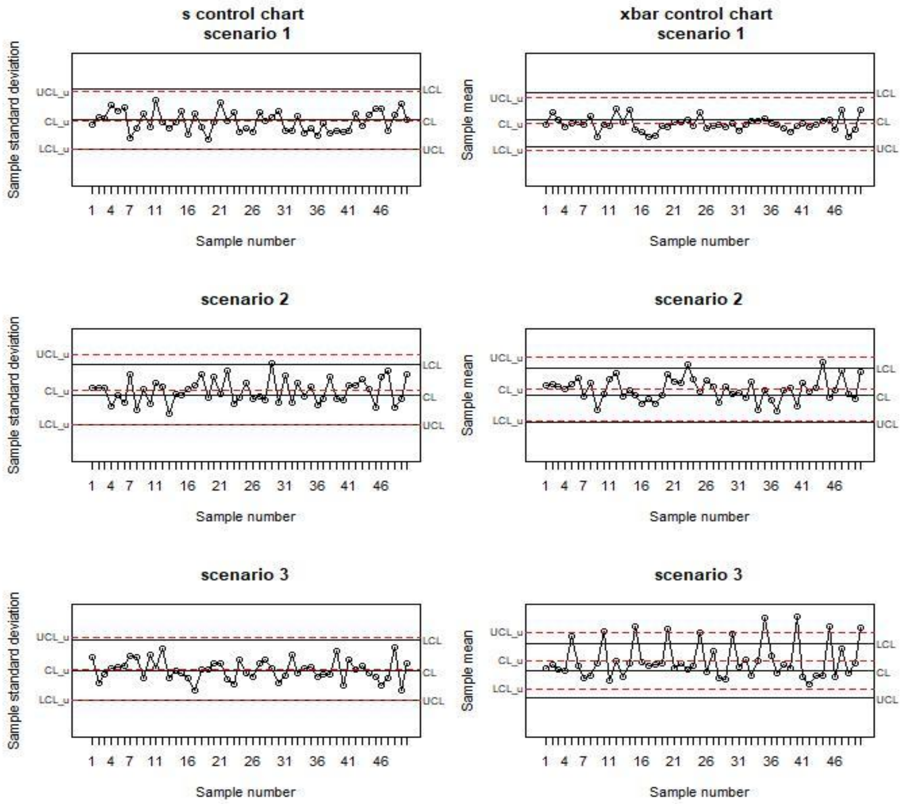

Figure 2 $\overline{x}$ **and** s **control chartsof a process. Samples are selected from a Normal distribution** $\mu = 10$ **and** $\sigma = 1$**. The production with incidence of 0.2 and its quality characteristic is affected by** $\varepsilon_{\mu} = 2$ **and** $\varepsilon_{\sigma} = 0$**.**

Fig. 1 contains the $\overline{x}$ and s control charts when the incidence in the process only affects the standard deviation of the product quality characteristic. In this case, we can see in the s control chart sufficient indications to think that, in the second as in the third scenario, the process is out-of-control. In the second scenario, the sample standard deviation of the samples number 17, 23 and 24 are outside the range delimited by the control limits. Something similar is observed in the sample number 15 corresponding to the third scenario. On the other hand, the $\overline{x}$ control charts show for both scenarios several points close to the control limits, which may be an indication that the process is out-of-control.

Fig. 2 contains the $\overline{x}$ and s control charts when the incidence in the process has influence only on the average of the quality characteristic, i.e., $\varepsilon_{\mu} = 2$ and $\varepsilon_{\sigma} = 0$. We observed that, in the third scenario, the total of the sample means corresponding to the production with incidence in the process are above UCL, while the standard deviation seems to be under control. The process under the second scenario is also in an out-of-control situation. The standard deviation of sample number 29 is outside the range defined by the control limits. In addition, two points corresponding to samples 23 and 44 are outside the control limits of the control chart for the mean and we observed in samples 47 and 50 another situation to identify the process as out-of-control, one or more points to a warning or control limit.

The $\overline{x}$ and s control charts assuming values $\varepsilon_{\mu} = 2$ and $\varepsilon_{\sigma} = 0.5$ are shown in Fig. 3. In this case the incidence in the process affects both the mean and the standard deviation of the product quality characteristic. The control charts contain indications that the second and third scenario processes are out-of-control. Among these, highlights the existence of points outside the control limits of the $\overline{x}$ and s control chart in both processes.

Additionally, the estimated control limits have been represented in each control chart. The information contained in the samples represented in the control chart has been used to carry out the estimation of the control limits. Therefore, the control limits in the second and third scenarios have been estimated when the process is out-of-control situation. We observed that, in general, the estimated control limits are closer to the control limits when the parameters are known and when the process is in a control situation. In this situation, the conclusions obtained from the control charts are similar. Otherwise, the estimated control limits may be

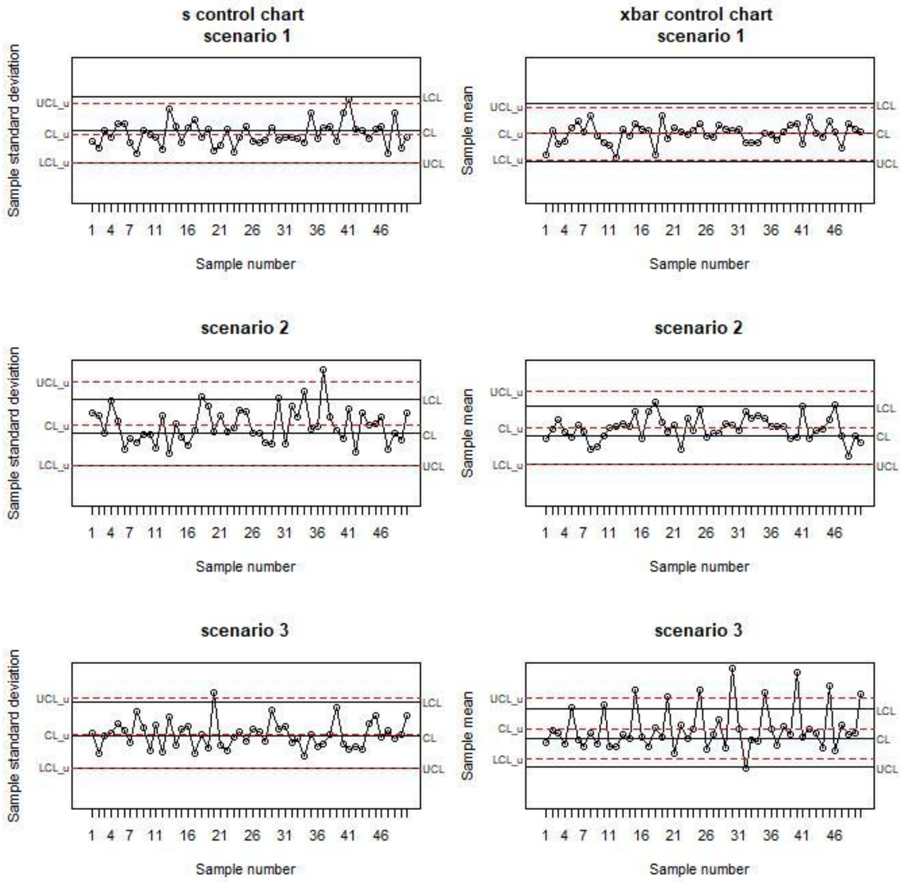

Figure 3 $\overline{x}$ and s control chartsof a process. Samples are selected from a Normal distribution $\mu = 10$ and $\sigma = 1$. The production with incidence of 0.2 and its quality characteristic is affected by $\varepsilon_{\mu} = 2$ and $\varepsilon_{\sigma} = 0.5$.

located far away from the control limits and the conclusions obtained in the control charts may be incorrect. For example, an assignable cause is present in the process of the second scenario that affects a proportion of 0.2. We observe in Fig. 2 the control charts when this assignable cause affects only the average of the process. Considering that the parameters are known and therefore the control limits, we observe several points outside the control limits in both graphs. However, if we consider the unknown parameters and the estimated control limits, all points of both graphs are within the control limits and then control charts give misleading information.

IV. CONCLUSION

In this paper we have analyzed the performance of control charts in the presence of assignable causes. To achieve this objective, a set of simulation studies were carried out, and the control charts corresponding to different production processes were elaborated. The analyzed control charts suffered from the problem of assignable causes, and this issue may have an impact on the production. Different consequences have been observed, and they are related to the frequency and the intensity of problems in the control charts. In each scenario, the $\overline{x}$ and s control charts have been developed assuming the case of known parameters. In addition, control limits were estimated under an assumption of unknown parameters, and the objective of this scenario is to analyze the impact on the control chart when the process is out-of-control situation.

According to the obtained results, we observe that control charts may be an highly effective tool in the detection of assignable causes in a given production process, since they have been able to identify indications that the process is out-of-control independently of the characteristics of the consequences produced for the assignable cause. This is really useful, since it allows to apply corrective actions as soon as possible, thus minimizing the losses generated by the manufacture of non-conforming articles. In addition, control charts report numerous advantages. For example, improve the productivity of the company, reduce the costs derived from an over-intervention in the process or provide information on the capacity of the process.

On the other hand, we have empirically analyzed the importance of the estimation of control limits when the the process is in-control. Results derived from this study indicate that the estimation of the parameters associated with samples from a process out-of-control could considerably provide different control limits, i.e., estimated control limits can differ from the real control limits. However, results also indicate that estimated control limits and based on a process in-control will be close to the real control limits. In conclusion, if the control limits are estimated using (small) samples from out-of-control processes, the control charts may lose effectiveness in the detection of variability due to assignable causes in the process, which could lead to erroneous conclusions or confusion when drawing conclusions.

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