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Labeling of Splitting Graph of a Bipartite Graph

G. Sathiamoorthy, K. R. Sekar

Abstract: ***The graph consisting Union of two vertex set with two vertices are adjacent if they correspond to adjacent edges of G or one edge is in one subset and the other is in another subset. This results split of a graph. Splitting of graph creates new variations and varieties in the original graph structure. In this paper, it is proved that a splitting graph of a complete bipartite graph satisfies graceful and alpha labeling.***

Index Terms: Alpha labeling, complete bipartite graph, Graceful labeling, splitting graph.MSC 2010: 05C78

I. INTRODUCTION

An α-labeling of a graph G is a graceful labeling of G with the additional condition that ∃ a number η ($0 \le \eta \le E(G)$) such that, for any edge e ∈ E(G) with terminal vertices u, v ∈ V(G), min [χ(u), χ(v)] ≤ η< max [χ(u), χ(v)]. η must be the smaller of the two vertex labels that yield the edge labeled 1. In Gallian's survey[1] it is denoted as S'(G) and in this paper, we follow this same notation. Split graph and a splitting graph of the bipartite graph was discussed by various authors [3], [4], [5], [7], [8] and application in various areas[6], [9].

In this paper, graphs are taken as simple, finite and undirected. A graph G with vertex set V(G) and edge set E(G) such that cardinality of vertex set is p and cardinality of edge set is q. For graceful labeling, definitions refer to the standard Gallian's survey article. The complete bipartite graph $K_{m,n}$ is the graph with p = m + n vertices and q = m n edges, obtained by connecting each of the m vertices in one set with each of the n vertices in another set in all possible ways.Splitting graph S(G) was introduced by Sampath Kumar and Walikar[2]. The splitting graph definition is given below.

The splitting graph Sp(G) of G is the graph with the vertex set V(G) U V'(G) and edge set{uv, u'v, uv': uv} in E(G).

II. MAIN RESULT

Theorem

Splitting graph of complete bipartite graph $K_{m,n}$ for all m, n satisfies graceful and α-labeling.

Proof:

A complete bipartite graph $K_{m,n}$ in which u_a, a = 1, 2, ..., m refers to the number of vertices in subset A and v_h, h = 1, 2, ..., n refers to the number of vertices in subset B, such that V(G) = A U B and A ∩ B = ϕ. Any vertex in set A is connected to all vertices in set B by an edge, which is shown in the following figure.

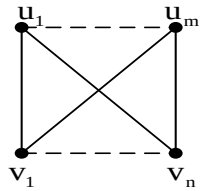

Figure 1: complete bipartite graph

Introducing splitting graph of the complete bipartite graph are as follows:-

The splitting graph S'(G) consists of 2p vertices and 3q edges, which is shown in the following figure.

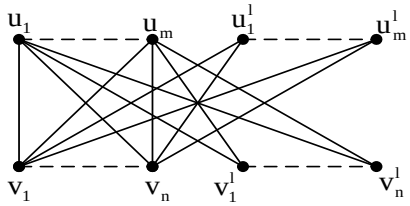

Figure 2: splitting graph of a complete bipartite graph

Consider the splitting graph of the complete bipartite graph with m, n vertices, in which the original segment graph consists of u_a, a = 1, 2, ..., m and v_h, h = 1, 2, ..., n vertices respectively. The split graph segment consists of u'_a, a = 1, 2, ..., m and v'_h, h = 1, 2, ..., n vertices respectively. The split graph of the bipartite graph has 3mn edges in total(q). The labeling of vertices are as follows:-

The first set of vertices are labeled as follows:-

$\varphi(u_a') = a - 1$, a = 1, 2, ..., m.

First set of splitted vertices are labeled as follows:-

$\varphi(u_a) = m + (a - 1)$, a = 1, 2, ..., m.

Second set of vertices are labeled as follows:-

$\varphi(v_h) = q - 2m(h - 1)$, h = 1, 2, ..., n.

Second set of splitted vertices are labeled as follows:-

$\varphi(v_h') = m(2 + n - h)$, h = 1, 2, ..., n.

Now, the resultant edge values are given below.

$g(v_h u'_a) = q - (a - 1) - 2m(h - 1)$, a = 1, 2, ..., m, h = 1, 2, ..., n.

$g(v_h u_a) = q - (a - 1) - m(2h - 1)$, a = 1, 2, ..., m, h = 1, 2, ..., n.

$g(v'_h u_a) = q - (a - 1) - m(2n + h - 1)$, a = 1, 2, ..., m, h = 1, 2, ..., n.

To prove no two vertices receives the same label value, the following procedure is adopted.

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* Correspondence Author
G. Sathiamoorthy*, Department of Mathematics, SASTRA eemed University, India.
K. R. Sekar, School of Computing, SASTRA Deemed University, India..

To prove φ is a bijection, consider any two vertices $e_1 = v_h u_a$ and $e_2 = v_a u_h$.

Let $\varphi(e_1) = |\varphi(u_a) - \varphi(v_h)|$ and $\varphi(e_2) = |\varphi(u_h) - \varphi(v_a)|$.

Now, $|\varphi(e_1) - \varphi(e_2)| = |(|\varphi(u_a) - \varphi(v_h)|) - (|\varphi(u_h) - \varphi(v_a)|)| \geq (\varphi(u_a) - \varphi(v_h) - \varphi(u_h) + \varphi(v_a))$, since $|a - b| \geq |a| - |b|$ and $|a| = a$ if $a \geq 0$.

let $\varphi(v_h) = \varphi(v_a) - 2m$, $\varphi(u_h) = \varphi(u_a) + 1$ for $h = a + 1 \geq (2m - 1)$.

that is, $e_1 \neq e_2$ if and only if $\varphi(e_1) \neq \varphi(e_2)$. Similarly the other cases can be proved. Thus φ is a bijection and hence a graceful labeling.

To obtain α-labeling, η must be the smaller of the two vertex labels that yield the edge labeled 1 which results in $g(u_n v'_n)$ for m, n. End proof.

Example:

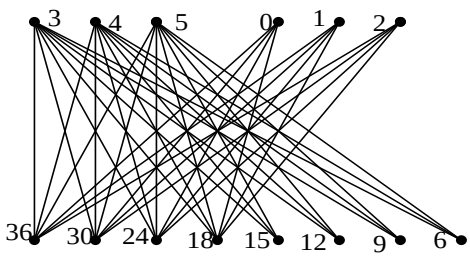

Figure 3: splitting graph of K3,4 and η = 5

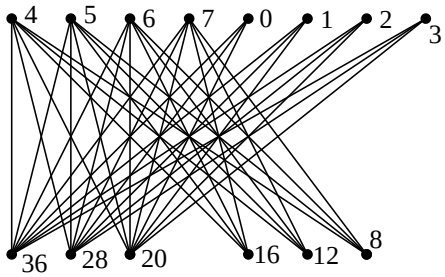

Figure 4: splitting graph of K4,3 and η = 7

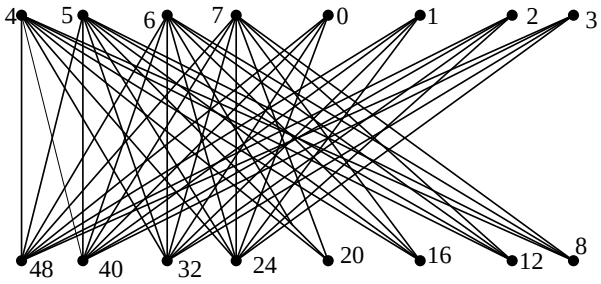

Figure 5: splitting graph of $K_{4,4}$$ and η = 7

III. RESULT AND DISCUSSIONS

Splitting number of complete bipartite graphs makes interesting visualize new set graphs by introducing split. From the above complete bipartite graph varieties, the general form of complete bipartite graph $K_{m,n}$ can be split, results in a combination of any number vertices in two sets. It is possible to create new varieties complete bipartite graphs by the split, which has lot applications in computer science like bipartite graph partitioning, architecture and data clustering, Clustering bipartite graphs in terms of approximate formal concepts and sub-contexts etc.

IV. CONCLUSION

A variety of labeling definitions applied over a Splitting graph of bipartite graph results new set labeling results, which yields a new window to solve application related problems in architecture and data clustering. Hence, we conclude that splitting graph of complete bipartite graph satisfies graceful and α-labeling.

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AUTHORS PROFILE

Dr.G.Sathiyamoorthy, working as an Assistant Professor, currently working in Graph theory and Labeling. His research work is always commendable to the core. He published plenty of papers in SCOPUS and SCI/SCIE indexed journals. He has got 15+ years of experiment in the field of mathematics. Nowadays guiding plenty of students in his arenas.

Dr.K. R.Sekar., currently working as an Assistant professor in a SASTRA Deemed University. He has got an affluent knowledge in teaching for the past 18 years and his research work includes Software Component, Big data analytics, Machine Learning, IoT, Block Chain, Clinical applications , and Soft computing Techniques