Haar Critical Path Method To Solve Fuzzy Critical Path Problems

S. Dhanasekar, S. Hariharan, A. Manivannan, And E. Umamaheswari

Abstract: The critical path method (CPM) is an important tool for the planning and control of the large projects and projects that are complex in nature. Clear determination of time duration will help to implement the CPM successfully. But in real life, the time duration cannot be determined crisply. Hence there is always an uncertainty about the time duration of activities which leads to the development of fuzzy critical path method. In this article, a wavelet based ranking namely, Haar ranking for fuzzy numbers is applied to order the fuzzy numbers. In this algorithm first the fuzzy parameters are converted in to Haar tuples using Haar wavelet technique and critical path method is applied to get the solution. The proposed method is easy to understand and implement, as it follows the standard steps of the regular critical path method. A Practical example is given and the solution strongly suggests that the proposed method gives us the fuzzy critical path and also identifies the critical activities and red flag activities.

Keywords: fuzzy critical path, fuzzy floats, fuzzy ranking, Haar ranking

I. INTRODUCTION

Network problems are often used in solving problems in Engineering and Management Science. The main objective of the network problem is to find the total project duration and classify the activities of the project as critical and non-critical. An activity is said to be critical if there is no leeway in determining its starting and finishing time. A non-critical activity allows some scheduling slack so that the start time of the activity can be advanced or delayed within the limits without affecting the completion time of the entire period. The longest path of the network is called the critical path. The purpose of the CPM is to identify the critical activities on the critical path so that the resources may be concentrated on those activities in order to reduce the time length of the project. Critical Path Method is developed by Morgan R. Walker and James E. Kelley Jr.[12]. If the time or cost of the activity is uncertain, the modelling of the problem needs to be done using fuzzy numbers. The network problems involving fuzzy numbers will be called as fuzzy network problems and the corresponding critical path will be called as fuzzy critical path. Hapake [10] used fuzzy arithmetic operations to compute the earliest starting time of each activity. In Dubois et al [7], applied extension of the fuzzy arithmetic operations to compute the latest starting time of each activity in a project network. Nasution [15] used interactive fuzzy subtraction to compute fuzzy critical path. Chanas and Zielinski [1] proposed a fuzzy CPM based on the extension principle [26, 23]. In their work, the classical criticality notion is treated as a function of activity duration time in the network. Then Slyeptsov and Tyshchuk [18] presented an efficient method to compute the fuzzy latest starting and finishing time. Zielinski [27] developed a polynomial algorithm for deter- mining the intervals of the Latest Starting Time in networks. The concept of fuzziness in network problems are studied by Dubois [7], Kutcha [13], Mon et al [14] and Rommelfanger [16]. Ravishankar. et al [17] used a new de-fuzzification formula to find the critical path. Elizebeth. et al [8] introduced new ranking method for finding the fuzzy critical path. Although the critical path distance can be obtained by using these algorithms, a critical path corresponding to the distance may not necessarily exists. If the net- work is bigger and complex then it is tedious to get critical path by the method of comparing all the possible paths. The main problem is the comparison between fuzzy numbers which corresponds to the fuzzy path. It should be taken into account that most of the existing ranking methods gives different results for different α-cut values. The results are not comparable with the previous methods. The choice of ranking technique is important since the choice of the path is depends upon it. Yuan [24], Wang et al [21] and Chien et al [5] proposed the properties of the ranking of fuzzy numbers to be satisfied. Wavelet analysis is similar to Fourier analysis in that it allows a target function over an interval to be represented in terms of an orthonormal basis. Haar wavelet is a sequence of re scaled"square-shaped" functions which together form a wavelet family or basis. Alfred Haar [9] introduced the Haar sequence in order to give an example of an orthonormal system for the space of square- integrable functions in the unit interval (0, 1) in the year 1909. The Haar wavelet is the simplest possible wavelet. The main drawback of Haar wavelet is it is not continuous and therefore not differentiable. This property may be an advantage for the analysis of discrete signals. For a detailed introduction to wavelet theory, refer to Strang [20], and Hernandez and Weiss [11]. In this paper, the ranking technique based on Haar wavelet is used [6], which satisfies the properties of linearity and additivity. The advantage of using this ranking technique is that, it converts a given fuzzy number into average and detailed coefficients using down sampling. The uniqueness of the Haar ranking method is that, the fuzzification from the de-fuzzied value is very easy to obtain through up sampling. It not only gives the Haar tuple to make use of the crisp value to decide the critical path but also gives back the original fuzzy number using de-fuzzification to calculate the fuzzy distance of the critical path. In this paper Haar ranking is utilized to convert the given fuzzy number into a Haar tuple and applied the critical path in order to identify the fuzzy critical path of a given network. The fuzzification from the de-fuzzied value is very easy to compute using Haar ranking method and is unique. The proposed algorithm ensures the conditions for critical activities. i.e., Fuzzy earliest starting time and latest finishing time together with total float should be a zero fuzzy number. It also gives the red flag activities, three floats. These parameters are helpful for the decision maker to use his resources optimally. In this paper, Section 2 deals with fuzzy

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preliminaries followed by Section 3 in which the proposed algorithm is given in detail. In Section 4 and Section 5 the disadvantages of the existing algorithms and advantages of the pro- posed algorithm is explained. The implementation of the algorithm through examples is discussed in Section 6 and Section 7 describes the conclusion.

II. PRELIMINARIES

Definition 2.1. A *fuzzy set* can be mathematically constructed by assigning to each possible individual in the universe of discourse a value representing its grade of membership.

Definition 2.2. A *fuzzy number* $\tilde{A}$ is a fuzzy set whose membership function $\mu_{\bar{A}}(x)$ satisfies the following condition

(1) $\mu_{\tilde{A}}(x)$ piecewise continuous

(2) $\mu_{\tilde{A}}(x)$ is convex

(3) $\mu_{\tilde{A}}(x)$ is normal (i.e.,) $\mu_{\tilde{A}}(x_0) = 0$.

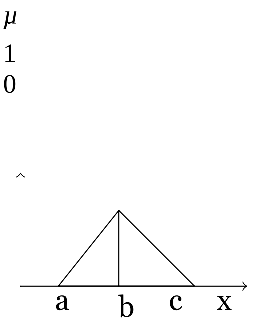
Definition 2.3. A fuzzy number $\tilde{A}$ = (a, b, c) with membership function of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \frac{c-x}{c-b} & b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

is called a *triangular fuzzy number* and a fuzzy number $\tilde{A}$ = (*a*, *b*, *c*, *d*) with membership function of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{d-c} & c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

is called a *trapezoidal fuzzy number*.



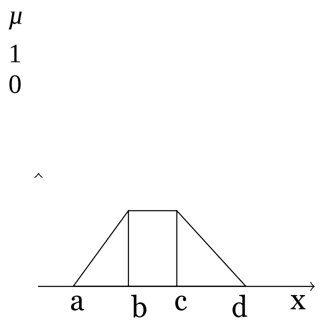


Fig 1. Triangular Fuzzy Number **Fig 2. Trapezoidal Fuzzy Number**

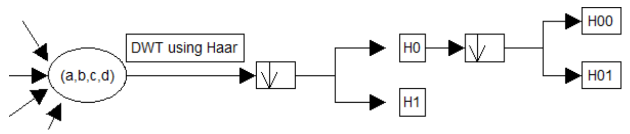
Definition 2.4. The *Fuzzy Operations* of fuzzy numbers are defined as Fuzzy Addition:

$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$ $(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

Fuzzy Subtraction:

$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$ $(a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$

Consider a Fuzzy Trapezoidal Number (*a*, *b*, *c*, *d*). When the fuzzy number is synthesized and analyzed using multi resolution analysis [20], the maximum information of a Fuzzy Number can be captured from the unique average coefficient through the down sampling process and the high fuzziness of the given fuzzy number can be analyzed using the detailed coefficients at each and every level of down sampling. It is represented diagrammatically as



MATLAB has been used to compute the average and detailed coefficients using Discrete Wavelet Transform (DWT) through Haar wavelet.

Definition 2.5. Haar ranking: [6]

For a given fuzzy number $\tilde{A} = (a, b, c, d)$ the average and detailed coefficients namely the scaling and wavelet coefficients can be calculated using $\alpha = ((a + b + c + d))/4$, $\beta = ((a + b) - (c + d))/4$, $\gamma = (a-b)/2$, $\delta = (c-d)/4$ and call this new 4-tuple as $R(\tilde{A}) = (\alpha, \beta, \gamma, \delta)$.

If it is a triangular fuzzy number (*a*, *b*, *c*) then it can be written as four tuples either by (*a*, *b*, *b*, *c*) or by zero padding, that is (*a*, *b*, *c*, 0). Then Haar definition is used to get the Haar tuple. In this paper the first one is used.

- $\tilde{A} \prec \tilde{B}$, if the first element of the ordered tuple of $R(\tilde{A})$ is less than the first element of the ordered tuple of $R(\tilde{B})$.
- $\tilde{A} \succ \tilde{B}$, if the first element of the ordered tuple of $R(\tilde{A})$ is greater than the first element of the ordered tuple of $R(\tilde{B})$.
- $\tilde{A} \approx \tilde{B}$ if and only if all the elements of $R(\tilde{A})$ and $R(\tilde{B})$ are term wise equal. (i.e) $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_1 = \gamma_2$, $\delta_1 = \delta_2$.

(1) Consider $\tilde{A} = (5, 10, 15, 20)$ and $\tilde{B} = (10, 15, 20, 25)$. The Haar tuple for $\tilde{A}$ is given by $(12.5, -5, -2.5, -2.5)$. The Haar tuple for $\tilde{B}$ is given by $(17.5, -5, -2.5, -2.5)$. Here $12.5 \le 17.5$, so $\tilde{A} \prec \tilde{B}$ (2)

(2) Consider $\tilde{A} = (5, 10, 15)$ and $\tilde{B} = (10, 15, 20)$. This should be changed to $\tilde{A} = (5, 10, 10, 15)$ and $\tilde{B} = (10, 15, 15, 20)$. The Haar tuple for $\tilde{A}$ is given by $(7.5, -2.5, -2.5, -2.5)$. The Haar tuple for $\tilde{B}$ is given by $(15, -2.5, -2.5, -2.5)$. Here $7.5 \le 15$, so $\tilde{A} \prec \tilde{B}$.

- Two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are said to be equal if they are element- wise

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equal.

- Two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are said to be equivalent if their crisp values ($R(\tilde{A}) = R(\tilde{B})$) are equal.

Definition 2.6. Ordered pair Addition:

$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$

Ordered pair Subtraction:

$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2)$ $(a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2)$

Definition 2.7. A *Fuzzy Network problem* [19] is defined as

$$\text{Max } \tilde{D} \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{T}_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{n} x_{ij} = \sum_{k=1}^{n} x_{ki} \text{ for } i = 2,3 \ldots . n-1,$$

$$\sum_{k=1}^{n} x_{kn} = 1, \text{for all } x_{ij} \geq 0, (i,j) \in A,$$

Where G = (N, A) is a directed and connected network, and N is the set of n nodes, and A is the set of $(i, j) \in A$ arcs. Denote $\tilde{T}_{ij}$ as the activity fuzzy time of activity $(i, j) \in A$.

III. PROPOSED ALGORITHM

Step 1. Using Haar ranking, convert all the fuzzy durations into Haar tuples.

Step 2. Considering the initial node as '1' and the terminal node as 'n', compute forward pass as follows

Initial step:

Set $\langle HEST_1 \rangle = \langle 0, 0, 0, 0 \rangle$ which indicates that the project starting time at node '1' is 0. (HEST = Haar tuple earliest starting time)

General step:

Compute $HEST_j$ as follows:

$HEST_j = MAX\{HEST_p + HD_{pj}, HEST_q + HD_{qj}, \cdots HEST_v + HD_{vj}\}$,

where HD_{pj}, HD_{qj} and HD_{vj} represents the Haar tuple of fuzzy duration time of activities between p- j, q- j and v- j. Here the nodes p, q…and v are linked directly to node j by incoming activities (p, j), (q,j)… and (v,j).

Complete the forward pass for all the nodes.

Backward pass:

Following the completion of the forward pass, the backward pass computation starts at node 'n' and ends at node '1'.

Initial step:

Set $HLFT_n = HEST_n$ to indicate the earliest and latest occurrences of the last node of the project. ($HLFT_n$ = Haar tuple of fuzzy latest finishing time of node 'n')

General step: Calculate

$HLFT_j = Min\{HLFT_p — HD_{jp}, HLFT_q \quad HD_{jq}, HLFT_v — HD_{jv}$. Here the nodes p, q and v are linked directly to node j through the outgoing activities (j, p), (j, q) and (j, v).

Compute backward pass for all the nodes and ensure that

$HLFT_1 = HEST_1 = \langle 0, 0, 0, 0 \rangle$

Step 3. An activity (i, j) will be called as critical if $HLFT_j = HEST_j$, $HLFT_i = HEST_i$, $HLFT_j – HLFT_i = HEST_j – HEST_i$

Step 4. Calculate the fuzzy floats as follows:

Haar Total Float: $HTF_{ij} = HLFT_j$ - $HEST_i$ - HD_{ij}

Haar Free Float $HFF_{ij} = HTF_{ij}$ - $HLFT_j$ - $HEST_j$

Haar Independent Float $HIF_{ij} = HFF_{ij}$ - $HLFT_i$ - $HEST_i$.

Based on the floats, if $HFF_{ij} \approx\_HTF_{ij}$, then the activity can be scheduled anywhere between $(HEST_j, HLFT_j)$ without affecting the schedule span. Suppose $HFF_{ij} \prec HTF_{ij}$, then the start of the activity can be delayed by at most HFF_{ij}. This is called the red flagged activity which is non-critical.

IV. ADVANTAGES OF THE PROPOSED ALGORITHM

- Algorithm proposed by Chen and Hsueh [4] uses the linear programming formulation of fuzzy critical path problems and implemented the algorithm of crisp linear programming problems to solve them. But solving optimization problems tabular methods are preferred as compared to linear programming techniques. The proposed method is tabular method.
- The existing algorithms are not ensuring the condition for critical activities. But the proposed algorithm ensures the conditions for critical activities. i.e., Fuzzy earliest starting time and latest finishing time together with total float should be a zero fuzzy number.
- The existing algorithms are designed just to find the critical path and not the floats which is very much important to the decision maker to make decisions. But all the three floats can be calculated using the proposed algorithm. From the floats, it is also possible to identify the red flag activities. This will help the decision maker to decide to continue the activity or utilize the resources for some other activity.
- This algorithm is easy to understand and implement since it follows the steps of crisp critical path problem.
- This algorithm ensures that the decision maker get all the results in terms of fuzzy numbers and the decision can be made accordingly. This algorithm does not require the knowledge of linear programming technique and fuzzy linear programming technique.
- The proposed algorithm can be implemented for network problems with generalized fuzzy numbers.

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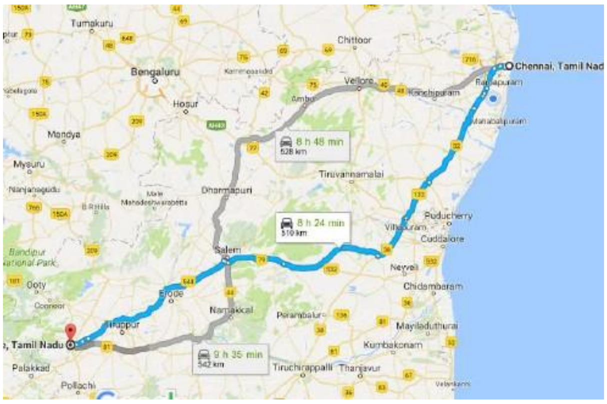
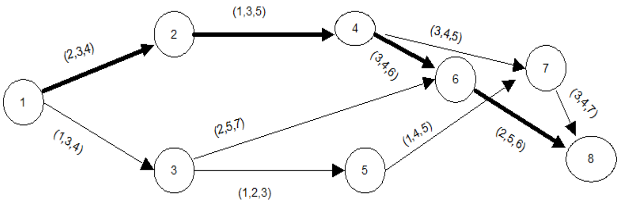
V. NUMERICAL EXAMPLES

Example 5.1. To validate the proposed approach, a real time application is taken [8], which treats with the construction of a road connection between Chennai and Coimbatore Tamil Nadu state, India. The linguistics terms like ”approximately between” and “around” can be modeled by the approximate reasoning of fuzzy set theory. The duration of the task are represented by triangular fuzzy numbers which seems to be adequate. The project manager wishes to construct a possible route from Chennai (s) to Coimbatore (d). The map of Tamil Nadu is given as follows.



Given a road map of Tamil Nadu on which the time taken between each pair of successive intersection are marked, to determine the critical path from source vertex ‘s’ to the destination vertex ‘d’.

Let vertex 1 be Chennai, vertex 2 be Vellore, vertex 3 be Villupuram, vertex 4 be Dharmapuri, vertex 5 be Perambalur, vertex 6 be Erode, vertex 7 be Karur and Vertex 8 be Coimbatore.

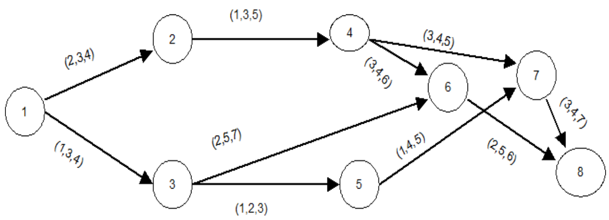


Activity	Time Duration
1-2	about 3 months ⟨2, 3, 4⟩
1-3	about 3 months ⟨1, 3, 4⟩
2-4	about 3 months ⟨1, 3, 5⟩
3-5	about 2 months⟨1, 2, 3⟩
3-6	about 5 months ⟨2, 5, 7⟩
4-6	about 4 months ⟨3, 4, 6⟩
4-7	about 4 months ⟨3, 4, 5⟩
5-7	about 4 months⟨1, 4, 5⟩
6-8	about 5 months⟨2, 5, 6⟩
7-8	about 4 months ⟨3, 4, 7⟩

Activity	**Time Duration**	**HD**	**HEST**	**HEFT**
1-2	⟨2, , 3, 4⟩	(3,-.5,-.5,-.5)	(0,0,0,0)	(3,-.5,-.5,-.5)
1-3	⟨1, 3, 4⟩	(2.75,-.75,-1,-.5)	(0,0,0,0)	(2.75,-.75,-1,-.5)
2-4	⟨1, 3, 5⟩	(3,-1,-1,-1)	(3,-.5,-.5,-.5)	(6,-1.5,-1.5,-1.5)
3-5	⟨1, 2, 3⟩	(2,-.5,-.5,-.5)	(2.75,-.75,-1,-.5)	(4.75,-1.25,-1.5,-1)
3-6	⟨2, 5, 7⟩	(4.75,-1.25,-1.5,-1)	(2.75,-.75,-1,-.5)	(7.5,-2,-2.5,-1.5)

HLST	HLFT	HTF	HFF	HIF
(0,0,0,0)	(3,-.5,-.5,-.5)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
(2,0,0,0)	(4.75,-.75,-1,-.5)	(2,0,0,0)	(0,0,0,0)	(0,0,0,0)
(3,-.5,-.5,-.5)	(6,-1.5,-1.5,-1.5)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
(4.75,-.75,-1,-.5)	(6.75,-1.25,-1.5,-1)	(2,0,0,0)	(0,0,0,0)	(-2,0,0,0)
(5.5,-1,-.5,-1.5)	(10.25,-2.25,-2,-2.5)	(2.75,-.25,.5,-1)	(2.75,-.25,.5,-1)	(0.75,-.25,.5,-1)
(6,-1.5,-1.5,-1.5)	(10.25,-2.25,-2,-2.5)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
(6.25,-1.75,-2.5,-1)	(10.25,-2.25,-3,-1.5)	(.25,-.25,-1,-.5)	(0,0,0,0)	(0,0,0,0)
(6.75,-1.25,-1.5,-1)	(10.25,-2.25,-3,-1.5)	(2,0,0,0)	(1.75,.25,1,-.5)	(-.25,.25,1,-0.5)
(10.25,-1.25,-2,-2.5)	(14.75,-3.25,-3.5,-3)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
(10.25,-2.25,-3,-1.5)	(14.75,-3.25,-3.5,-3)	(.25,-.25,-1,.5)	(.25,-.25,-1,.5)	(.25,-.25,-1,.5)

Applying the proposed algorithm the HEST and HEFT values are obtained as follows.



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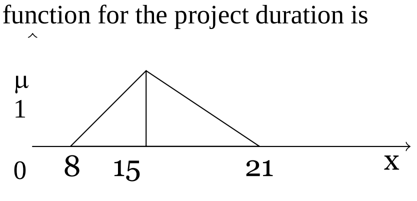
4-6	⟨3, 4, 6⟩	(4.25,-.75,-.5,-1)	(6,-1.5,-1.5,-1.5)	(10.25,-2.25,-2,-2.5)
4-7	⟨3, 4, 5⟩	(4,-.5,-.5,-.5)	(6,-1.5,-1.5,-1.5)	(10,-2,-2,-2)
5-7	⟨1, 4, 5⟩	(3.5,-1,-1.5,-.5)	(4.75,-1.25,-1.5,-1)	(8.25,-2.25,-3,-1.5)
6-8	⟨2, 5, 6⟩	(4.5,-1,-1.5,-.5)	(10.25,-2.25,-2,-2.5)	(14.75,-3.25,-3.5,-3)
7-8	⟨3, 4, 7⟩	(4.5,-1,-.5,-1.5)	(10,-2,-2,-2)	(14.5,-3,-2.5,-3.5)

The critical path is 1-2-4-6-8. The project duration is (14.75,-3.25,-3.5,-3). The corresponding fuzzy time duration is (8, 15, 21) and it matches with the result obtained in [8]. The critical path is represented as thick lines in the following diagram.

VI. RESULTS AND DISCUSSIONS

The critical path is *Chennai - Vellore - Dharmapuri - Erode - Coimbatore*. The project duration is about 15 months. The routes Chennai - Vellore, Vellore - Dharmapuri, Dharmapuri - Erode and Erode - Coimbatore are criti- cal activities. So the duration for laying these routes should not be delayed. The Red flag Activities are 1-3, 4-7, 3-5 and 5-7. i.e) *Chennai-Villupuram, Dharmapur-Karur, Villupuram-Perambalur* and *Perambalur-Karur*. The start of route laying between Chennai to Villupuram can be delayed by at most (0, 0, 0) duration. The start of route laying between Dharmapuri to Karur can be delayed by at most (0, 0, 0) duration.The start of route laying between Villupuram to Perambalur can be delayed by at most (0, 0, 0) duration. Since $HFF_{36} \approx HTF_{36}$ the route laying between Villupuram to Erode can be scheduled anywhere be- tween (6, 10, 15) without affecting the schedule plan. Similarly the route laying between Karur to Coimbatore can be scheduled anywhere between (8, 15, 21). These analysis gives freedom to utilize the resources effectively. Furthermore the membership function for the project duration is



- According to the decision maker the project duration will lie between 8 months and 21 months.
- The overall level of satisfaction of the decision maker about the statement that the project duration will be 15 months is 100 percent.
- The overall level of satisfaction of the decision maker for the remaining values of minimum time can be obtained as follows:

Let x_0 represent the minimum time then the overall level of satisfaction of the decision maker for x_0 is $\mu_{\tilde{A}}(x_0) \times 100$, where

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-8}{7} & 8 \le x \le 15 \\ 1 & x = 15 \\ \dfrac{21-x}{6} & 15 \le x \le 21 \\ 0 & \text{otherwise} \end{cases}$$

VII. CONCLUSIONS

In this paper, A new method namely, Haar ranking based Fuzzy Critical Path Method is proposed and implemented to solve Fuzzy Network Problems. The proposed method is effective, easy to understand and implement to identify the fuzzy critical path. There is no computational complexity in the proposed method since it involves order pair arithmetic. Using the proposed method, the critical activities can be identified and also find the red flag activities which can be delayed without affecting the project duration. The comparative study between the existing method and the proposed method confirms the efficiency of the proposed method and it is given as tabular form

Example	**Existing method Optimal Solution [8]**	**proposed method Optimal Solution**
5.1	(8, 15, 21)	(8, 15, 21)
	Critical activity conditions are not satisfied	Critical activity conditions are satisfied
		Identification of red flag activities

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