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Digital Generation of Harmonic Signals

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***Abstract:** Generation methods of high accuracy signals based on generalized algorithm of digital interpolation are proposed. Operation of such integer algorithms results in a sequence of digital readout of sine and cosine curves. Optimum subprogram in micro assembler is given oriented at microprocessor implementation (without multiplication and division).*

***Index Terms:** integer algorithms, microprocessors, machine control.*

I. INTRODUCTION

Wide application of scientific researches of complicated engineering systems required for modern software [1] on the basis of computing technique. The major role is played by microprocessors (MP) and appropriate algorithms [2,3]. The architecture of MPwith its requirements for maximum hardware speed should be combined with high algorithm speed [4]. This property is characteristic for integer algorithms which exclude multiplication, division, and other so called long operations [5,6]. This will be accompanied by refuse to transfer directly analytic expression of implementable function of data processing to algorithm structure. With this aim it is recommended to implement algorithms on the basis of model properties of functions. It means transfer either to vector approximation (algorithms by Volder[5] or Meggit[6]), or to geometrical constructions known in descriptive geometry and engineering drawing, or to digital interpolation of straight lines and curves used in software systems of machine control [7]. Its essence is that the plot of curve (straight line) is traced only in predetermined discretely set points (with integer coordinates) spaced from neighboring points not more than by unity. Herewith, the next adjacent point is selected by the sign (+ or –) of specially introduced estimation function which characterizes actual deviation from the curve (straight line) in a given point. (Further on they are referred to as interpolation nodes). Digital interpolation algorithm makes it possible to generate special functional signals required upon implementation of automated researching systems: development of special high accuracy signals during testing of complicated engineering systems.

II. GENERALIZED ALGORITHM OF DIGITAL INTERPOLATION

At first let us describe the algorithm of digital linear interpolation.

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Its initial parameters are $\Delta X$and ΔY – the increments of linear coordinates of the origin and end of straight line (in integer format). For all interpolation nodes the estimation function is introduced: $F_i(x_i, y_i) = F_0 + +|\Delta X_i| \cdot y_i - |\Delta Y_i| \cdot x_i$, where $x_i, y_i$are the coordinates of the i-th interpolation node, $i \in \{0,1,2,\dots,\Delta X\}$are the numbers of nodes approximating the straight line,$F_0 = \Delta X - 2 \cdot \Delta Y$ *is* the constant providing the minimum approximation error of straight line (for the case $\Delta X \geq \Delta Y > 0$).

Hence, the algorithm is as follows:

$$\delta_y(i) = \begin{cases} 0, \text{if} F_i \geq 0; \\ 1, \text{if} F_i < 0; \end{cases}$$

(1)

$$\delta_x(i) = 1,$$

$$F_{i+1} = \begin{cases} F_i - 2 \cdot \Delta Y, & \text{if} F_i \geq 0; \\ F_i + 2 \cdot (\Delta X - \Delta Y), & \text{if} F_i < 0; \end{cases}$$

$$x_{i+1} = x_i + \delta_x(i);$$

$$y_{i+1} = y_i + \delta_y(i).$$

For other three combinations of the signs of $\Delta X$and ΔYthere exist similar equations containing $|\Delta X|$and$|\Delta Y|$as well as two sign functions $sign\Delta X$and $sign\Delta Y$.

$$y = sign x = \begin{cases} +1, \text{ if } x \geq 0; \\ -1, \text{ if } x < 0. \end{cases} \quad (2)$$

Other four cases when $|\Delta X|$<$|\Delta Y|$are described by symmetric variants with mutual renaming of coordinate axes.

For subsequent considerations it should be mentioned that the estimation function$F_i = F(x_i, y_i)$is adjusted after variation of current interpolation node.

The generalized algorithm of digital interpolation of curve is based on the algorithm of linear interpolation of tangential to the curve near current interpolation node [8]. During motion along the curve the tangential parameters are subsequently updated in each interpolation node, that is, $\Delta X_i, \Delta Y_i$, F_0, and F_i. $\Delta X_i$and $\Delta Y_i$are defined as the appropriate integers, the ratio of which equals to derivative of the curve function near current interpolation node ($f^/(x_i, y_i) = \frac{\Delta Y_i}{\Delta X_i}$).

Let us formulate the mathematical model of generalized algorithm of digital interpolation of random curves.

1) Initial value of estimation functionF_0

a) at $|\Delta X_0| \geq |\Delta Y_0| F_0 = |\Delta X_0|-2|\Delta Y_0|$; (3a)

(3)

b) at $|\Delta X_0| < |\Delta Y_0| F_0 = |\Delta Y_0|-2|\Delta X_0|$; (3b)

2) The selection rule of the next *i*-th step to adjacent node:

a) at $|\Delta X_i| \geq |\Delta Y_i|$:

(4)

$$\delta_x(i) = sign\Delta X_i,\ \delta_y(i) = \begin{cases} 0, \text{if} F_i \geq 0; \\ sign\Delta Y_i, \text{if} F_i < 0. \end{cases}$$

b) at $|\Delta X_i| < |\Delta Y_i|$: (5)

$\delta_y(i) = sign\Delta Y_i,\ \delta_x(i) = \begin{cases} 0, \text{if} F_i \geq 0; \\ sign\Delta X_i, \text{if} F_i < 0. \end{cases}$

3) Calculation of new coordinates of the curve

$x_{i+1} = x_i + \delta_x(i);$ $y_{i+1} = y_i + \delta_y(i);$ (6)

4) Adjustment of estimation function $F_i$due to variation of coordinates of the*(i+1)*-th interpolation node

a) at $|\Delta X_i| \geq |\Delta Y_i|$:

$F_{i+1}^{**} = \begin{cases} F_i - 2 \cdot |\Delta Y_i|, & if\ F_i \geq 0; \\ F_i + 2(|\Delta X_i| - |\Delta Y_i|), & if\ F_i < 0. \end{cases}$ (7)

b) at $|\Delta X_i| < |\Delta Y_i|$:

$F_{i+1}^{**} = \begin{cases} F_i - 2 \cdot |\Delta X_i|, & if\ F_i \geq 0; \\ F_i + 2(|\Delta Y_i| - |\Delta X_i|), & if\ F_i < 0. \end{cases}$ (8)

5) Adjustment of estimation function $F_i$due to variation of tangential steepness in the new *(i+1)*-th node

a) at $|\Delta X_i| \geq |\Delta Y_i|$:

$F_{i+1}^{*} = F_{i+1}^{**} - 2(|\Delta Y_{i+1}| - |\Delta Y_i|) + (|\Delta X_{i+1}| - |\Delta X_i|$(9)

b) at $|\Delta X_i| < |\Delta Y_i|$:

$F_{i+1}^{*} = F_{i+1}^{**} - (|\Delta Y_{i+1}| - |\Delta Y_i|) + 2(|\Delta X_{i+1}| - |\Delta X_i|)$(10)

6) Adjustment of estimation function $F_i$due to variation of estimation function.

a) upon transfer from $|\Delta X_i| \geq |\Delta Y_i|$to $|\Delta X_{i+1}| < |\Delta Y_{i+1}|$:

$F_{i+1} = F_{i+1}^{*} - (|\Delta X_{i+1}| - 2|\Delta Y_{i+1}|) + (2|\Delta X_i| - |\Delta Y_i|)$(11)

b) upon transfer from $|\Delta X_i| < |\Delta Y_i|$to $|\Delta X_{i+1}| \geq |\Delta Y_{i+1}|$:

$F_{i+1} = F_{i+1}^{*} + (|\Delta X_{i+1}| - 2|\Delta Y_{i+1}|) + (2|\Delta X_i| - |\Delta Y_i|)$(12)

c) in other cases (most of them):$F_{i+1} = F_{i+1}^{*}$. (13)

Combining adjustments by items 4) and 5), as well as reducing similar terms and considering that tangential to the curve is in the point with its coordinate corresponding to similar coordinate of the *i*-th node, we have the following recurrent equation of adjustment of estimation function F_i.

At $|\Delta X_i| \geq |\Delta Y_i|$:

$F_{i+1}^{*} = F_i - 2|\Delta Y_{i+1}| + |\Delta X_{i+1}| - |\Delta X_i| \cdot signF_i$(14)

At $|\Delta X_i| < |\Delta Y_i|$:

$F_{i+1}^{*} = F_i - 2|\Delta X_{i+1}| + |\Delta Y_{i+1}| - |\Delta Y_i| \cdot signF_i$(15)

It should be mentioned that for actual curves ΔX and ΔYcan be independent on the number *i* which simplifies Eqs. (14) and (15).

An inherent portion of this integer algorithm is calculation of $\Delta X$and $\Delta Y$by the curve steepness in each *i*-th interpolation node. As a rule, these are the functions $\Delta X = \gamma(x, y)$ and $\Delta Y = \varphi(x, y)$. They are significantly simpler than the derivative of function $y = f(x)$defining the curve type. Hence, $\gamma$and $\varphi$are calculated more easily, including integer equations when required.

For instance, for $y = A \cdot e^{x/B} \Delta X = B,\ \Delta Y = y,$

for $y = Aln(1 + \frac{x}{B}) \Delta X = B + x, \Delta Y = A,$

for $y = A(1 - e^{-x/B}) \Delta Y = A - y,\ \Delta X = B,$

where *A*and *B*are some scaling constants of curve plot.

III. ALGORITHMS OF DIGITAL GENERATION OF SPECIAL SIGNALS

Let us exemplify the algorithm of digital interpolation for generation of digital signals of certain type (curve shape, accuracy of amplitude and frequency). These are special testing signals for complicated engineering systems. They comprise technical foundation of scientific research automation. Let us show some examples of synthesis of the required integer algorithms implemented usually by microprocessors.

Test signal $\boldsymbol{y = Ax \cdot e^{-x/B}}$

This signal has bell-curve shape with amplitude fading in time.Let us develop the algorithm of digital interpolation, that is, let us determine ΔX and $\Delta Y$for this signal.

Let us find derivative of the function $y = A \cdot x \cdot e^{-x/B}$

$$y^{/} = (A \cdot x \cdot e^{-x/B})^{/} = \frac{z-y}{B}, \quad (16)$$

where $z = A \cdot B \cdot e^{-x/B} = C \cdot e^{-x/B}$.

That is, it is required to generate two curves: $y = f(x)$and $z = \varphi(x)$, and the values of the second function for each $x$should be substituted into ΔX and ΔYupon interpolation of the first function.

Thus, let us determine derivative of the second function so that to use it for setting $\Delta Z$and ΔX – the parameters of its digital interpolation.

Hence, the derivative of $z = C \cdot e^{-x/B}$is:

$z^{/} = -\frac{C}{B} \cdot e^{-x/B} = -\frac{z}{B}$ (17)

That is,

$$\frac{\Delta Z}{\Delta X} = -\frac{z}{B}. \quad (18)$$

Respectively, the steepness (the angle coefficient) of the tangential is ${}^{-z}/_{B}$.

Let us apply another approach in order to increase the approximation accuracy. Let us assume that with probability $P_z = \frac{z + \frac{P_z}{2}}{B}$the amplitude z will decrease by –1. Hence, the average value of z in a unit interval will be $z + \frac{P_z}{2} \cdot (-1)$. Let us write the equation:

$P_z = -\frac{z}{B} - \frac{P_z}{2B}$ (19)

Solving it with regard to P_z , we have:

$$-P_z = -\frac{2z}{2B+1} . \quad (20)$$

Therefore, $\overleftarrow{\Delta Z} = -2z,\ \overleftarrow{\Delta X_z} = 2B + 1$,where $\overleftarrow{\Delta Z},\ \overleftarrow{\Delta X_z}$are the adjusted parameters of digital interpolation of the exponent $z = A \cdot B \cdot e^{-x/B}$.

Let us transfer to the first function$y = A \cdot x \cdot e^{-x/B}$. Its derivative is:

$$y^{/} = A \cdot e^{-x/B} - \frac{A}{B} \cdot x \cdot e^{-x/B}. \quad (21)$$

Respectively, with consideration for Eq. (16):

$$\frac{\Delta Y}{\Delta X} = \frac{A \cdot B \cdot e^{\frac{-x}{B}} - y}{B} = \frac{z-y}{B} \quad (22)$$

Let us determine the adjusted parameters of its digital interpolation :

$$\frac{\overleftarrow{\Delta Y}}{\overleftarrow{\Delta X_y}} = \frac{Z - \frac{z}{2B+1} - (y + \frac{\overleftarrow{\Delta Y}}{2\overleftarrow{\Delta X_y}})}{B}. \quad (23)$$

Let us consider the steepness of the tangential $f_y$to the first curve:

$$P_y = \frac{Z - \frac{z}{2B+1} - (y + \frac{P_y}{2})}{B} \quad . \qquad (24)$$

Solving Eq. (24) with regard to f_y, we have:

$$P_y = \frac{2BZ - y(2B+1)}{(2B+1)^2/_2} \quad (25)$$

Finally, we have:

$$\overline{\Delta Y} = 2BZ - y(2B+1), \qquad (26)$$

$$\overline{\Delta X_y} = {(2B+1)^2}/_2 \quad .$$

Some important remarks to the obtained results. All scale constants *A*, *B*should be assigned on the basis of the required interpolation accuracy and the condition that absolute magnitude of the tangential steepness should not exceed 1 (that is, $|P| \leq 1$).

The second condition will provide that during interpolation the increments will be either ± 1or 0. This fact would permit to solve the issue of multiplication upon calculation of $2BZ$, $y(2B+1)$and similar issues of other curves using the recurrent equation :

$BZ_{i+1} = BZ_i \pm B$, ifZ was modified.

$y_{i+1} \cdot (2B+1) \;\; = y(2B+1) \;\; \pm 2B + 1$, if$Y$ was modified

F_0, $2B+1$ and ${(2B+1)^2}/_2$are calculated once by operator before starting the program of generation of special signal.They are program constants.

Harmonic signal $\boldsymbol{y = A \cdot \sin\left({x}/_{B}\right)}$

Let us generate sinusoid using its auxiliary function – its derivative – cosine curve [9]. That is, the sine curve $M \cdot \sin({i}/_{M})$and cosine curve $M \cdot \cos({i}/_{M})$ will be generated simultaneously, where iis the node number. Then, the increments of $S_i$and $C_i$in the interval $(i; i+1)$of argument will be as follows:

$$\Delta S_i = \frac{1}{M} \cdot \frac{C_i + (C_i + \Delta C_i)}{2}, (27)$$

$$\Delta C_i = \frac{1}{M} \cdot \frac{S_i + (S_i + \Delta S_i)}{2}. (28)$$

Equations (27) and (28) are obtained using averaged derivatives at the origin and the end of unit segment of argument between adjacent interpolation nodes. Herewith, it is considered that the sine derivative is cosine and the cosine derivative is minus sine.

After simplification we have the following set of equations:

$$2c_i + \Delta c_i = 2M\Delta s_i \qquad (29)$$

$$-(2s_i + \Delta s_i) = 2M\Delta c_i \qquad (30)$$

Solving it with regard to $\Delta S_i$and ΔC_iwe obtain:

$$\Delta S_i = \frac{4MC_i - 2S_i}{4M^2+1}; \quad \Delta C_i = -\frac{4MS_i + 2C_i}{4M^2+1}$$

(31)

Equations (31) make it possible to assign the following parameters of digital interpolation (on the basis of Eqs. (3*a*) and (14); since the derivative functions $\frac{1}{M} \cdot \sin({x}/_{M})$and $\frac{1}{M} \cdot cos({x}/_{M})$in any interpolation node are not higher than 1):

$$\Delta \bar{Y}_s(i) = 4Mc_i - 2s_i \quad (32)$$

$$\Delta \bar{Y}_c(i) = -(4Ms_i + 2c_i)(33)$$

$$\Delta \bar{X}_s = \Delta \bar{Y}_c = 4M^2 + 1(34)$$

It is possible to improve the accuracy for Eqs. (32), (33), and (34), ifthe steepness of curves $S_i$and $C_i$is averaged not by trapezoidal integration but analytically. This can be performed using the following equations:

$$\Delta \bar{Y}_s(i) = 6Mc_i - 3s_i, \qquad (35)$$

$$\Delta \bar{Y}_c(i) = -(6Ms_i + 3c_i), (36)$$

$$\Delta \bar{X}_s = \Delta \bar{Y}_c = 6M^2 + 1. (37)$$

The values of $6M^2$, $6M$ are calculated by subprogram prior to initiation of generation, either calculated by software designer and preset as constants into software or hardware upon implementation. Multiplication of $6MS_i$and $6MC_i$are performed by addition to previous $(i-1)$values of $\pm$6M, provided that sine or cosine were implemented by ± 1.

IV. RESULTS AND DISCUSSION

The proposed method of digital generation of special signals is based on generalized algorithm of digital interpolation of arbitrary curves. The advantages of this method are as follows:

a) high accuracy of approximation (error not higher than one least significant register bit);

b) mobility in variation of curve shapes and their parameters since only subprograms and their constants are modified;

c) digital form of the obtained results simplifies integration with tested engineering system;

d) simple programming in microassembler with subsequent implementation using rapid microprocessors due these integer algorithms.

Let us exemplify microassembler subprogram for generation of harmonic signals (Table 1).

Table 1. Microassembler subprogram

LOR:	SUB R4;R0	subtraction
	BPL KOR	conditional transfer
	SUB #T,R4	subtraction
DOL:	DEC R4	increment by 1
	DEC R5	increment by 1
	SUB #P,R1	subtraction
	ADD#Q,R0	addition
SAL:	ADD#T,R4	addition
	HALT	stop
KOR:	SUB R1;R3	(input) subtraction
	BPL LOR	conditional transfer
	DEC R1	increment by 1
	INC R2	reduction by 1
	ADD#Q,R3	addition
	SUB R4;R0	subtraction
	BPL SAR	conditional transfer
	BR DOL	unconditional transfer

Initial register values (for $M < \sqrt{2^{15} - 1}$)are as follows: $(RO) = M^2$; $(R1) = 2My_0 - x_0$; $(R2) = x_0$; $(R3) = M^2$; $(R4) = 2Mx_0 + y_0$; $(R5) = y_0$; where $(x_0, y_0)$is the initial interpolation node.

Current coordinates (x_i, y_i)in registers R2and R5 (after stop) as constants are as follows:$P = 2M$, $Q = 2M^2$, $T = M$.

V. CONCLUSION

The proposed generation methods of special signals, rapid and adopted to microprocessor implementation, are capable to perform tests of complicated engineering systems and complexes, including automatic control and monitoring systems, radio engineering (radio location) devices, vibroseis prospecting methods, as well as other applications of computers [10], [11], [12].

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