Analysis of Incremental and Component of Value-at-Risk in the Stocks Investment Portfolio

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Abstract: ***In the formation of investment portfolios in stock assets, investors often raise questions: actually how much component each stock contributes to portfolio risk. Also, every time a portfolio structure is changed, is there a risk change in the investment portfolio. This paper aims to determine the incremental and component of Value-at-Risk in the formation of an investment portfolio. To solve these problems used several methods as follows: Incremental Value-at-Risk (Ivar) and Component Value-at-Risk (CVaR) used for the measurement of investment risk on some stocks. IVaR to measure changes in the value of a portfolio against changes in the composition or weight of the allocation of funds. Whereas CVaR for identifying elements and composition in the portfolio. IVaR assessment on stock portfolios using the before and after approach, and the delVaR approach. Based on the results of the analysis it can be shown that IVaR estimation using the delVaR approach is more efficient and practical compared to the before and after approach. So the delVaR approach is seen as more practical in its use in incremental measurements and Value-at-Risk components.***

Index Terms: Before & after approach, component, $delVaR$ ***, value-at-risk, incremental.***

I. INTRODUCTION

In the world of investment in stock assets, Value-at-Risk (*VaR*) gives investors an indication of portfolio risk, *IVaR* provides an indication of how these risks change when there is a change in portfolio investment [1], [2]. In practice, it is often related to changes in portfolio risk when investors make changes to new investments. In this case, Incremental Value-at-Risk (*IVaR*) is a change in the *VaR* portfolio that is related to each addition of new investments in the portfolio. The relationship between *IVaR* and new investments is very informative [3], [4]. Size *IVaR* can be used as an aid in making risk control decisions. For example, investors can use *IVaR* to determine the expected return on investment plans, and to determine investment limits. While Component Value-at-Risk (*CVaR*) is very useful for identifying high risk sources and also the opposite, and for determining investment constraints, making investment decisions, determining capital requirements, etc. [5].

Regarding the risk measurement of the stock investment portfolio, several authors have conducted research on the use of *VaR* to measure the risk of investment portfolios. Among them, in [6] estimate Value-at-Risk (*VaR*) on several stocks in Indonesia using the ARMA-FIGARCH model approach. The results show that the estimation of *VaR* with the ARMA-FIGARCH model has a very good performance, which has a Quadratic Probability Score (*QPS*) of each analyzed stock close to zero. Also, in [7] measures portfolio risk for several stocks in Indonesia, using Condistional Value-at-Risk under liability assets with non-constant volatility. The results show that the risk measurement in this study also has good performance. In [8] examines the relationship of Incremental Value-at-Risk (*IVaR*) with Value-at-Risk (*VaR*). He said that *IVaR* is approximately equal to *VaR* which is multiplied by the beta coefficient of the stock. The formula reliability he said is strong, even for small changes in the composition of the investment portfolio. Furthermore, in [9] formulated the Value-at-Risk Contribution (*VaRC*) under Asset Liability Models by Using Exponential Weighted Moving Average Approaches. This approach is used to measure the risk of investment portfolios of several stocks traded on the capital market in Indonesia. The results of the formulation show that the approach of this model is very useful in measuring the risk of the stock investment portfolio.

In this paper, we intend to determine *IVaR* and *CVaR* using the before and after approach and the *delVaR* approach. The aim is to show an increase and decrease in risk, and how portfolio changes can change investment risk. Furthermore, risk measurement of investment portfolios using this approach is used to analyze several stocks traded on the capital market in Indonesia.

II. RISK MEASUREMENT MODEL

A. Value-at-Risk

Suppose that t is the time of financial investment, and contains risks for the next period of time throughout l. Suppose also $\Delta V(l)$ is a change in the value of assets in

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financial investment over time t until $t+l$. This quantity is measured in rupiah and is a random variable in the time index t. Cumulative distribution function (CDF) of $\Delta V(l)$ symbolized by $F_l(x)$. Defined *VaR* of investment along the time horizon l end with probability φ like:

$$p = \Pr[\Delta V(l) \le \text{VaR}] = F_l(\text{VaR}) \quad (1)$$

Because the holder as long as financial investment suffers losses $\Delta V(l) < 0$, The inner $VaR(l)$ is a special assumption for negative values if φ small. A negative sign is loss. From the definition, the probability that the holder will suffer losses is greater or equal to *VaR* along the time horizon l is φ. *VaR* can also be interpreted as follows; with probability $(1-\varphi)$, the potential loss suffered by the holder in financial investment throughout the time horizon l is smaller or equal to *VaR*.

Holders who are not owners of their own capital (short positions) suffer losses if assets increase, i.e. $\Delta V(l) > 0$. Then, *VaR* is defined as:

$$\varphi = \Pr[\Delta V(l) \ge \text{VaR}] = 1 - \Pr[\Delta V(l) \le \text{VaR}] = 1 - F_l(\text{VaR}) \quad (2)$$

For p small, *VaR* in short positions is a special assumption for positive values. A positive sign is loss. Various definitions show that *VaR* is often associated with the tail behavior of the cumulative distribution function $F_l(x)$. For the owner of his own capital (long position), tail left $F_l(x)$ is important. While for short positions focusing on the right tail $F_l(x)$. The definition of *VaR* in (1) is then used in short positions when using distribution $-\Delta V(l)$. Here only the *VaR* calculation method is discussed using long position [10], [11].

For any univariate cumulative distribution function $F_l(x)$ and probability φ, where $0 < \varphi < 1$, quantity:

$$x_\varphi = \inf\{x \mid F_l(x) \ge \varphi\} \quad (3)$$

called quantile to-φ of $F_l(x)$, where "inf" states the smallest real number that satisfies $F_l(x) \ge \varphi$. If the cumulative distribution function $F_l(x)$ in (1) it is known, then quantile to -φ is *VaR* = x_φ. If the cumulative distribution function is unknown, the *VaR* study must begin with an estimate of the cumulative distribution function and or here quantile, specifically the tail behavior of the cumulative distribution function [10], [12].

B. Estimation of IVaR

B.1. 'Before & After' approach

Suppose an available portfolio p, with *VaR* portfolios namely $VaR(p)$. Then, there are trade candidates a, this gives *VaR* new portfolio, say $VaR(p+a)$. *IVaR* related to the amount of investment a, $IVaR(a)$, then estimated as the difference between the two [8], [13]:

$$IVaR = VaR(p+a) - VaR(p) \quad (4)$$

B.2. *delVaR* approach

Suppose a portfolio p and want to estimate *IVaR* related to the increase in the amount of investment a on available portfolios. Starting with mapping p and a on the set of n instrument. Portfolio p then have an investment weight vector in each stock (instrument) $[w_1,\ldots,w_n]$, where w_1 is the investment weight mapped by instrument 1, and so on, and the new portfolio has an investment weight vector $[w_1+\Delta w_1,\ldots,w_n+\Delta w_n]$. If a relatively small against p, we can estimate *VaR* of a new portfolio, $VaR(p+a)$, with the first order Taylor series approach around $VaR(p)$:

$$VaR(p+a) \approx VaR(p) + \sum_{i=1}^{n} \frac{\partial VaR}{\partial w_i} dw_i \quad (5)$$

where is the partial derivative, $\partial VaR / \partial w_i$, declare marginal changes in *VaR* relating to marginal changes in the elements of the relevant cash flow. Equation (5) can be written in the matrix equation:

$$IVaR(a) \approx \nabla VaR(p) dw \quad (6)$$

where dw is transpose of a vector $[dw_1,\ldots,dw_n]$ and $\nabla VaR(p)$ declare a partial derivative vector $VaR(p)$ associated with w_i. $\nabla VaR(p)$ known as *delVaR*. Approach of *delVaR* **v**ery useful because it makes it easy to assess and use *IVaR* in real time [3].

C. Estimation of CVaR

First of all selected criteria decomposition - whether to decompose *VaR* with instruments, asset classes, cash, etc. Portfolio *VaR* is a homogeneous linear function of investment in instruments. This linear homogeneity leads to the use of the Euler theorem, which is explained as follows:

$$VaR = \sum_{i=1}^{n} w_i \frac{\partial VaR}{\partial w_i} = \nabla VaR(p) w \quad (7)$$

Next, define *VaR* for instruments i, $CVaR_i$, as:

$$CVaR_i = w_i \frac{\partial VaR}{\partial w_i} \quad (8)$$

Substitution (7) into the (8) obtained:

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$$VaR = \sum_{i=1}^{n} CVaR_i \tag{9}$$

If in (9) is divided by VaR, then obtained:

$$1 = \frac{1}{VaR} \sum_{i=1}^{n} CVaR_i = \sum_{i=1}^{n} \%CVaR_i \tag{10}$$

Percentage of $CVaR$, written as $\sum_{i=1}^{n} \%CVaR_i$, declare component VaR as a total percentage VaR [3], [14].

As a numerical illustration, formulations which have been described above, is then used to analyze the risk of the investment portfolio on a few stocks in Indonesia.

III. RESULTS AND DISCUSSION

A. Estimation of IVaR with the 'Before & After' Approach

Suppose an investor wants to add 4 new assets one by one to his initial investment portfolio as given in Table 1.

Table 1: Structure of instruments / investment assets in portfolios

Portfolio	Instrument / Asset of Investment
Portfolio 1	MERK
Portfolio 2	MERK, SMGR
Portfolio 3	MERK, SMGR, INCO
Portfolio 4	MERK, SMGR, INCO, MLBI
Portfolio 5	MERK, SMGR, INCO, MLBI, GGRM

Suppose that the investor diversified his investment by making a new portfolio composition of 0.6 for the initial portfolio and 0.4 additional new investments. Suppose that the investor will invest funds (S) of IDR 1,000,000.00 in each of the portfolios above, then VaR can be calculated for each portfolio for the next 24 hours with a 95% confidence level. The calculations are carried out using nonparametric methods (historical simulation). The calculation results obtained by the VaR value for each portfolio:

$$VaR(p_1) = -\text{IDR}1{,}000{,}000.00 \times (-0.021506205) = \text{IDR}21{,}506.20$$
$$VaR(p_2) = -\text{IDR}1{,}000{,}000.00 \times (-0.021863365) = \text{IDR}21{,}863.36$$
$$VaR(p_3) = -\text{IDR}1{,}000{,}000.00 \times (-0.01921025) = \text{IDR}19{,}210.25$$
$$VaR(p_4) = -\text{IDR}1{,}000{,}000.00 \times (-0.01336927) = \text{IDR}13{,}369.27$$
$$VaR(p_5) = -\text{IDR}1{,}000{,}000.00 \times (-0.018590806) = \text{IDR}18{,}590.81$$

Furthermore, using in (4), $IVaR$ values are obtained as follows:

$$\begin{aligned} IVaR_1 &= VaR(p_2) - VaR(p_1) \\ &= \text{IDR}21{,}863.36 - \text{IDR}21{,}506.20 = \text{IDR}357.16 \\ IVaR_2 &= VaR(p_3) - VaR(p_2) \\ &= \text{IDR}19{,}210.36 - \text{IDR}21{,}863.36 = -\text{IDR}2{,}653.11 \\ IVaR_3 &= VaR(p_4) - VaR(p_3) \\ &= \text{IDR}13{,}369.27 - \text{IDR}19{,}210.25 = -\text{IDR}5{,}840.98 \\ IVaR_4 &= VaR(p_5) - VaR(p_4) \\ &= \text{IDR}18{,}590.81 - \text{IDR}13{,}369.27 = \text{IDR}5{,}221.54 \end{aligned}$$

The $IVaR$ values are calculated, when graphed as shown in Fig. 1.

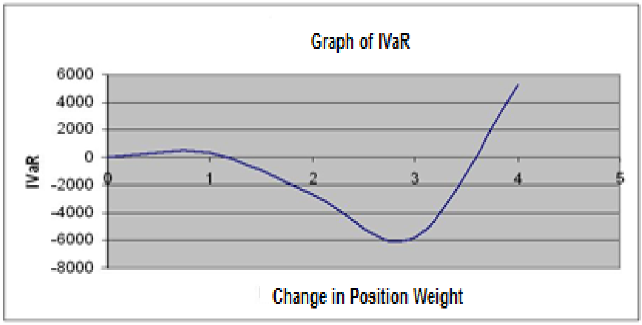

Fig. 1: Movement of *IVaR* values with 'before and after' approaches

Looking at the graph in Fig. 1, it appears that at first the graph of the $IVaR$ values rises slowly, then decreases with speed, and then rises sharply.

B. Estimation of IVaR with the delVaR Approach

Suppose an investor has the weight of the composition as given in Table 2. From the data return of the four portfolios given in Table 2, then the derivative vector is calculated from, and the calculation results obtained by the value of *delVaR* are as follows:

$$\nabla VaR(p) = [-0.014089581 \;\; -0.02521808 \;\; -0.02894496 \;\; -0.006352107 \;\; -0.021521448]$$

Table 2: Investment asset composition in portfolio

Investment Asset	Composition			
	Beginning of Portfolio	Portfolio 1	Portfolio 2	Portfolio 3
MERK	0.2	0.3	0.1	0.2
SMGR	0.2	0.1	0.3	0.1
INCO	0.2	0.4	0.4	0
MLBI	0.2	0.1	0.2	0.3
GGRM	0.2	0.1	0	0.4

From the data return of the four portfolios given in Table 2, then the derivative vector is calculated from, and the calculation results obtained by the value of *delVaR* are as follows:

$$\nabla VaR(p) = [-0.014089581 \;\; -0.02521808 \;\; -0.02894496 \;\; -0.006352107 \;\; -0.021521448]$$

Using the search for changes to the initial portfolio composition weighting to the new portfolio, then the calculation is done using equation (6), and the results obtained are $IVaR$ values as follows:

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$$IVaR_1 \approx -\text{IDR}1{,}888.79\,;\ IVaR_2 \approx -\text{IDR}2{,}597.55\,;$$
$$IVaR_3 \approx \text{IDR}3{,}371.30\,.$$

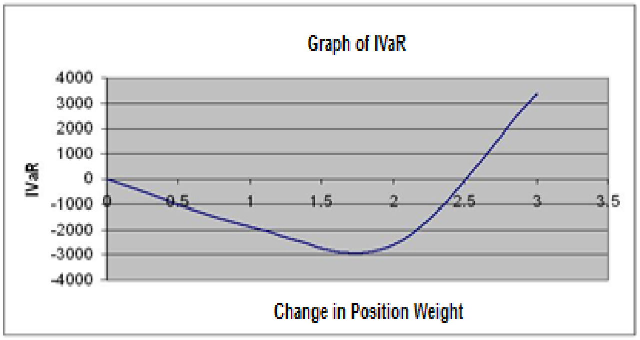

Fig. 2: *IVaR* value movement in the second way of the *delVaR* approach

Looking at the graph presented in Fig. 2, it appears that the *IVaR* values graph initially decreases with speed, then rises drastically.

C. Calculation of CVaR Decomposition

Furthermore, using the same composition weights as given in Table 2, by calculating *delVaR*, using in (9) the calculation results obtained component values of *VaR* as given in Table 3.

Table 3: The *VaR* decomposition values in portfolio 1, 2, 3 and 4

$CVaR_i$	Portfolio 1 (IDR)	Portfolio 2 (IDR)	Portfolio 3 (IDR)	Portfolio 4 (IDR)
$CVaR_1$	1,398.61	0.00	0.00	0.00
$CVaR_2$	8,844.09	391.39	1,188.12	1,268.52
$CVaR_3$	0.00	13,735.20	19,175.60	0.00
$CVaR_4$	0.00	5,826.89	0.00	0.00
$CVaR_5$	4,878.29	- 3,371.11	0,00	16,756.37
VaR	15,120.99	16,582.37	20,363.72	18,024.89

The *CVaR* value in Table 3, if expressed as a percentage using equation (10), can be shown in Table 4 as follows:

Table 4: Percentage of *CVaR* in portofolio 1, 2, 3 and 4

$CVaR_i$	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
$CVaR_1$	9.25%	0%	0%	0%
$CVaR_2$	58.49%	2.36%	5.83%	7.04%
$CVaR_3$	0%	82.83%	94.17%	0%
$CVaR_4$	0%	35.14%	0%	0%
$CVaR_5$	32.26%	-20.33%	0%	92.96%
VaR	100%	100%	100%	100%

Based on the values of Table 4, it can be seen that the *VaR* value of each portfolio is the sum of the *CVaR* values. This shows that *VaR* can be broken down into several risk components or *CVaR*.

IV. CONCLUSION

In this paper, Incremental and Value-at-Risk Component analysis has been carried out for investment portfolios in several stocks in Indonesia. Based on the results of the analysis it can be concluded as follows: *IVaR* estimation using the *delVaR* approach is more efficient and practical compared to the before and after approach. With the *delVaR* approach we only need to find a set of estimators to find the value of *IVaR*. But the lack of a *delVaR* approach is that it cannot estimate the *IVaR* value if the value of the new investment is added as in the before and after approach. The main factor that influences *IVaR* is the amount of capital allocation weight (composition) used in the portfolio. So that losses in investing can be minimized by composing portfolios appropriately. The portfolio risk value can be composed into risk components or *CVaR*. These components are very useful in identifying risks in the portfolio, so that investment planning and investment loss prevention can be carried out.

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