Optimization of Channel Precoding for MM wave Massive MIMO using Hybrid Precoding

A.Mounika Durga, Shaik.Jakeer Hussain

Abstract-mm wave communication system encounters the higher path loss than the microwave communication system. To defeat the path loss problem massive number of antennas with low wavelength are deployed at transmitter and receiver side. To transmit multiple data streams and to get better spectral efficiency precoding is required. Developing the hybrid Precoding is economically high and consumes more power and it is divided in to analog and digital parts. Due to the presence of large antennas and analog part in the hybrid precoding, mm wave massive MIMO requires some special algorithms to do the channel estimation and precoding. To construct the sparse Precoding and combining problems in mm wave massive MIMO we are considering the channel as spatial structure. In this paper sparse precoding is designed based on the orthogonal basis pursuit algorithm for mm wave massive MIMO by using optimal unconstrained precoder.

Keywords–mm wave Massive MIMO, Channel estimation, Precoding, orthogonal basis pursuit.

I. INTRODUCTION

In future 5G is expected to give 1000 times capacity more than the present 4G. To get this lot of technologies has been proposed. In that mm wave is one of the technology playing crucial role [1] in 5G cellular mobile communications, because of availability of large bandwidth. Major difference between the mm wave and present wireless communication system is the carrier frequency. Carrier frequency in the mm wave is ten times greater to the present wireless systems. It causes the more path loss in mm wave systems. However it is the drawback in the mm wave systems but the interesting point is wavelength of antenna is very low. So we can place more number of antennas arrays. mm wave combining with massive MIMO improves the beam forming gain, spectral efficiency [2],[3] and helps to overcome the path loss in the mm wave system. Initially conventional MIMO exploits the digital precoder to transmit data, it controls both signal’s amplitude and phase. In digital precoder structure each and every antenna requires one RF chain compulsory. There are several digital precoding techniques are introduced. In that simplest one is the Matching Filter (MF) precoding [4]. In the MF precoding SNR rate is increased at the mobile station side. But the drawback is interference is high between the input data streams. Zero forcing (ZF) precoding [4] is introduced to overcome the interference problem in MF, but performance loss is more. To get high SNR and low interference wiener filter (WF) [4] is introduced. There are still more digital precoding methods known as optimal DPC, near optimal Tomlinson Harashima precoding [5] are introduced, but they are very difficult to compute. Digital precoding fulfills the requirement to get better performance by controlling both signals amplitude and phase but it consumes more power and hardware cost is high because ,requirement of more number of RF chains. Analog beam forming is implemented with less number of RF chains by using phase shift networks [7],[8],[9] and it is developed initially for point to point communication systems [6]. Drawback of analog beam forming is, it can manipulate only phase.

To defeat the issues in analog beam forming and digital precoding new era is introduced, that is hybrid precoding [10]. Hybrid precoding is divided in two parts, one is the digital precoder is used to remove the interference between the data streams and the other part is analog beam former used to gain the more antenna gain. Basically Hybrid beamforming structure is two types, they are spatially sparse hybrid precoding and successive interference cancelation (SIC). Spatially sparse precoding is the fully connected architecture [11] in which the every RF chain is connected to all base station antennas through the phase shifts. SIC is the sub connected architecture [12] in which RF chain is connected to only subset of antennas. Several authors carry out research on hybrid precoding. Considering the hardware limitations by using more RF chains and analog to digital converters precoding in [13] is implemented by using phase shift networks and finite precision ADCs for processing the signal at receiver. In [14] DFT based processing is used for hybrid precoding where the base station knows perfectly about the channel and zero forcing method is used for precoding to get better spectral efficiency. In this paper we are designing precoder by taking input as optimal unconstrained precoder, and combining is designed by considering MMSE receiver.

The subsequent sections of this paper is arranged as follows. Introduction of mmwave massive mimo system and channel model is provide in section II. In section III, designing of preocder using optimal unconstrained preocder is explained. Combiner design for mmwave massive MIMO using MMSE receiver is presented in section IV. Simulation results are presented for different types of antenna array dimensions in section V. Conclusion of this paper is presented in section VI.

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II. SYSTEM AND CHANNEL MODEL

Let us consider single user mm wave massive MIMO system which is shown in Fig. 1 in [16] contains transmitter with N_T antennas, receiver with N_R antennas. Here transmitter transmits the N_S data streams to the receiver. To allow multi data stream transmission transmitter is endow with N_T^{RF} chains with condition $N_S \leq N_T^{RF} \leq N_T$. In this architecture transmitter applies baseband precoder $\mathrm{F_B}$ of size $N_T^{RF} \times N_S$ by using N_T^{RF} RF chains, followed by RF precoder $\mathrm{F_R}$ of size $N_T \times N_T^{RF}$ and it is implemented by using phase shift networks. Input data stream S is the size of $N_S \times 1$ and it should satisfy the condition $\mathrm{E[SS^*]} = \frac{\mathrm{I_{Ns}}}{\mathrm{N_S}}$. After precoding the signal which is to be transmitted to receiver is in form of discrete time can be defined as $\mathrm{X = F_R F_B S}$. Elements of the RF precoder has to satisfy $\left(\mathrm{F_R^{(i)} F_R^{(i)*}}\right)_{l,l} = N_T^{-1}$ that means all elements in the RF preocder matrix should have equality in norm. Power constraint of transmitter is done by normalizing baseband precoder and the condition to satisfy is $\left\|\mathrm{F_R F_B}\right\|_F^2 = N_s$.

For our simplicity, we are considering the narrowband system [15] because the mm wave systems coherence bandwidth is large. Then received signal $N_R \times 1$ can be represented as

$$y = \sqrt{\rho} H \mathrm{F_R F_B S} + \mathrm{n} \qquad (1)$$

Here H is channel matrix of size $N_R \times N_T$ and it has to satisfy condition $\mathrm{E}[\|\mathrm{H}\|_F^2] = N_T N_R$, n is noise with mean 0 and variance σ_n^2, $CN(0, \sigma_n^2)$ and ρ is average of received power. In mm wave to allow precoding the transmitter and receiver should perfectly familiar with channel H. In real time systems receiver should know the channel state information by doing some training methods and that information shares with transmitter. Receiver is constructed with N_R^{RF} RF chains with size of $N_S \leq N_R^{RF} \leq N_R$ and phase shifters to receive the precoding signal and that is

$$\tilde{\mathrm{y}} = \sqrt{\rho} \mathrm{W_B^* W_R^* H F_R F_B S} + \mathrm{W_B^* W_R^* n} \qquad (2)$$

Here $\mathrm{W_R}$ is the RF combining matrix of size $N_T \times N_T^{RF}$ and $\mathrm{W_B}$ is the baseband combiner matrix at receiver side similar to base station. Elements of $\mathrm{W_{RF}}$ matrix should have equal norm same as analog RF precoding. When the signal is transmitted over the mmwave massive MIMO channel, spectral efficiency can be represented as

$$R = \log_2\left(\left| I_{N_S} + \frac{\rho}{N_S} R_n^{-1} \mathrm{w_B^* W_R^* H F_R F_B} \times \mathrm{F_B^* F_R^* H W_R W_B} \right|\right) \qquad (3)$$

Where R_n is matrix which contains the noise covariance and it will get after the combining procedure at receiver side. Here we are considering narrowband channel because path loss is more in mmwave system due to high carrier frequency and antenna correlation is happened when large number of antennas are placed at transmitter. Combination of mmwave and massive MIMO causes fading. In narrow band clustered channel model, channel matrix H is defined as sum total of N_{SC} scattering cluster paths and N_{PP} propagation paths.

$$H = \eta \sum_{j,l} \alpha_{jl} \wedge_R (\phi_{jl}^R, \theta_{jl}^R) \wedge_T (\phi_{jl}^T, \theta_{jl}^T) a_R(\phi_{jl}^R, \theta_{jl}^R) a_T(\phi_{jl}^T, \theta_{jl}^T)^* \qquad (4)$$

Where η is normalization factor, defined as $\eta = \sqrt{N_T N_R / N_{SC} N_{PP}}$ and α_{jl} is the gain of the lth ray and jth scattering cluster. Where $\phi_{jl}^R, \theta_{jl}^R$ are azimuthal and elevation angles of arrival. $\phi_{jl}^T, \theta_{jl}^T$ are azimuthal and elevation angles of departure. $a_R(\phi_{jl}^R, \theta_{jl}^R), a_T(\phi_{jl}^T, \theta_{jl}^T)$ are the transmitter and receiver antenna array responses over azimuthal and elevation angles. Azimuthal and elevation angle of departure at transmitter side and receiver side are randomly distributed with mean cluster of angles and angular spread (kd). More number of distributions are proposed but here we are using the laplacian distribution, because it is good for propagation channel models. In order to find the transmitter antenna array response over azimuthal and elevation angles we are taking the unit gain condition of $\phi_{jl}^T \in \left[\phi_{\min}^T, \phi_{\max}^T\right]$ and $\theta_{jl}^T \in \left[\theta_{\min}^T, \theta_{\max}^T\right]$. Similarly same we are doing for receiver antenna array response. Antenna array responses are function of transmitter and receiver antenna array structure. There are two types of antenna array structures existing, they are uniform linear array and uniform planar array. ULA is only two dimensional and UPA is 3D. Array response for N elements with respect to y axis is defined as

Mathematical representation of the FBMC the transmitted signal is

$$a_{ULA} = \frac{1}{\sqrt{N}} \left[1, e^{jkd\sin(\phi)}, \ldots\ldots, e^{j(N-1)kd\sin(\phi)}\right]^T \qquad (5)$$

Array response vector for N=W*H elements with respect to y and z axis defined as

$$a_{UPA}(\phi, \theta) = \frac{1}{\sqrt{N}} \begin{bmatrix} 1, .., e^{jkd(m\sin(\phi)\sin(\theta) + n\cos(\theta))}, ..., \\ e^{j(W-1)kd\sin(\phi)\sin(\theta) + (H-1)\cos(\theta))} \end{bmatrix}^T \qquad (6)$$

Here m, n are the index for y and z axis.

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III. SPARSE PRECODING FOR SINGLE USER MMWAVE MASSIVE MIMO CHANNEL

We have to design mmwave massive MIMO precoder to get maximum spectral efficiency. In this we are not directly maximizing optimization of preocder to increase spectral efficiency by using equation (3), it requires joint optimization because, existence of four matrix variables of precoder and combiner to maximize spectral efficiency. So we are decoupling the transmitter and receiver optimization problem for simplification and concentrating on precoder design initially. We are designing precoder to maximize the mutual information obtained by Gaussian signal over clustered channel.

$$I(\mathrm{F_R}, \mathrm{F_B}) = \log_2\left(\left| \mathrm{I} + \frac{\rho}{\mathrm{N_S}\sigma_n^2} \mathrm{H F_R F_B F_B^* F_R^* H^*} \right|\right) \quad (7)$$

Designing of precoder problem in mmwave massive MIMO by using optimization technique can be expressed as

$$(\mathrm{F_R^{opt}}, \mathrm{F_B^{opt}}) = \underset{\mathrm{F_B, F_R}}{\mathrm{argmax}}\ \mathrm{I}(\mathrm{F_R}, \mathrm{F_B}),$$
$$\mathrm{s.t}\ \ \mathrm{F_R} \in F_R$$
$$\left\| \mathrm{F_R} \mathrm{F}_B \right\|_F^2 = N_S \quad (8)$$

Where f_R is set of realizable precoders. There are no solutions to found for approximation (8) for feasibility constraint of nonconvex. So here we are finding near optimal precoder to the approximations (8).here we have to find optimal unconstrained preocoder of channel $\mathrm{F_{opt}}$. To do the precoder we have to find singular value decomposition of channel. Mutual information of SVD of channel can be expressed as

$$I(\mathrm{F_R}, \mathrm{F_B}) = \log_2\left(\left| \mathrm{I} + \frac{\rho}{\mathrm{N_S}\sigma_n^2} \Sigma^2\, \mathrm{V^* F_R F_B F_B^* F_R^* V} \right|\right) \quad (9)$$

Here Σ is the diagonal matrix consisting two elements Σ_1 and Σ_2 of size $N_S \times N_S$. And V= [$\mathrm{V_1}$ $\mathrm{V_2}$] of dimension $\mathrm{N_T \times N_S}$. Optimal unconstrained precoder of channel H is defined as $\mathrm{F_{opt}} = \mathrm{V_1}$. Here we are considering the eigen values of matrix $\mathrm{I_{N_S}}$ are small [16] that means $\mathrm{V_1^* F_R F_B F_B^* F_R^* V_1}$ are small. In mmwave massive MIMO precoding it is expressed as $\mathrm{V_1^* F_R F_B} \approx \mathrm{I_{N_S}}$. And singular values of $\mathrm{V_2^* F_R F_B}$ are small that means $\mathrm{V_2^* F_R F_B} \approx 0$. Finally the mutual information can be expresses as

$$I(\mathrm{F_R}, \mathrm{F_B}) = \log_2\left(\left| \mathrm{I_{N_S}} + \frac{\rho}{N_S \sigma_n^2} \Sigma_1^2 \right|\right) - \left(N_S - \left\| \mathrm{V_1^* F_R F_B} \right\|_F^2 \right) \quad (10)$$

Here to get maximum mutual information we have to maximize the $tr(\mathrm{V_1^* F_R F_B})$. Instead of maximizing the trace we can also minimize $\left\| \mathrm{F_{opt}} - \mathrm{F_R F_B} \right\|_F$. Precoder problem can be restated as

$$(\mathrm{F_R^{opt}}, \mathrm{F_B^{opt}}) = \underset{\mathrm{F_B, F_R}}{\mathrm{argmin}} \left\| \mathrm{F_{opt}} - \mathrm{F_R F_B} \right\|$$
$$\mathrm{s.t}\ \ \mathrm{F_R} \in F_R$$
$$\left\| \mathrm{F_R} \mathrm{F}_B \right\|_F^2 = N_S \quad (11)$$

Here set of precoders are restricted to take from the array response of elevation and azimuthal angle of departure. And the above problem (11) can be rewrite as

$$(\mathrm{F_R^{opt}}, \mathrm{F_B^{opt}}) = \underset{\mathrm{F_B, F_R}}{\mathrm{argmin}} \left\| \mathrm{F_{opt}} - \mathrm{F_R F_B} \right\|$$
$$\mathrm{s.t}\ \ \mathrm{F_R^j} \in \{a_T(\phi_{jl}^T, \theta_{jl}^T), \forall j, l\}$$
$$\left\| \mathrm{F_R} \mathrm{F}_B \right\|_F^2 = N_S \quad (12)$$

Algorithm to find spatial sparse precoding using orthogonal matching pursuit

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Input: $\mathrm{F_{opt}}$

$\mathrm{F_R}$ =empty matrix

$\mathrm{F_{re}} = \mathrm{F_{opt}}$

for $i \le N_T^{RF}$ do

$\psi = \mathrm{A_T^* F_{re}}$

$k = \underset{l=1,\ldots,N_{CS}N_{PP}}{\arg\max}\, (\varphi\varphi^*)_{l,l}$

$\mathrm{F_R} = \left[\mathrm{F_R 1 A_T^k}\right]$

$\mathrm{F_B} = (\mathrm{F_R^* F_R})^{-1} \mathrm{F_R^* F_{opt}}$

$\mathrm{F_{re}} = \dfrac{\mathrm{F_{opt}} - \mathrm{F_R F_B}}{\left\| \mathrm{F_{opt}} - \mathrm{F_R F_B} \right\|_F}$

end for

$\mathrm{F_B} = \sqrt{N_S}\, \dfrac{\mathrm{F_B}}{\left\| \mathrm{F_R F_B} \right\|_F}$

return $\mathrm{F_R}$ $\mathrm{F_B}$

IV. SPARSE COMBINER DESIGN FOR SINGLE USER MMWAVE MASSIVE MIMO CHANNEL

In the previous section we have discussed about the precoder design that increases the mutual information at transmitter side. In this section we are going to design the combiner. Receiver decodes the signal with help of $N_R \times 1$ dimension receive signal. In massive MIMO systems using such type of decoder increases complexity, so to overcome this problem we are using MMSE receiver. Here we have to design combiners, which reduces the mean square error between transmitted signal and processed signal. Combiner problem can be expressed as

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$$(W_R^{opt}, W_B^{opt}) = \underset{W_R, W_B}{\operatorname{argmax}} \; E\left[\left\|S - W_B^* W_R^* y\right\|_2^2\right]$$
$$s.t \;\; W_R \in \mathcal{W}_R \qquad (13)$$

$\mathcal{W}_R$ is the set of combiners and exact solution for the problem (13) can get by using MMSE [17] and the solution is

$$W_{MMSE}^* = \frac{1}{\sqrt{\rho}}(F_B^* F_R^* H^* H F_R F_B + \frac{\sigma_n^2 N_S}{\rho} I N_S)^{-1} F_B^* F_R^* H^* \qquad (14)$$

By using MMSE we can rewrite the combiner optimization problem (13) as follows

$$(W_R^{opt}, W_B^{opt}) = \underset{W_R, W_B}{\operatorname{argmin}} \left\| E\left[yy^*\right]^{0.5} (W_{MMSE} - W_R W_B \right\|_F$$
$$s.t \;\; W_R \in \mathcal{W}_R \qquad (15)$$

Algorithm to find MMSE spatial sparse combining using orthogonal matching pursuit

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Input: W_{MMSE}

1: W_R =empty matrix

2: $W_{re} = W_{MMSE}$

3: for $i \le N_R^{RF}$ do

4: $\psi = A_R^* E[yy^*] W_{re}$

5: $k = \underset{l=1,\ldots,N_{CS}N_{PP}}{\arg\max} \, (\varphi\varphi^*)_{l,l}$

6: $W_R = \left[W_R 1 A_R^k\right]$

7: $W_B = (WE[yy^*]W_R)^{-1} W_R^* E[yy^*] W_{MMSE}$

8: $W_{re} = \dfrac{W_{MMSE} - W_R W_B}{\left\|W_{MMSE} - W_R W_B\right\|_F}$

9: end for

10: return $W_R \; W_B$

V. SIMULATION RESULTS

In this section we are going to discuss the simulation results obtained by doing the precoding in section iii and combining in section IV. Here we are taking the scattering clusters 8 and propagation paths 10 per each cluster in channel. Fig.1. shows the mmwave massive MIMO spectral efficiency with the antenna array of 64 transmitters and 16 receivers with one input data stream. Here both transmitter and receiver are using the 4 RF chains for precoding and combining. From simulation results it is observed that spectral efficiency is better for optimal unconstrained precoder and sparse precoding combining than beam steering. In Fig.2.shows the mmwave massive MIMO spectral efficiency with the antenna array of 256 transmitters and 64 receivers with one input data stream. Here both transmitter and receiver are using the 6 RF chains for precoding and combining. From simulation results it is observed that spectral efficiency is better for optimal unconstrained precoder and sparse precoding combining than beam steering In Fig.3.shows the mmwave massive MIMO spectral efficiency with the antenna array of 64 transmitters and 64 receivers with one input data stream. Here both transmitter and receiver are using the 4 RF chains for precoding and combining. From graph it is observed that spectral efficiency is better for optimal unconstrained precoder and sparse precoding combining than beam steering. Fig.4. shows the mmwave massive MIMO spectral efficiency with the antenna array of 1024 transmitters and 256 receivers with one input data stream. Here both transmitter and receiver are using the 6 RF chains for precoding and combining. From simulation results it is observed that spectral efficiency is better for optimal unconstrained precoder than sparse precoding combining and beam steering.

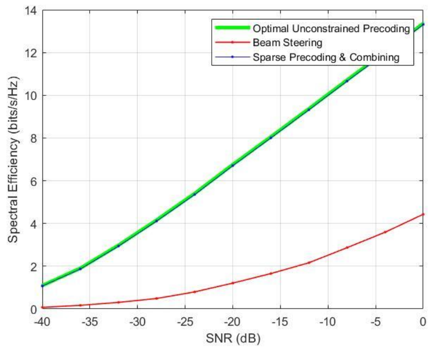

Fig.1. mmwave massive MIMO spectral efficiency for different precoding algorithms for 64 transmitters and 16 receivers.

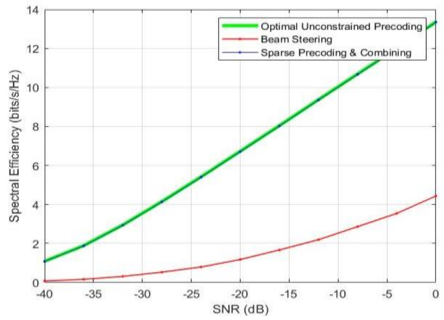

Fig.2. mmwave massive MIMO spectral efficiency for different precoding algorithms for 256 transmitters and 64 receivers.

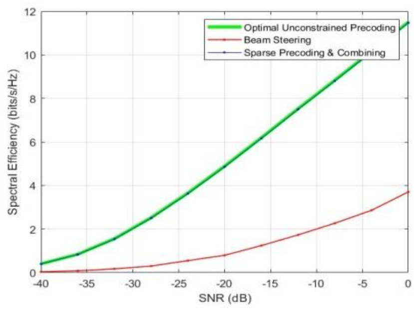

Fig.3. mmwave massive MIMO spectral efficiency for different precoding algorithms for 64 transmitters and 64 receivers.

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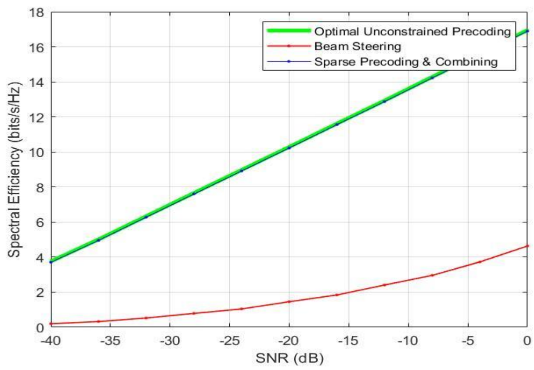

Fig.4. mmwave massive MIMO spectral efficiency for different precoding algorithms for 1024 transmitters and 256 receivers

VI. CONCLUSION

In this paper we have designed precoding and combining for single user mmwave massive MIMO system. Here we have used spatial structure of channel and formulated the precoding problem as sparse nature, combiner is designed by using the MMSE receiver. We have simulated the both precoding and combining problems for different types of antenna arrays using different precoding algorithms and simulation results shows that spectral efficiency is improved by increasing the antenna array size. And from simulation results it is observed that optimal unconstrained precoder solution is close to the sparse precoding and combining algorithm using orthogonal matching pursuit, and also it is observed that the spectral efficiency is improved when antenna array size is increased.

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