Reliability for a Multicomponent System Using Mixture of Two Weibull Distributions

P. Ashok, N. Swathi, M. Tirumala Devi, T. S. Uma Maheswari

ABSTRACT: The reliability is derived for a multi component Series system, Parallel system and standby system is considered for where stress-strength follow weibull distribution. The general expression for the reliability of a multi component standby system is obtained and the system reliability is computed numerically for different values and parameters.

Key words: Weibull Distribution, Series system, Parallel system, Stress-strength Model, Standby System.

I. INTRODUCTION

If X denotes the strength of a component and Y is the stress imposed on it. The component operates as long Y is less than X and the Reliability of the component may therefore be defined as R=(X<Y) The Probability of the failure of a system depends upon the stress and strength of the system .Kapur and Lamberson [1]. The reliability of an n-cascade system with stress attenuation was proposed by Pandit and Sriwastav [2].The reliability for multi component systems when stress-strength follows exponential distributions. Sandhya and Uma mheshwaei [3].Sriwastav and Kokati [4] used cascade system for reliability estimation by considering that the components stress-strength identically distributed random variable. The purpose of this paper is to study the variations in system reliability for different parameter values in a multicomponent strength-stress based on X and Y being two independent random variables and follow weibull distribution.

II. STATISTICAL MODEL

2.1. Reliability of the series system

Let the random variable $X_1, X_2, \ldots, X_n$ be the strengths of the n components arranged in series and Y is the common stress imposed on the n components respectively with the (p.d.f) $f_i(x)$ and $g(y)$ then the system reliability R_n of the series system is given by.

$$R_n = P(Y < \min(X_1, X_2 \cdots X_n))$$

2.2. Reliability of the parallel system

Let $X_1, X_2, \ldots, X_n$ be the strengths of the n components arranged in parallel and Y be the stress imposed on the n components respectively with the (p.d.f) $f_i(x)$ and $g(y)$ then the system reliability R_n of the parallel system is given by.

$$R_n = P(Y < \max(X_1, X_2 \cdots X_n))$$

2.3. Reliability of the standby system

Consider a system of n-components, out of which only one is working under the impact of stress and the remaining (n-1) components are standbys. Whenever the working component fails, one from standby components takes the place of a failed component and is subjected to impact of stress then the system works. The system fails whenever all the components fail. Let $X_1, X_2, \ldots X_n$ be the strengths of the n components arranged in order of activation in the system. And let $Y_1, Y_2 \cdots Y_n$ the stresses on the n components respectively then the system reliability R_n of the system is given by.

$$R(n) = P\left[\left(\bigcap_{i=1}^{n-1} X_i < Y_i\right) \bigcap (X_n > Y_n)\right]$$

Let $f_i(x)$ and $g_i(y)$ are the probability density functions of X_i & Y_i $i = 1,2,\cdots n$ respectively then.

$$R(n) = \int_{-\infty}^{\infty} F_1(y) g_1(y)dy \int_{-\infty}^{\infty} F_2(y) g_2 dy \cdots \int_{-\infty}^{\infty} \overline{F_n}(y) g_n(y)dy$$

Where $F_i(y) = \int_0^y f_i(x)dx$ **and** $\overline{F_i}(y) = 1 - F_i(y)$

III. RELIABILITY COMPUTATIONS

Let X is the strength and Y is the stress of a system with p.d.f of weibull distribution is given by.

$$f_i(x) = \lambda_i \alpha x^{\alpha-1} e^{-\lambda_i x^\alpha} \quad x > 0, \lambda_i, \alpha > 0, i = 1,2,\cdots n$$

$$g_i(y) = \mu_i \alpha y^{\alpha-1} e^{-\mu_i y^\alpha} \quad y > 0, \mu_i, \alpha > 0, i = 1,2,\cdots n$$

3.1. Series system

The system reliability

$$R_n = \int_0^\infty g(y)\left\{\prod_{i=1}^n [1 - F_i(y)]\right\}dy$$

$$= \int_0^\infty \mu \alpha y^{\alpha-1} e^{-\mu y^\alpha} \left\{\prod_{i=1}^n e^{-\lambda_i y^\alpha}\right\}dy$$

Revised Manuscript Received on January 25, 2019.

P. Ashok, Department of Mathematics, Kakatiya University, Warangal, Telangana State, India.
N. Swathi, Department of Mathematics, Kakatiya University, Warangal, Telangana State, India.
M. Tirumala Devi, Department of Mathematics, Kakatiya University, Warangal, Telangana State, India.
T. S. Uma Maheswari, Department of Mathematics, Kakatiya University, Warangal, Telangana State, India.

System reliability of n components arranged in series is

$$R_n = \frac{\mu}{\mu + \sum_{i=1}^{n} \lambda_i}$$

3.2. Parallel system

$$R_n = 1 - \int_0^\infty \left[g(y) \prod_{i=1}^{n} F_i(y) \right] dy$$

$$= 1 - \int_0^\infty \mu \alpha y^{\alpha-1} e^{-\mu y^\alpha} \prod_{i=1}^{n} (1 - e^{-\lambda_i y^\alpha}) dy$$

System reliability of n components arranged in parallel is

$$R_n = \sum_{i=1}^{n} \frac{\mu}{\mu + \lambda_i} - \sum_{i<j=1}^{n} \frac{\mu}{\mu + \lambda_i + \lambda_j} + \cdots + (-1)^{n+1} \frac{\mu}{\mu + \sum_{i=1}^{n} \lambda_i}$$

3.3 Standby systems

3. 1. Strength and Stress follows Weibull distribution.

$$R(1) = \int_0^\infty \overline{F_1}(y) g_1(y) dy$$

$$= \int_0^\infty e^{-\lambda_1 y^\alpha} \mu_1 \alpha y^{\alpha-1} e^{-\mu_1 y^\alpha} dy$$

$$R(1) = \frac{\mu_1}{\lambda_1 + \mu_1}$$

$$R(2) = \left[\int_0^\infty F_1(y) g_1(y) dy \right] \left[\int_0^\infty \overline{F_2}(y) g_2(y) dy \right]$$

$$= \left[\int_0^\infty \left[1 - e^{-\lambda_1 y^\alpha} \right] \mu_1 \alpha y^{\alpha-1} e^{-\mu_1 y^\alpha} dy \right] \left[\int_0^\infty \left[e^{-\lambda_2 y^\alpha} \right] \mu_2 \alpha y^{\alpha-1} e^{-\mu_2 y^\alpha} dy \right]$$

$$R(2) = \left[\frac{\lambda_1}{\lambda_1 + \mu_1} \right] \left[\frac{\mu_2}{\lambda_2 + \mu_2} \right]$$

$$R(3) = \left[\int_0^\infty F_1(y) g_1(y) dy \right] \left[\int_0^\infty F_2(y) g_2(y) dy \right] \left[\int_0^\infty \overline{F_3}(y) g_3(y) dy \right]$$

$$= \left[\int_0^\infty \left[1 - e^{-\lambda_1 y^\alpha} \right] \mu_1 \alpha y^{\alpha-1} e^{-\mu_1 y^\alpha} dy \right] \left[\int_0^\infty \left[1 - e^{-\lambda_2 y^\alpha} \right] \mu_2 \alpha y^{\alpha-1} e^{-\mu_2 y^\alpha} dy \right] \left[\int_0^\infty e^{-\lambda_3 y^\alpha} \mu_3 \alpha y^{\alpha-1} e^{-\mu_3 y^\alpha} \right]$$

$$R(3) = \left[\frac{\lambda_1}{\lambda_1 + \mu_1} \right] \left[\frac{\lambda_2}{\lambda_2 + \mu_2} \right] \left[\frac{\mu_3}{\lambda_3 + \mu_3} \right]$$

In general

$$R(n) = \left[\prod_{i=1}^{n-1} \left[\frac{\lambda_i}{\mu_i + \lambda_i} \right] \right] \left[\frac{\mu_n}{\lambda_n + \mu_n} \right]$$

3.2. Strength follows weibull distribution and stress follows mixture of two weibull Distributions.

$$R(1) = \int_0^\infty \overline{F_1}(y) g_1(y) dy$$

$$= \int_0^\infty e^{-\lambda_1 y^\alpha} \left(p_1 \mu_1 \alpha y^{\alpha-1} e^{-\mu_1 y^\alpha} + (1 - p_1) \mu_2 \alpha y^{\alpha-1} e^{-\mu_2 y^\alpha} \right) dy$$

$$R(1) = p_1 \frac{\mu_1}{\lambda_1 + \mu_1} + (1 - p_1) \frac{\mu_2}{\lambda_1 + \mu_2}$$

$$R(2) = \left[\int_0^\infty F_1(y) g_1(y) dy \right] \left[\int_0^\infty \overline{F_2}(y) g_2(y) dy \right]$$

$$= \left[\int_0^\infty \left[1 - e^{-\lambda_1 y^\alpha} \right] \left[p_1 \mu_1 \beta y^{\alpha-1} e^{-\mu_1 y^\alpha} + (1 - p_1) \mu_2 \alpha y^{\alpha-1} e^{-\mu_2 y^\alpha} \right] dy \right] \times$$
$$\left[\int_0^\infty \left[e^{-\lambda_2 y^\alpha} \right] \left[p_3 \mu_3 \beta y^{\alpha-1} e^{-\mu_3 y^\alpha} + (1 - p_3) \mu_4 \alpha y^{\alpha-1} e^{-\mu_4 y^\alpha} \right] dy \right]$$

$$R(2) = \left[1 - p_1 \frac{\mu_1}{\lambda_1 + \mu_1} - (1 - p_1) \frac{\mu_2}{\lambda_1 + \mu_2} \right] \times \left[p_3 \frac{\mu_3}{\lambda_2 + \mu_3} + (1 - p_3) \frac{\mu_4}{\lambda_2 + \mu_4} \right]$$

$$R(3) = \left[\int_0^\infty F_1(y) g_1(y) dy \right] \left[\int_0^\infty F_2(y) g_2(y) dy \right] \left[\int_0^\infty \overline{F_3}(y) g_3(y) dy \right]$$

$$= \left[\int_0^\infty \left[1 - e^{-\lambda_1 y^\alpha} \right] \left[p_1 \mu_1 \alpha y^{\alpha-1} e^{-\mu_1 y^\alpha} + (1 - p_1) \mu_2 \alpha y^{\alpha-1} e^{-\mu_2 y^\alpha} \right] dy \right] \times$$
$$\left[\int_0^\infty \left[1 - e^{-\lambda_2 y^\alpha} \right] \left[p_3 \mu_3 \alpha y^{\alpha-1} e^{-\mu_3 y^\alpha} + (1 - p_3) \mu_4 \alpha y^{\alpha-1} e^{-\mu_4 y^\alpha} \right] dy \right] \times$$
$$\left[\int_0^\infty \left[e^{-\lambda_3 y^\alpha} \right] \left[p_5 \mu_5 \alpha y^{\alpha-1} e^{-\mu_5 y^\alpha} + (1 - p_5) \mu_6 \alpha y^{\alpha-1} e^{-\mu_6 y^\alpha} \right] dy \right]$$

$$R(3) = \left[1 - p_1 \frac{\mu_1}{\lambda_1 + \mu_1} - (1 - p_1) \frac{\mu_2}{\lambda_1 + \mu_2} \right] \times \left[1 - p_3 \frac{\mu_3}{\lambda_2 + \mu_3} - (1 - p_3) \frac{\mu_4}{\lambda_2 + \mu_4} \right]$$
$$\times \left[p_5 \frac{\mu_5}{\lambda_3 + \mu_5} + (1 - p_5) \frac{\mu_6}{\lambda_3 + \mu_6} \right]$$

In general

$$R(n) = \left[\prod_{i=1}^{n-1} \left[1 - p_{2i-1} \frac{\mu_{2i-1}}{\lambda_i + \mu_{2i-1}} - (1 - p_{2i-1}) \frac{\mu_{2i}}{\lambda_i + \mu_{2i}} \right] \right] \left[p_{2n-1} \frac{\mu_{2n-1}}{\lambda_n + \mu_{2n-1}} + (1 - p_{2n-1}) \frac{\mu_{2n}}{\lambda_n + \mu_{2n}} \right]$$

3.3. Strength and stress Follows Mixture of two weibull distributions.

$$R(1) = \int_0^\infty \overline{F_1}(y) g_1(y) dy$$

$$= \int_0^\infty \left(p_1 e^{-\lambda_1 y^\alpha} + (1 - p_1) e^{-\lambda_2 y^\alpha} \right) \left(p_1' \mu_1 \alpha y^{\alpha-1} e^{-\mu_1 y^\alpha} + (1 - p_1') \mu_2 \alpha y^{\alpha-1} e^{-\mu_2 y^\alpha} \right) dy$$

$$R(1) = \left[p_1 p_1' \frac{\mu_1}{\lambda_1 + \mu_1} + p_1'(1 - p_1) \frac{\mu_1}{\lambda_2 + \mu_1} + p_1(1 - p_1') \frac{\mu_2}{\lambda_1 + \mu_2} + (1 - p_1)(1 - p_1') \frac{\mu_2}{\lambda_2 + \mu_2} \right]$$

$$R(2) = \left[\int_0^\infty F_1(y) g_1(y) dy \right] \left[\int_0^\infty \overline{F_2}(y) g_2(y) dy \right]$$

$$= \left[\int_0^\infty \left(1 - p_1 e^{-\lambda_1 y^\alpha} - (1 - p_1) e^{-\lambda_2 y^\alpha} \right) \left(p_1' \mu_1 \alpha y^{\alpha-1} e^{-\mu_1 y^\alpha} + (1 - p_1') \mu_2 \alpha y^{\alpha-1} e^{-\mu_2 y^\alpha} \right) dy \right] \times$$
$$\left[\int_0^\infty (p_3 e^{-\lambda_3 y^\alpha} + (1 - p_3) e^{-\lambda_4 y^\alpha}) \left(p_3' \mu_3 \alpha y^{\alpha-1} e^{-\mu_3 y^\alpha} + (1 - p_3') \mu_4 \alpha y^{\alpha-1} e^{-\mu_4 y^\alpha} \right) dy \right]$$

$$R(2) = \left[1 - p_1 p_1' \frac{\mu_1}{\lambda_1 + \mu_1} - p_1'(1 - p_1) \frac{\mu_1}{\lambda_2 + \mu_1} - 1 - p_1(1 - p_1') \frac{\mu_2}{\lambda_1 + \mu_2} - (1 - p_1)(1 - p_1') \frac{\mu_2}{\lambda_2 + \mu_2} \right] \times$$
$$\left[p_3 p_3' \frac{\mu_3}{\lambda_3 + \mu_3} + p_3'(1 - p_3) \frac{\mu_3}{\lambda_4 + \mu_3} + p_3(1 - p_3') \frac{\mu_4}{\lambda_3 + \mu_4} + (1 - p_3)(1 - p_3') \frac{\mu_4}{\lambda_4 + \mu_4} \right]$$

$$R(3) = \left[\int_0^\infty F_1(y) g_1(y) dy \right] \left[\int_0^\infty F_2(y) g_2(y) dy \right] \left[\int_0^\infty \overline{F_3}(y) g_3(y) dy \right]$$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$=\left[\int_0^\infty \left(1-p_1e^{-\lambda_1 y^\alpha}-(1-p_1)e^{-\lambda_2 y^\alpha}\right)\left(p_1'\mu_1\alpha y^{\alpha-1}e^{-\mu_1 y^\alpha}+(1-p_1')\mu_2\alpha y^{\alpha-1}e^{-\mu_2 y^\alpha}\right)dy\right]\times$$

$$\left[\int_0^\infty (1-p_3e^{-\lambda_3 y^\alpha}-(1-p_3)e^{-\lambda_4 y^\alpha})\left(p_3'\mu_3\alpha y^{\alpha-1}e^{-\mu_3 y^\alpha}+(1-p_3')\mu_4\alpha y^{\alpha-1}e^{-\mu_4 y^\alpha}\right)dy\right]\times$$

$$\left[\int_0^\infty (p_5e^{-\lambda_5 y^\alpha}+(1-p_5)e^{-\lambda_6 y^\alpha})\left(p_5'\mu_5\alpha y^{\alpha-1}e^{-\mu_3 y^\alpha}+(1-p_5')\mu_6\alpha y^{\alpha-1}e^{-\mu_6 y^\alpha}\right)dy\right]$$

$$R(3)=\left[1-p_1p_1'\frac{\mu_1}{\lambda_1+\mu_1}-p_1'(1-p_1)\frac{\mu_1}{\lambda_2+\mu_1}-1-p_1(1-p_1')\frac{\mu_2}{\lambda_1+\mu_2}-(1-p_1)(1-p_1')\frac{\mu_2}{\lambda_2+\mu_2}\right]\times$$

$$\left[1-p_3p_3'\frac{\mu_3}{\lambda_3+\mu_3}-p_3'(1-p_3)\frac{\mu_3}{\lambda_4+\mu_3}-1-p_3(1-p_3')\frac{\mu_4}{\lambda_3+\mu_4}-(1-p_3)(1-p_3')\frac{\mu_4}{\lambda_4+\mu_4}\right]\times$$

$$\left[p_5p_5'\frac{\mu_5}{\lambda_5+\mu_5}+p_5'(1-p_5)\frac{\mu_5}{\lambda_6+\mu_5}+p_5(1-p_5')\frac{\mu_6}{\lambda_6+\mu_6}+(1-p_5)(1-p_5')\frac{\mu_6}{\lambda_6+\mu_6}\right]$$

In general

$$R(n)=\left[\prod_{i=1}^{n-1}\left[\begin{array}{l}1-p_{2i-1}p_{2i-1}'\frac{\mu_{2i-1}}{\lambda_{2i-1}+\mu_{2i-1}}-(1-p_{2i-1})p_{2i-1}'\frac{\mu_{2i-1}}{\lambda_{2i-1}+\mu_{2i}}+\\ 1-p_{2i-1}(1-p_{2i-1}')\frac{\mu_{2i}}{\lambda_{2i}+\mu_{2i-1}}-(1-p_{2i-1})(1-p_{2i-1}')\frac{\mu_{2i}}{\lambda_{2i}+\mu_{2i}}\end{array}\right]\right]\times$$

$$\left[\begin{array}{l}p_{2n-1}p_{2n-1}'\frac{\mu_{2n-1}}{\lambda_{2n-1}+\mu_{2n-1}}+p_{2n-1}'(1-p_{2n-1})\frac{\mu_{2n-1}}{\lambda_{2n-1}+\mu_{2n}}+\\ p_{2n-1}(1-p_{2n-1}')\frac{\mu_{2n}}{\lambda_{2n}+\mu_{2n-1}}+(1-p_{2n-1})(1-p_{2n-1}')\frac{\mu_{2n}}{\lambda_{2n}+\mu_{2n}}\end{array}\right]$$

IV. NUMERICAL CALCULATIONS

Table-1: System reliabilities of series and parallel systems which follows weibull distribution.

μ_1	λ_1	λ_2	λ_2	R_3	R_3
0.1	0.1	0.1	0.1	0.25	0.75
	0.2	0.2	0.2	0.142	0.542
	0.3	0.3	0.3	0.1	0.421
0.2	0.1	0.1	0.1	0.4	0.9
	0.2	0.2	0.2	0.25	0.75
	0.3	0.3	0.3	0.181	0.631
0.3	0.1	0.1	0.1	0.5	0.95
	0.2	0.2	0.2	0.333	0.847
	0.3	0.3	0.3	0.25	0.75
0.4	0.1	0.1	0.1	0.571	0.971
	0.2	0.2	0.2	0.4	0.9
	0.3	0.3	0.3	0.307	0.821
0.5	0.1	0.1	0.1	0.625	0.982
	0.2	0.2	0.2	0.454	0.930
	0.3	0.3	0.3	0.357	0.868

Figure-1: Series System reliability R_3 for stress-strength follows weibull distribution.

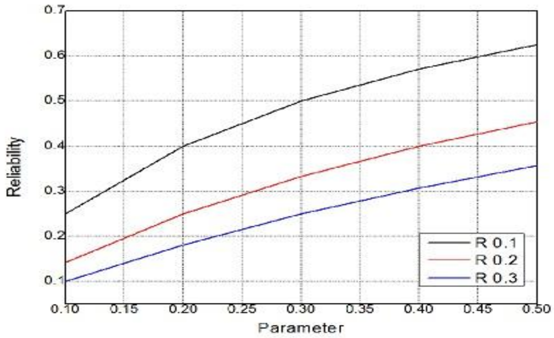


Figure-2: Parallel System reliability R_3 for stress-strength follows weibull distribution.

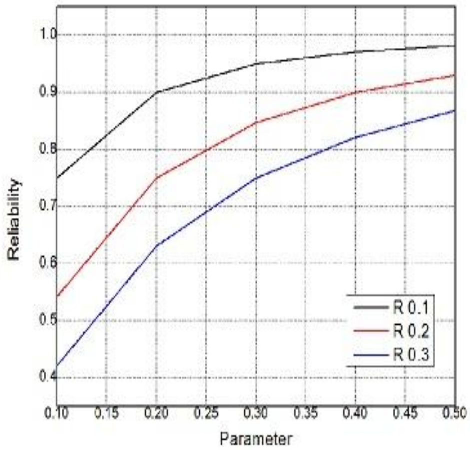
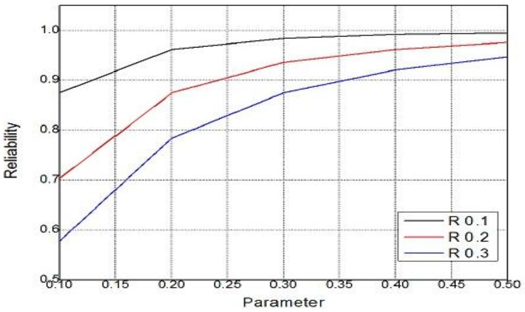


Table-2: Marginal reliabilities system reliabilities R_2, R_3 when strength and stress follow weibull distribution.

μ_1	λ_1	R(1)	R(2)	R(3)	R_2	R_3
0.1	0.1	0.5	0.25	0.125	0.75	0.875
	0.2	0.333	0.222	0.148	0.555	0.703
	0.3	0.25	0.187	0.140	0.327	0.578
0.2	0.1	0.666	0.222	0.074	0.888	0.962
	0.2	0.5	0.25	0.125	0.75	0.875
	0.3	0.4	0.24	0.144	0.64	0.784
0.3	0.1	0.75	0.187	0.046	0.937	0.984
	0.2	0.6	0.24	0.096	0.84	0.936
	0.3	0.5	0.25	0.125	0.75	0.875
0.4	0.1	0.8	0.16	0.032	0.96	0.992
	0.2	0.666	0.222	0.074	0.888	0.962
	0.3	0.571	0.244	0.104	0.815	0.921
0.5	0.1	0.833	0.138	0.0231	0.971	0.995
	0.2	0.714	0.204	0.0583	0.918	0.976
	0.3	0.625	0.234	0.0878	0.859	0..947

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Figure-3: Standby System reliability R_3 for stress-strength follows weibull distribution.



V. CONCLUSION

The general expressions for system reliability have been derived for the Series; Parallel & Standby system models. The system reliability is computed numerically for different values of the stress and strength parameters. It is observed that if the scale parameter α is same for stress and strength random variables the reliability is same as if the stress and strength follow exponential distribution and α does not affect the reliability. Numerical calculations for Marginal reliabilities R (i) and Total reliabilities R_i i=1, 2, 3 have been computed for the above three particular cases when stress and strength are identically distributed. It has been observed from the graphs, the value of reliability increases when strength parameter decreases & the stress parameter increases.

REFERENCES

1. Kapur, K.C. and Lamberson, L.R. (1977): Reliability in Engineering Design, John Wiley and Sons, Inc.
2. Pandit. S.N.N and Sriwastav, G.L. (1975). Studies in Cascade Reliability-I, IEEE Transastions on Reliability, Vol.R-24.No.1p.53-57.
3. Raghava char, A. C.N. kesava Rao, B and pandit S.N.N. (1987).The Reliability of a cascade system with Normal Stress and Strength distribution, AsR.vol.2, p.49-54.
4. Sandhya, K. and Umamaheswari, T.S (2013) Reliability of a multicomponent stress strength model with standby system using mixture of two exponential. Journal of reliability and Statistical Studies, Vol, 6, Issue 2, p.105-113.
5. Sriwastav, G.L and Kakati, M.C. (1981).A Stress-Strength Model with Redundancy, IAPQR Trans.6 No .1, p.21-27.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication