An Implementation of Differential Evolution Algorithm for Optimal Water Allocation Problem

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Abstract*: *Differential evolution algorithm is an optimization technique which is very efficient and simple for global optimization over continuous spaces. This paper applies differential evolution algorithm in water resource allocation and distribution problems in order to allocate water resources in an optimal way. The algorithm considers the optimal allocation as a simulated biological evolution process. The main aim of this paper is to implement differential evolution algorithm, to allocate water resources optimally and to check its efficiency through a case study. The objective is to meet the water demand of the users by minimizing the total water supply from public water source and to encourage the use of other water sources especially rain water harvesting. An optimal water allocation model is considered and the results show that it is simple, accurate in producing the results, adaptable and reliable.

Index Terms: Evolutionary Algorithms, Differential Evolution Algorithm, Mutation

I. INTRODUCTION

Optimal water allocation is to scientifically and rationally distribute water by analysing water resources in a particular region, and to find the present economic status so that the future economic development can be predicted. The aim of optimal water allocation is to achieve economic, ecological and social benefits by utilization of water resources in a sustainable manner, for the harmonized development of resources, economy and environment. It helps in the balanced use of the inadequate resources of water and is capable of providing a consistent source of water for a particular region's agricultural, industrial, life and ecology[1]. Water resources allocation is a nonlinear optimization problem with space-time variability and incorporates the environmental, ecological, sociological and economic factors [2]. A class of direct search algorithm called evolutionary algorithms is extensively used to solve real world optimization problems including data mining problems [3]. One of the most popular evolutionary algorithms is differential evolution algorithm which coordinates all the objective functions to find the optimal solution set [1]. Differential evolution algorithm is a variant of genetic algorithm and is used to solve complex problems were inter dependencies exists between input parameters [4]. Some of the advantages of differential evolution algorithm are easy, quick, easy to use, adjustable for integer and discrete optimization, population based, efficient in nonlinear constraint optimization, stochastic function minimization, optimizes multi-modal search spaces, finds the true global minimum irrespective of the initial parameter values, quick convergence, and uses not many control

parameters (Storn and Price, 1997) [5].For non-linear function optimization Angira and Babu (2005) used differential evolution algorithm. Vasan and Raju (2007) used differential evolution algorithm for Mahi Bajaj Sagar Project in India. Reddy and Kumar (2007) applied multi-objective differential evolution algorithm for optimal cropping pattern in a multicrop irrigation system[5, 6]. This paper aims at implementing differential evolution algorithm for optimal allocation of water resource, and checks its effectiveness through a case study.

II. OPTIMAL WATER ALLOCATION MODEL

The aim of this model is to maximize the region's social benefits and to obtain the equivalent allocation of water resources under a known water-resource and social system. In order to solve the conflicting water supply-demand an examination of the demand and supply with respect to a particular region based on their social aspects, climatic conditions, available water from different sources etc, can be used.

Let us assume that the research district has k sub divisions. Independent water sources for each subdivision is represented using i(k) and the water users by j(k). Let m represent the public water sources. The decision variables represents the amount of water allocated in kth sub-division to user j from public water source m and independent water source i. Represents jth user's water demand in sub division k. Represents the maximum value of water available in subdivision k from independent water source i. Denotes the maximum value of water supply from public water source m to subdivision k. denotes the maximum value of water consumption in sub division k from public water source m. Represents the lower limit of change in demand of water from user j in sub division k and denotes the higher limit of change in demand of water from user j in sub division k [7].

A. Objective Function

The social objective of optimal water allocation is to minimize the volume of regional water allocation so that the water is allocated fairly to all the users. The objective function is described using (1), subject to the following constraints.

$$f(x) = \min\left(\sum_{k=1}^{K}\sum_{j=1}^{j(k)}\left(D_j^k - \left(\sum_{i=1}^{i(k)} X_{ij}^k + \sum_{m}^{M} X_{mj}^k\right)\right)\right) \quad (1)$$

1)Constraints

The constraints associated with the amount of water supply are given in (2) and (3).

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$$\sum_{i=1}^{i(K)} X_{ij}^{k} \leq W_{i}^{k} \quad (2)$$

$$\sum_{m=1}^{M} X_{mj}^{k} \leq W_{m}^{k} \quad (3)$$

The constraint associated with the consumption of water is given in (4).

$$W(c,k) \leq P_{m}^{k} \quad (4)$$

The constraints associated with the change in supply and demand of water consumption is given in (5).

$$L_{j}^{k} \leq \sum_{i=1}^{i(K)} X_{ij}^{k} + \sum_{m=1}^{M} X_{mj}^{k} \leq H_{j}^{k} \quad (5)$$

Non-negative constraints are given in (6).

$$X_{ij}^{k}, X_{mj}^{k} \geq 0 \quad (6)$$

III. DIFFERENTIAL EVOLUTION ALGORITHM

Differential evolution algorithm was proposed by Rainer and Kenneth Price in the year 1995 to find a solution for polynomial problem. Complex optimization problems which are difficult to solve using conventional mathematical programming methods can be solved using differential evolution algorithm. Differential evolution algorithm is a global optimization, stochastic direct search algorithm which generates the whole set of solutions in a single run through multiple non-inferior solutions by population evolution. From these multiple solutions it is possible to make different choices. Differential evolution algorithm performs global search based on population, encodes real-number, performs mutation operation based on differentiation and uses a one-to-one competition survival technique. After generation of initial population and evaluation; mutation, recombination, evaluation and selection is carried out until the termination condition is met. Differential evolution has close resemblance with other Evolutionary Algorithms like Evolutionary Programming, Genetic Algorithm and Particle Swarm Optimization [8].

Differential evolution algorithm maintains a population of candidate solutions by recombination, evaluation, and selection. Two population members are selected randomly and based on their weighted difference a new candidate solution is generated and this process is called recombination. Differential evolution algorithm individuals are originated based on the notion of chromosome, and parameter vectors of dimension Dare used to represent them. A population of NP real valued vectors of D-dimension is maintained by two arrays. The population of current vector is stored in the primary array and the selected vectors for the next generation in the secondary array [5, 9]. Dimension of the objective space is represented using D and population size by NP. Individuals are represented by D-dimensional vectors, Xi = Xi,1, Xi,2, ----------Xi,D where i = 1, 2, ----- NP is the sequence of the individual. Number of evolution generations is represented using G.

The steps used by differential evolution algorithm are as follows:

Step 1: Initialization

The decision variables are the amount of allocated water to users from different sources of water. A feasible solution set is formed from decision variables by encoding them based on the survival of the fittest principle. This iterative process continues until an optimal solution is generated [10]. Initialization of the swarm is done to begin the optimal searches initial point, which is generated randomly within a given scope. Let the parameter variable scope be

$$X_{ij}^{l} \leq X_{ij} \leq X_{ij}^{u}$$

where i = 1, 2, 3, ------------, NP and j = 1, 2, 3, ------, D. Then the initial population X is given using (7).

$$X_{ij} = X_{ij}^{l} + rand(0,1)\left(X_{ij}^{u} - X_{ij}^{l}\right) \quad (7)$$

Where xi, j is the ith individuals jth variable and is the lower bound and is the upper bound. Uniform random number within 0 and 1 is produced by Rand(0,1) function.

Step 2 : Mutation

The objective is to reduce the amount of water allocated from public source to users based on other sources of water. Mutation maintains the diversity of population and steers the optimization process. The mutant vectors are created by the mutation operation using (8) by perturbing the difference between a randomly selected vector and two other vectors selected randomly [11].

$$V_{i}(g+1) = X_{r_1}(g) + F\left(X_{r_2} - X_{r_3}(g)\right) \quad (8)$$

Where i, r1, r2, r3 are integers whose value ranges from 1 to NP. r1, r2 and r3 are selected randomly for each operation. The deviation variables amplification capacity is controlled by the scale factor, F whose value lies between 0 and 1[12].

The adopted configuration of differential evolution algorithm is described using DE/x/y/z. The solution to be perturbed like best or random is specified using x. Two distinct members of the population is randomly selected and their difference is represented by the difference vector. Difference vector numbers used in the perturbation of x is represented by y. The recombination operator used like binomial, exponential etc., is specified by z. Different mutation strategies has been proposed by R. Stone and K. Price, out of which DE/rand/1 is the most popular and commonly used.

Step 3: Crossover.

The perburbance parameter's diversity can be increased by including the crossover parameter. A crossover operation is performed on viand xi to generate the new undetermined offspring individual ui using (9) and Uji,g+1 is obtained from (10).

$$U_{1i,g+1}, U_{2i,g+1}, ---------- U_{iD,g+1} \quad (9)$$

$$U_{ji,g+1} = \begin{cases} V_{ji,g+1} & ; if\ rand\left(b_j\right) \leq CR\ or\ j = rnb\left(r_i\right) \\ X_{ji,g+1} & ; if\ rand\left(b_j\right) \geq CR\ and\ j \neq rnb\left(r_i\right) \end{cases} \quad (10)$$

Rand (bj) generates the random number between 0 and 1 for the estimated value of j.

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The sequence from 1 to D is generated randomly by rnb(ri), in order to make sure that Uji,g+1containsno less than one parameter from Vji,g+1. CR is the crossover operator, whose value lies between 0 and 1.

Step4: Selection

Selection is used to choose the best vector from the trial vector and the updated target vector by comparing the value of their objective functions. In minimization problems, if the trial vectors value is better than the objective function value, then it is replaced by the simplified value [11]. The selection to the next generation is based on the minimum value of objective function. Based on the principle of greediness a one-to-one comparison is done among the parent Xi,g and the newly generated child individual Ui,g+1. The selection into the next generation is given in (11) where, Xi's is the offspring of Xi for the generation next.

$$X_{i,g+1} = \begin{cases} U_{i,g+1} & ;if\ U_{i,g+1} \le f(X_{i,g}) \\ X_{i,g+1} & ;otherwise \end{cases} = \begin{cases} U_{i,g+1} & ;if\ U_{i,g+1} \le f(X_{i,g}) \\ X_{i,g+1} & ;otherwise \end{cases}$$

Step 5 : Boundary Constraints

Inside the feasible region new individuals are generated by tracing the parameter values based on boundary constraints. For that the new individuals which do not suit the boundary constraints are replaced with a parameter vector generated randomly in the feasible region based on the following statement. If Uji,g+1 is less than or Uji,g+1is greater than then Uji,g+1 is equal to rand(j)($X_j^u - X_j^l$) + X_j^l

Mutation, crossover and selection are repeated until termination conditions are met[1,13].

IV. CASE STUDY

The above model is applied to optimize the allocation of water resources in Bengaluru City. Bengaluru's sustainable economic development and liveability relies on effective and efficient water infrastructure and services. The main aim is to meet the growing demand for water due to increase in population and economic development and to minimize the wastage of water resources. Effective water management presents an excellent opportunity for system optimisation and being more future prepared. BWSSB -Bangalore Water Supply and Sewerage Board is an autonomous Water Utility, whose service area covers approximately 800 square kilometers with more than nine million customers. Bangalore relies severely on river Cauvery for its water supply. Reducing dependence on the River Cauvery for water supplies, and moving towards the conjunctive water management is very important because by 2050, the population of Bangalore City is projected to be more than 20 million, increasing the water demand up to 45 TMC from the existing 20 TMC. To meet this water demand, identification of new water resources and consequent infrastructure creation through careful consideration of technical, environmental, and societal aspects along with prudent and proactive financial decision-making is very important. The region's water sources are classified as public water and independent water sources. Public water source is the water supplied by BWSSB. Independent water sources are groundwater extracted through bore wells and rain water harvested water.

A.Water Data

The primary objectives were to capture publicized suggested water management strategies and to identify the current culture around water use and management in Bengaluru. This information was then used to construct a matrix of initiatives that are needed for the city to effectively deal with its long term water needs. The city's historic approach to increasing demand has been to expand infrastructure and to draw more from its three primary sources of water — surface, ground and rain. Table 1 features Bangalore's population estimation and water demand/supply data for five consecutive decades starting with 2011.

Year	Population (Million)	Water Demand (MLD)	Present Supply (MLD)	Shortfall in Demand (MLD)
2011	8.499	1400	950	450
2021	10.581	2100	1450	650
2031	14.296	2900	2070	1450
2041	17.085	3400	2070	1950
2051	20.561	4100	2070	2650

Table 1: Bengaluru's population estimation and water demand/supply data (Source: bwssb.gov.in)

Table 2 specifies the water quantity estimation under different consumption types. It shows that through the public source only 50.75 % of water could be allocated where in the demand was over 8,740 Lakhs.

Consumption Type	No. of Connections	Consumption (ML)	Percentage of Water Accounted	Demand (Lakhs)
Domestic	740000	16992	44.67	4718
Non Domestic	42100	1623	3.84	1961
Partial Non Domestic	36300	2194	5.2	1291
Industries, BIAL and others	2641	618	1.56	715
Sanitary Connections	49100	0	0	54
Total	870141	21427	50.75	8740

Table 2:Water Quantity Estimation (Source : bwssb.gov.in)

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In order to ensure that the city does not face water crisis the citizens of has to use the water judiciously and save this precious resource by depending on other sources of water, especially rain water harvesting.

V. RESULTS

Z.Y. Yang et al. [15] have proposed the parameters for differential evolution algorithm and the different control parameters used in the algorithm are listed out below.

Initial pop: In the optimization procedure the starting population is represented as a matrix called initial pop where each row represents a population member. It is generated randomly within the lower and upper boundaries.

Population size: Population size should be large enough so that local optima or stagnation can be avoided. If population size is large more computations are required. Based on the problems dimension population size is usually 5 to 10 times the size of the problem's dimension NP = 10 times D, in order to have enough mutation vectors in differential evolution.

Mutation operator (Scale factor): The deviation variables amplification capacity is controlled by the mutation operator F and is relative to the population range. To globalize the search positive values are used and a negative value localizes the search between difference vectors.

Crossover operator: CR value ranges within 0 and 1. Higher success rate is obtained with a lower value of CR and the difference between parent and children are very less. Fast convergence is obtained with a high value of CR and children will not possess any of their parent's parameter values.

Evolution generations: The algorithm terminates based on the maximum number of evolution generations and the optimal solution is the best individual in that generation.

Termination condition: Other than the number of evolution generations differential evolution algorithm also uses other termination criteria's. Usually a threshold value is considered and if the threshold value is less than the objective functions value the program terminates.

The characteristics of objective functions and the value of NP determine the optimal values of F and CR. The speed of convergence of the search process and its robustness is determined by the constants F, CR and NP [1].

The volume of water obtained in sub-division k for ward 1 through rain water harvesting was calculated and is plotted in figure 1 with variation of crossover constant, CR from 0.2 to 0.6 and Mutation factor, F from 0.2 to 1for population size, NP = 10D (D = 100) for strategy DE/rand-1-bin. The minimum value was obtained when CR = 0.2 and F = 0.6. Likewise for all 169 wards under subdivision k was computed. It was noted that when the number of users are more the amount of water obtained varies considerably based on the type of rain water harvesting used.

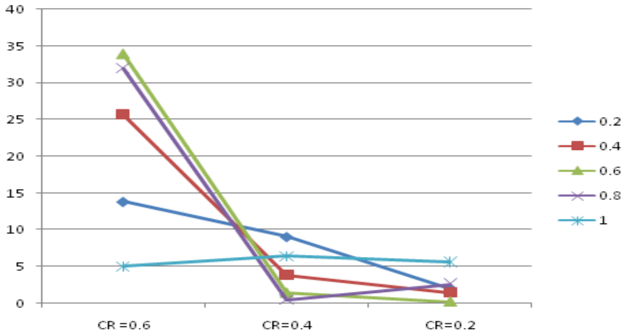

Figure 1: Obtained Results for the function with CR from0.2 to 0.4 and F from 0.2 to 1

Ward Wise Water Consumption and current demand of 199 wards for the month of July 2015 was considered and the differential evolution algorithm was applied on this data to calculate the optimal allocation using rain water harvesting. For the 199 wards the number of users is the number of metered connections and current demand was considered as the user demand. Upper limit is the water consumption and the lower limit is taken as 0. The control parameters used in the algorithm for the function [F] = DE in MATLAB is given in table 3.

Control Parameters	Chosen values
D	10
NP	100
F	0.9
CR	0.5
Gen	100
Mutation Strategy	DE/rand/1/bin

Table 3: Parameter settings for the model

Optimal allocation was calculated and current demand Vs. optimal allocation is given in figure 2. The fit line was constructed with 95% confidence bounds and the Goodness of fit values are SSE: 1.052e+06, R-square: 0.7737, Adjusted R-square: 0.7623, RMSE: 114.7.

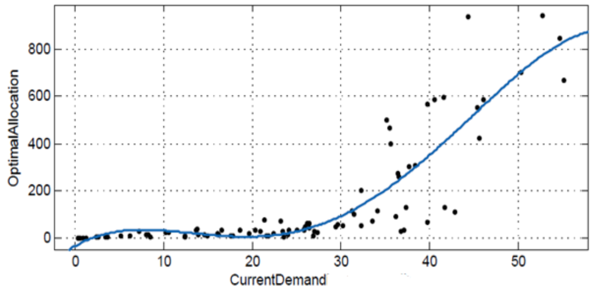

Figure 2: Current Demand Vs Optimal Allocation

Figure 3 shows the optimal water resource allocation for different wards based on the number of metered connections. The number of metered connections and the optimal allocation is represented in the x-axis and y-axis respectively.

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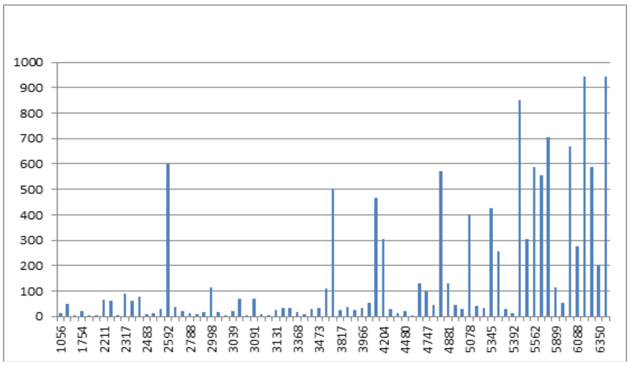

Figure 3: Optimal water resource allocation

The demand supply gap of water from public water source for the year 2020, 2035 and 2050 is represented in figure 4. Using differential evolution algorithm we observed that optimal water allocation was possible to meet the demands of the users and could minimize the water supply gap between the user demand and available water with BWSSB. The supply gap for the years 2020, 2035 and 2050 are -1223, -2325, and -3937 in MLD. Based on the optimal allocation it was observed that by the year 2050 the supply gap from BWSSB can be reduced by at least fifty percent.

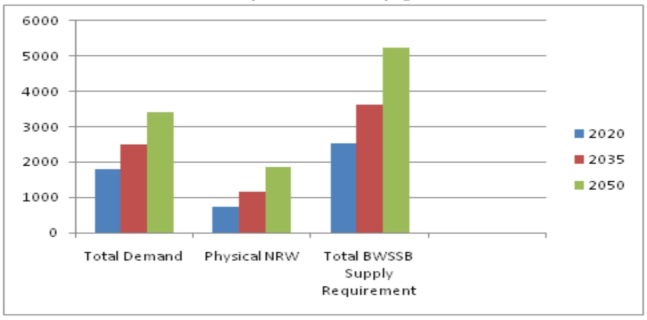

Figure 4: Demand Supply gap

VI. CONCLUSIONS

A regions social and economic development is highly dependent on the water resources available. Water shortages are increasing all over the world due to the population growth, climate change, unaccounted usage and wastage of water. Optimization of water resource allocation is very much required to ease out water shortages. Differential evolution algorithm has been effectively applied in this paper to optimal water allocation problem. The results of the optimal water allocation to the particular case study show good agreement with the reality of water resource exploitation and utilization. The study shows that by depending on other sources of water it is possible to preserve this precious resource by optimally utilizing the different sources of water resources.

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