International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-7 Issue-5S2, January 2019

Zero Forcing in Snake Graph

J. Anitha

***Abstract:** A dynamic coloring of the vertices of a graph G starts with an initial subset S of colored vertices, with all remaining vertices being non-colored. At each discrete time interval, a colored vertex with exactly one non-colored neighbor forces this non-colored neighbor to be colored. The initial set S is called a forcing set (zero forcing set) of G if, by iteratively applying the forcing process, every vertex in G becomes colored. The zero forcing number of G, denoted Z(G), is the minimum cardinality of a zero forcing set of G. In this paper, obtain the zero forcing number for hexagonal chain torus, alternate quadrilateral snake and double quadrilateral snake. AMS Subject Classification--- 05C69, 05C85, 05C90 and 05C20.*

***Keywords:** Zero Forcing Set, Hexagonal Chain Torus, Alternate Quadrilateral Snake, Double Quadrilateral Snake.*

I. INTRODUCTION

The idea of *zero forcing* on graphs is a current thought that is part of a program study on minimum ranks of matrices with particular combinatorial limitations (American Institute of Mathematics 2008). Zero forcing is utilized to study inverse eigen value problems, PMU placement problems, and quantum control problems [1]. Zero forcing is also called *graph infection* or *graph propagation* in the zones identified with quantum dynamics and control theory of quantum mechanical systems [2]. By the monotonous utilization of a similar quantum transformation, this reality has been used to accomplish noise protection, cooling, state preparation, quantum state transfer and in computer science in the context of fast-mixed searching [2].

Recently, there has been a lot of interest in studying the forcing number of graphs for its own sake and its relation to other graph parameters, such as the path cover number, connected domination number, and the chromatic number. Among others, zero forcing number of a graph contain upper bounds in terms of its degrees [3]. It is easy to see that the trivial lower bound on the zero forcing number of a graph is $Z(G) \geq \delta - 1$.

Definition 1.1. [4] *For $v \in V(G)$, the open neighbourhood of v, denoted as $N_G(v)$, is the set of vertices adjacent with v; and the closed neighbourhood of v, denoted by $N_G[v]$, is $N_G(v) \cup \{v\}$. For a set $S \subseteq V(G)$, the open neighbourhood of S is defined as $N_G(S) =^{S} N_G(v)$ and the closed neighbourhood of S is defined as $N_G[S] = N_G(S) \cup S$. For brevity, $v \in S$ we denote $N_G(S)$ by $N(S)$ and $N_G[S]$ by $N[S]$.*

Definition 1.2. [4] *The* zero forcing process is a coloring game on a graph. If u is a blue vertex and exactly one neighbor w of u is white, then change the color of w to blue; this is called the color change rule and we say that u forces w. A zero forcing set for G is a subset of vertices B such that if initially the vertices in B are colored blue and the remaining vertices are colored white, then repeated application of the color change rule can color all vertices of G blue. A minimum zero forcing set is a zero forcing set of minimum cardinality, and the zero forcing number Z(G) of G is the cardinality of a minimum zero forcing set.

II. MAIN RESULTS

In this section, we obtain the zero forcing number for hexagonal chain torus, alternate quadrilateral snake and double quadrilateral snake.

2.1. Linear Chain Torus

Definition 2.1. [5] *Let H^n denote a linear chain of n hexagons. H^n with wraparound edges is called linear chain torus and is denoted by H_T^n.*

Lemma 2.2. *Let G be a linear chain torus H_T^n. Then $Z(G) \geq 4$.*

Proof. Let *S*be a power dominating set of *G*. Suppose $|S| = 3$. Without loss of generality, let $S = \{v_i : 1 \leq i \leq 3\}$. The worst case arises when v_1, v_2, v_3 are adjacent vertices in *G*. Then atleast one vertex $u \in N(S)$, $deg(u) > 2$, a contradiction. Therefore, $Z(G) \geq 4$.

The Algorithm given below computes the zero forcing number in Linear Chain Torus H_T^n. *Zero Forcing Algorithm in Linear Chain Torus*

Input: Linear chain torus $H_T^n$of length $n \geq 4$.

Algorithm: Label the vertices in the 4 rows of the H_T^nas follows: vertices in 1^{st} row as

1,2,...,*n*, 2^{nd} row as $a_1, a_2, \ldots, a_{n+1}$, 3^{rd} row as $a_1', a_2', \ldots, a_{n+1}'$ and 4^{th} row as $1', 2', \ldots, n'$

from left to right. Select the $\{2, 2', a_2, a_2'\}$ in *S* vertices. See Figure 1(*a*).

Output: $Z(H_T^n) = 4$.

Proof of Correctness: Let *S* be the set of vertices labeled $\{2, 2', a_2, a_2'\}$ in *S* . See Figure 1(*a*).

For every vertex in *S* is adjacent to exactly exactly one non-colored vertex. Proceeding inductively, at every inductive step *i*, $i \geq 2$. Now $|S| = 4$. This implies that, $Z(H_T^n)$=4. Hence the proof.

Theorem 2.3. *Let G be a linear chain torus $H_T^n$of length $n \geq 4$. Then $Z(G) = 4$.*

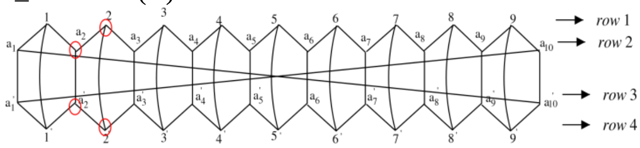


Figure 1: Circled vertices indicate a zero forcing set of Linear chain torus H_T^9

2.2. Double Triangular Snake Graph

A triangular cactus is a connected graph all of whose blocks are triangles. A triangular snake is a triangular cactus whose block-cut point graph is a path.

Manuscript received January 25, 2019.

J. Anitha, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai, Tamilnadu, India

Equivalently, it is obtained from a path $P = v_1,v_2,...,v_{n+1}$ by joining $v_i$and v_{i+1} to new vertices $u_1,u_2,...,u_n$. A triangular snake has $2n$+1 vertices and $3n$ edges, where n is the number blocks in the triangular snake and is denoted by byT_n [6].

A double triangular snake $D(T_n)$ consists of two triangular snake that have a common path. That is, double triangular snake is obtained by joining to new vertices $v_i$and v_{i+1} to new vertices $u_i$and $w_i$for $1 \leq i \leq n$ [6].

Lemma 2.4. *Let G be a graph and H as shown in Figure 2(a) be a subgraph G with* $deg_H w_i = deg_G w_i = 2, \forall i, i = 1,2$. *Then H contains at least one vertex of any zero forcing set of G.*

Proof. Neither u nor v, when monitored, can further monitor any of w_i, i= 1,2, as $deg_H u = deg_H v = 2$.

Lemma 2.5. *Let G be a double triangular snake* $D(T_n)$ *of length* $n \geq 2$. *Then* $Z(G) \geq n$.

Proof. In $D(T_n)$, there are n copies of H as described in Lemma 2.4. Therefore, $Z(G) \geq n$.

The Algorithm given below computes the zero forcing number in double triangular snake $D(T_n)$.

Zero Forcing Algorithm in Double Triangular Snake Input: Double Triangular snake $D(T_n)$ of length $n \geq 2$.

Algorithm: Label the vertices of the diamond snake as $v_1,v_2,...,v_{n+1},u_1,u_2,...,u_n$, $w_1,w_2,...,w_n$ as shown

in Figure 2(b). Select the vertices $\{v_1\} \cup \{u_i\}$, $1 \leq i \leq n$ in S. See Figure 2(b).

Output: $Z(D(T_n)) = n + 1$.

Proof of Correctness: Let S be the set of vertices labeled $\{v_1\} \cup \{u_i\}$, $1 \leq i \leq n$. Then the vertices labeled as $\{u_1, v_1\}$ in S is adjacent to exactly one non-colored vertex say, $\{w_1,v_2\}$. Now change the vertex $\{w_1, v_2\}$ as blue. Then the vertex labeled say, $\{u_2, v_2\}$ is adjacent to exactly one non-colored vertex say, $\{w_2, v_3\}$ colored as blue in the next iterative step. Proceeding inductively, at every inductive step i, $i \geq 3$. Now $S = \{v_1\} \cup \{u_i\}$is a zero forcing set of double triangular snake graph $D(T_n)$. This implies that, $Z(D(T_n)) = n + 1$. Hence the proof.

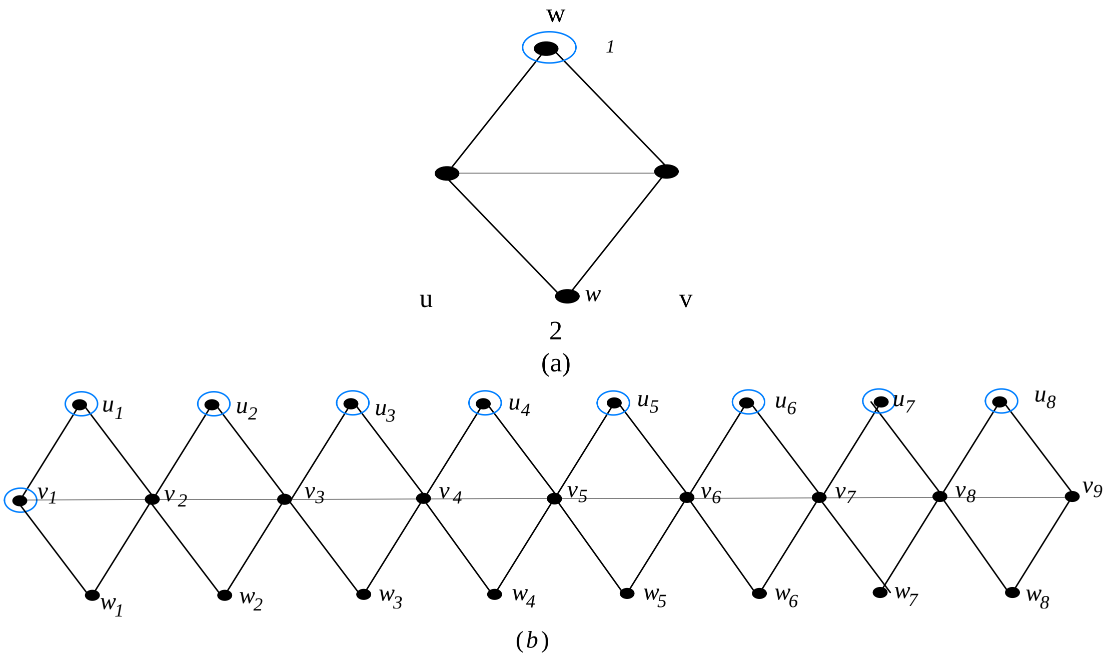


Figure 2: (a) Sub graph H of G (b) Circled vertices indicate a zero forcing set of double triangular snake D(T8)

Theorem 2.6. *Let G be a double triangular snake graph* $D(T_n)$ *of length* $n \geq 2$. *Then* $Z(G) = n + 1$.

Diamond Snake Graph

A Diamond snake $D_n$is obtained by joining to new vertices $v_i$and v_{i+1} to new vertices u_i and w_i for i= 1,2,...,n-1. A Diamond snake has $3n$+1 vertices and $4n$ edges, where n is the number blocks in the diamond snake and is denoted by byD_n. [6]

Lemma 2.7. *Let G be a graph and H as shown in Figure 2(a) be a subgraph G with* $deg_H w_i = deg_G w_i = 2, \forall i, i = 1,2$. *Then H contains at least one vertex of any zero forcing set of G.*

Proof. Neither u nor v, when monitored, can further monitor any of w_i, i= 1,2, as $deg_H u = deg_H v = 2$.

Lemma 2.8. *Let G be a diamond snake* D_n*of length* $n \geq 2$. *Then* $Z(G) \geq n$.

Proof. In D_n, there are n copies of H as described in Lemma 2.7. Therefore, $Z(G) \geq n$.

The Algorithm given below computes the zero forcing number in diamond snake D_n.

Zero Forcing Algorithm in Diamond Snake Input: Diamond snake $D_n$of length $n \geq 2$.

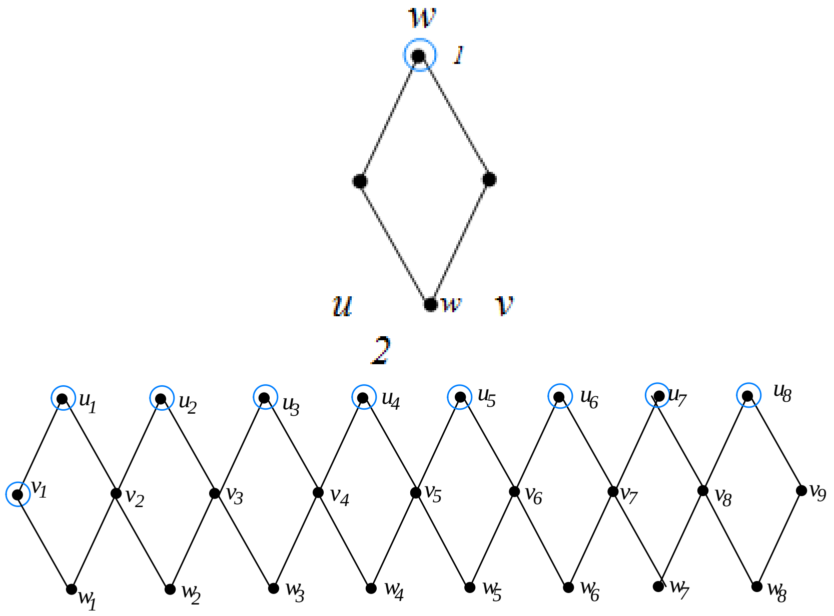
Algorithm: Label the vertices of the diamond snake as $v_1,v_2,...,v_{n+1}$, $u_1,u_2,...,u_n,w_1,w_2,...,w_n$ as

shown in Figure 3(b). Select the vertices $\{v_1\} \cup \{u_i$: $1 \leq i \leq n\}$ in S. See Figure 3(b).

Output: $Z(D_n) = n + 1$.

Proof of Correctness: Let S be the set of vertices labeled $\{v_1\} \cup \{u_i$: $1 \leq i \leq n\}$. Then the vertices labeled as $\{u_1,v_1\}$ in S is adjacent to exactly one non-colored vertex say, $\{w_1,v_2\}$.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication



(*b*)

Figure 3: (a) Sub graph H of G (*b*) Circled vertices indicate a zero forcing set of diamond snake D_8

Now change the vertex $\{w_1,v_2\}$ as blue. Then the vertex labeled say, $\{u_2,v_2\}$ is adjacent to exactly one non-colored vertex say, $\{w_2,v_3\}$ colored as blue in the next iterative step. Proceeding inductively, at every inductive step i, $i \geq 3$. Now $S = \{v_1\} \cup \{u_i\!: 1 \leq i \leq n\}$ is a zero forcing set of diamond snake graph D_n. This implies that, $Z(D_n)) = n + 1$. Hence the proof.

Theorem 2.9. *Let G be a diamond snake graph D_n of length $n \geq 2$. Then $Z(G) = n + 1$.*

Alternate Quadrilateral Snake Graph

An alternate quadrilateral snake $A(Q_n)$ is obtained from a path $u_1,v_1,u_2,v_2,\ldots,u_n,v_n$ by joining u_i, v_i to new vertices w_i, w_i' respectively and then joining w_i and w_i'. That is every alternate edge of a path is replaced by a cycle C_4 [6].

Lemma 2.10. *Let G be a graph and H as shown in Figure 4(a) be a subgraph G with $deg_H w_i = deg_G w_i = 2$, $\forall i$, $i = 1,2$. Then H contains at least one vertex of any zero forcing set of G.*

Proof. Neither u nor v, when monitored, can further monitor any of w_i, $i = 1,2$, as $deg_H u = deg_H v = 2$.

Lemma 2.11. *Let G be a alternate quadrilateral snake $A(Q_n)$ of length $n \geq 4$. Then $Z(G) \geq n$.*

Proof. In $A(Q_n)$, there are n vertex disjoint copies of H as described in Lemma 2.10.

Therefore, $Z(G) \geq n$.

The Algorithm given below computes the zero forcing number in alternate quadrilateral snake $A(Q_n)$.

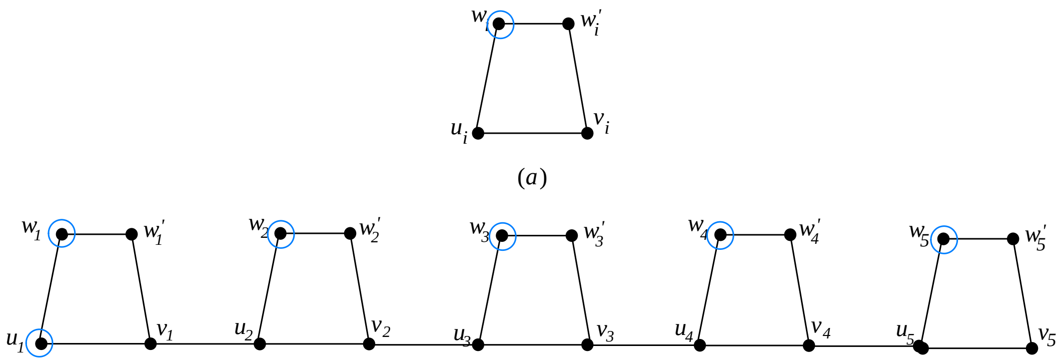
Zero Forcing Algorithm in Alternate Quadrilateral Snake

Input: Alternate quadrilateral snake $A(Q_n)$ of length $n \geq 4$.

Algorithm: Label the vertices of the path as $u_1,v_1,u_2,v_2,\ldots,u_n,v_n$ alternately from left to right and label the vertices by joining u_i,v_i new vertices as w_i, w_i'. Select the vertices $\{u_1\} \cup \{w_i\}$, $1 \leq i \leq n$ in S. See Figure 4(*b*).

Output: $Z(A(Q_n)) = n + 1$.

Proof of Correctness: Let S be the set of vertices labeled $\{u_1\} \cup \{w_i\}$. Then the vertices



(*a*)

(*b*)

Figure 4: (a) Sub graph H of G (*b*) Circled vertices indicate a zero forcing set of alternate quadrilateral snake $A(Q_5)$

labeled as $\{w_1\}$ in S is adjacent to exactly one non-colored vertex say, $\{w_1'\}$. Now change the vertex $\{w_1'\}$ as blue. Then the vertex labeled say, $\{w_1'\}$ is adjacent to exactly one non-colored vertex say, $\{u_1\}$ colored as blue in the next iterative step. Proceeding inductively, at every inductive step i, $i \geq 2$. Now $S = \{u_1\} \cup \{w_i\!: 1 \leq i \leq n\}$ is a zero forcing set of alternate snake graph $A(Q_n)$. This implies that, $Z(A(Q_n)) = n + 1$. Hence the proof.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Theorem 2.12. *Let G be a alternate snake graph $A(Q_n)$ of length $n \geq 4$. Then $Z(G) = n+1$.*

2.3. Double Quadrilateral Snake $D(Q_n)$

A double quadrilateral snake $D(Q_n)$ is obtained form a path $v_1,v_2,\ldots,v_nv_{n+1}$ by joining each of vertices v_i and v_{i+1}, $i= 1,2,\ldots,n-1$ to new vertices u_i, u_i' and to the new vertices w_i and w_i' respectively and adding an edge between each pair of vertices (u_i,w_i) and (u_i', w_i') [7].

Lemma 2.13. *Let G be a graph and H as shown in Figure 3(a) be a subgraph G with*

$$deg_H u_i, w_i, u_i', w_i' = deg_G u_i, w_i, u_i', w_i' = 2, \ \forall i, \ i$$

. Then H contains at least *one vertex of any zero forcing of G.*

Proof. Neither u nor v, when monitored, can further monitor any of u_i, w_i, u_i', w_i' $i = 1,2$, as $deg_H u = deg_H v = 3$.

Lemma 2.14. *Let G be a double quadrilateral snake $D(Q_n)$ of length $n \geq 4$. Then $Z(G) \geq n$.*

Proof. In $D(Q_n)$, there are n vertex disjoint copies of H as described in Lemma 2.13.

Therefore, $\gamma_p(G) \geq n$.

The Algorithm given below computes the power domination number in double quadrilateral snake $D(Q_n)$.

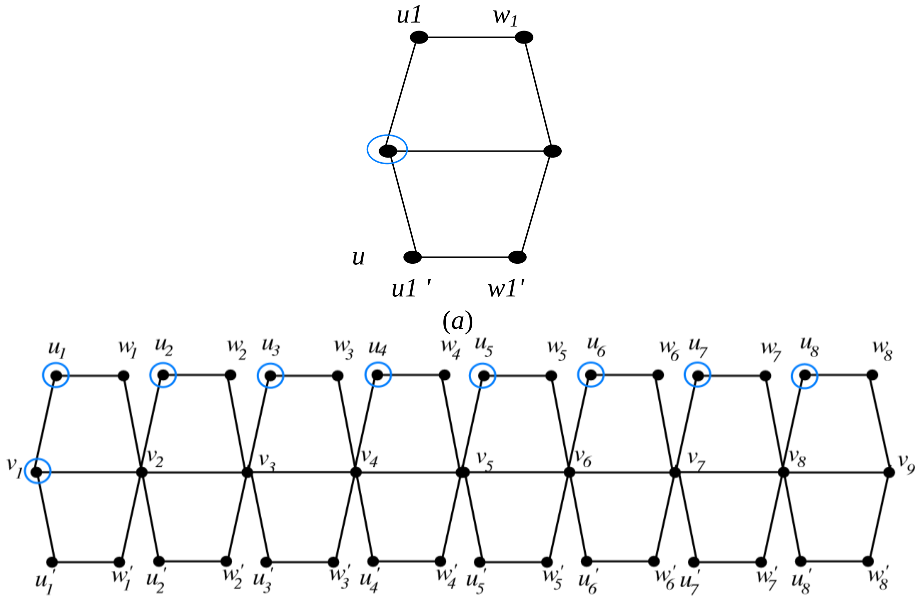
Zero Forcing Algorithm in Double Quadrilateral Snake

Input: Double quadrilateral snake $D(Q_n)$ of length $n \geq 4$.

Algorithm: Label the vertices of a path as $v_1,v_2,\ldots,v_nv_{n+1}$ alternately from left to right and label the vertices by joining each of vertices v_i and $v_{i+1}, i = 1,2,\ldots,n1$ to new vertices u_i,u_i0 and to the new vertices $w_i$and w_i0 respectively. Select the vertices $\{v_1\}\cup\{u_i\}, 1 \leq i \leq n$ in S. See Figure 5(b).

Output: $Z(D(Q_n)) = n + 1$.

Proof of Correctness: Let S be the set of vertices labeled $\{v_1\} \cup \{u_i\}$. Then the vertices



(*a*)

(*b*)

Figure 5: (a) Sub graph H of G (b) Circled vertices indicate a power dominating set of Double quadrilateral snake $D(Q_8)$

labeled as $\{u_1\}$ in S is adjacent to exactly one non-colored vertex say, $\{w_1\}$. Now change the vertex $\{w_1\}$ as blue. Then the vertex labeled say, $\{w_1\}$ is adjacent to exactly one non-colored vertex say, $\{v_1\}$ colored as blue in the next iterative step. Proceeding inductively, at every inductive step i, $i \geq 3$. Now $S = \{v_1\} \cup \{u_i\}$ is a zero forcing set of alternate snake graph $D(Q_n)$. This implies that, $Z(D(Q_n)) = n + 1$. Hence the proof.

Theorem 2.15. *Let G be a double quadrilateral snake $D(Q_n)$ of length $n \geq 4$. Then $Z(G) = n + 1$.*

III. CONCLUSION

In this paper, we have obtained the zero forcing numbers for hexagonal chain torus, alternate quadrilateral snake double triangular snake and double quadrilateral snake.

REFERENCES

1. AIM Special Work Group, Zero forcing sets and the minimum rank of graphs. Linear Algebra Appl. 428(7), 1628-1648 (2008).
2. D. Burgarth, V.Giovannetti, L. Hogben, S. Severini, and M. Young, Logic circuits from zero forcing, Natural Computing 14(3), 485-490 (2015).
3. M. Gentner, L.D. Penso, D. Rautenbach and U.S. Souza, Extremal values and bounds for the zero forcing number, Discrete Applied Mathematics, 214, 196-200 (2016).
4. K. F. Benson, D.Ferrero, M. Flagg, V. Furst, L. Hogben, V. Vasilevskak and B. Wissman, Zero forcing and power domination for graph products, https://arxiv.org/abs/1510.02421.
5. Randic, Milan, Tsukano, Yoko, Hosoya, and Haruo, On enumeration of kekule structures for benzenoid tori, *http ://hdl.handle.net/*10083/841.
6. S.K. Vaidya and R. M. Pandit, Edge domination in various snake graphs, International Journal of Mathematics and Soft Computing, 7(1) (2017), 43-50.
7. A. Manonmania and R. Savithiri, Double quadrilateral snakes on k-odd sequential harmonious labeling of graphs, Malaya Journal of Matematik, 3(4)(2015) 607611.