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An Incentive Inventory Model for Exponential Function of Cost with Maximum Life Time of Deteriorating Products

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***Abstract:** This paper investigates an inventory model for deteriorating products with maximum lifetime and constant demand. Shortages are allowed and backlogged them completely. This model assumes that (i) deteriorating products not only deteriorate continuously, and has a maximum lifetime, and (ii) deteriorating products having exponential function of holding cost, shortage cost and purchasing cost. The goal of this model is to determine the optimal decisions so that the seller's profit function is maximized. We provide simple analytical tractable procedures for deriving the model and give numerical examples to illustrate the solution procedure.*

***Keywords:** Shortages, Deteriorating Items, Exponential Function, Inventory Costs.*

I. INTRODUCTION

In recent years, many researchers have investigated on inventory models for deteriorating items. This is a common factor in daily life. Generally, deterioration indicates the damage, evaporation, spoilage and obsolescence of the products. In supply chain management, it is too difficult to preserve fruits, vegetables, medicines, volatile liquids, blood banks, high-tech products etc., for all business sectors and these types of items deteriorate with time.

In most of the papers, the authors considered constant deterioration rate. But, in real life situation, items may deteriorate due to expiration of their maximum life time i.e., deterioration rate is proportional with time and the maximum life time can be controlled by the production system, i.e., the manufacturer can fix the maximum life time of the product [9]. Bakker et al. [1] analyzed review of inventory systems with deterioration. Ghosh and Chaudhuri [3] developed an EOQ model with a quadratic demand, time-proportional deterioration and shortages in all cycles. Muniappan and Uthayakumar [6] analyzed mathematical analyze technique for computing optimal replenishment policies. Muniappan et al. [7] studied an economic lot sizing production model for deteriorating items under two level trade credit. Sarkar et al. [10] developed an inventory model with variable demand, component cost and selling price for deteriorating items. Goyal and Giri [4] developed production-inventory problem of a product with time varying demand, production and deterioration rates.

Wan-Chih Wang et al. [12] developed seller's optimal credit period and cycle time in a supply chain for deteriorating items with maximum lifetime. Chen and Teng [2] presented retailer's optimal ordering policy for deteriorating items with maximum lifetime under supplier's trade credit financing. Kreng and Tan [5] analyzed optimal replenishment decision in an EPQ model with defective items under supply chain trade credit policy. Sarkar [8] developed an EOQ model with delay in payments and time varying deterioration rate. Sarkar B and Sarkar S [9] studied an improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand. Teng et al. [11] developed economic order quantity model with trade credit financing for non-decreasing demand.

Combining the above arguments, only a few researchers take the maximum lifetime of a deteriorating item into consideration. The model assumes, (i) the deteriorating items not only deteriorate continuously, and has a maximum lifetime and (ii) The products having exponential function of holding cost, shortage cost, purchase cost. Also, above literature review of the mentioned topic and Table 1 show inventory model is not developed under situation in exponential function of various cost. So in this paper we investigate this issue together and derive a comprehensive inventory model to determine the seller's maximum profit.

The rest of the paper is organized as follows. In the next section, assumptions, notations and model formulation is given. In section 3, a numerical examples and sensitivity analysis are given in detail to illustrate the models. Finally conclusion and summary are presented.

II. MODEL FORMULATION

The mathematical model in this paper is developed with the following assumptions and notations:

Assumptions and Notations

1. The demand rate D is constant and known
2. r, F, c_3 denotes the ordering cost, rate of default risk, market price in dollars
3. The holding cost $h(m) = hk_1 e^{-\alpha_1 L_m}$, where k_1,α_1 are positive constants and L_m is maximum life time.
4. The purchase cost $c_1(m) = c_1 k_2 e^{-\alpha_2 L_m}$, where k_2 and α_2 are positive constants and L_m is maximum life time.
5. The shortage cost $s(m) = sk_3 e^{-\alpha_3 L_m}$, k_3 and α_3 are positive constants and L_m is maximum life time.

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6. All deteriorating items have their expiration rates. The physical significance of deterioration rate is the rate to be close 1 when the time is approaching to the maximum life time L_m. To make the problem tractable, we follow the same assumption as is Sarkar [8] as well as in Chen and Teng [2] that the deterioration rate is $\theta(t) = \frac{1}{1+L_m-t}, 0 \le t \le T$
7. Shortages are allowed and backlogged them completely
8. We assume that the replenishment cycle time T is less than or equal to L_m because $\theta(t) \le 1$

Decision Variables

L_m^* : Optimal maximum life time of products
T_1^* : The seller's optimal shortage time in years
T^* : The seller's optimal cycle time in years

The inventory level at time t decreases because of demand and deterioration. So according to this description the changes of inventory with respect to time can be shown using the following differential Equations;

$$\frac{dI_1(t)}{dt} = -D - \theta(t)I_1(t), \ 0 \le t \le T_1 \quad (1)$$

$$\frac{dI_2(t)}{dt} = -D, T_1 \le t \le T \quad (2)$$

with the boundary condition $I_1(0) = 0$, $I_2(T) = 0$, we obtain

$$I_1(t) = -D(1 + L_m - T_1)\log\left[\frac{1+L_m-t}{1+L_m-T_1}\right], 0 \le t \le T_1 \quad (3)$$

$$I_2(t) = D[T - t], T_1 \le t \le T \qquad (4)$$

Therefore, the maximum order quantity is

$$Q_0 = I_1(0) + I_2(0)$$

$$Q_0 = D(1 + L_m - T_1)\log\left[\frac{1+L_m}{1+L_m-T_1}\right] + DT \quad (5)$$

The seller's annual profit function consists of the following components

(i) Average ordering cost is $\frac{r}{T}$

(ii) Average stock holding cost is $\frac{h(m)}{T}\left\{\int_0^{T_1} I_1(t)dt\right\}$

$$= \frac{hk_1e^{-\alpha_1 L_m}D}{T}\left\{(1 + L_m - T_1)\left[T_1 - (1 + L_m)\log\left[\frac{1+L_m-T_1}{1+L_m}\right]\right]\right\}$$

(6)

(iii) Annual purchasing cost or selling price is $\frac{c_1(m)Q_0}{T}$

$$= \frac{c_1k_2e^{-\alpha_2 L_m}D}{T}\left\{(1 + L_m - T_1)\log\left[\frac{1+L_m}{1+L_m-T_1}\right] + T\right\} \quad (7)$$

(iv) Average shortage cost is $\frac{s(m)}{T}\left\{\int_{T_1}^{T} I_2(t)dt\right\}$

$$= \frac{sk_3e^{-\alpha_3 L_m}D}{2T}(T - T_1)^2 \quad (8)$$

(v) Average annual revenue after default risk is $Dc_3(1 - F)$ (9)

Hence, the seller's annual profit per unit time is given by

$$z(L_m, T_1, T) = Dc_3(1 - F) - \frac{r}{T} - \frac{h(m)}{T}\left\{\int_0^{T_1} I_1(t)dt\right\} - \frac{c_1(m)Q_0}{T} - \frac{s(m)D}{2T}\left\{\int_{T_1}^{T} I_2(t)dt\right\}$$

$$= Dc_3(1 - F) - \frac{r}{T} - \frac{Dhk_1e^{-\alpha_1 L_m}}{T}\left\{(1 + L_m - T_1)\left[T_1 - (1 + L_m)\log\left[\frac{1+L_m-T_1}{1+L_m}\right]\right]\right\} - \frac{k_2c_1e^{-\alpha_2 L_m}D}{T}\left\{(1 + L_m - T_1)\log\left[\frac{1+L_m}{1+L_m-T_1}\right] + T\right\} - \frac{Dsk_3e^{-\alpha_3 L_m}}{2T}(T - T_1)^2$$

$= D[c_3(1 - F) - c_1k_2e^{-\alpha_2 L_m}] - \frac{r}{T} - \frac{D}{T}\{hk_1e^{-\alpha_1 L_m}(1 +$ Lm−T1T1+1+Lm−T1log1+Lm−T11+Lmhk11+Lme−α1 Lm+c1k2e−α2Lm+sk3e−α3Lm2T−T12 (10)

Now, by solving the following equations simultaneously we get the optimal decision variables (say L_m^*, T_1^*, T^*) to maximize the seller's total profit.

$$\frac{\partial z(L_m,T_1,T)}{\partial L_m} = 0, \frac{\partial z(L_m,T_1,T)}{\partial T_1} = 0, \frac{\partial z(L_m,T_1,T)}{\partial T} = 0 \quad (11)$$

provided they satisfy the sufficient conditions $\frac{\partial^2 z(L_m,T_1,T)}{\partial {L_m}^2} \le 0$, $\frac{\partial^2 z(L_m,T_1,T)}{\partial {T_1}^2} \le 0$, $\frac{\partial^2 z(L_m,T_1,T)}{\partial T^2} \le 0$. Equation (9) is equivalent to

$$\frac{\partial z(L_m,T_1,T)}{\partial L_m} = 0$$

i.e., $Dc_1k_2\alpha_2e^{-\alpha_2 L_m} - \frac{D}{T}\{-k_1\alpha_1e^{-\alpha_1 L_m}(1 + L_m - T_1)T_1 +$ k1e−α1LmT1+1+Lm−T1log1+Lm−T11+Lm−k1α11+L me−α1Lm−k2α2e−α2Lm+k1e−α1Lm+k11+Lme−α1Lm +k2e−α2Lm−1+Lm−T11+Lm+log1+Lm−T11+Lm−α3k 3e−α3Lm2T−T12=0

$$\text{i.e.,}\ L_m = \frac{\frac{sD\alpha_3k_3(T-T_1)^2}{2}+hDk_1(1-T_1^2)+c_1Dk_2[T_1\alpha_2(T_1-1)-1]-TDc_1k_2\alpha_2}{hDk_1(\alpha_1T_1(1-2T_1)+\alpha_1-1-2T_1)+c_1Dk_2(T_1\alpha_2(1-\alpha_2+\alpha_2T_1)-T_1-\alpha_2)+\frac{sD\alpha_3^2k_3(T-T_1)^2}{2}-TDc_1k_2\alpha_3^2}$$

(12)

and $\frac{\partial z(L_m,T_1,T)}{\partial T_1} = 0$

$\Rightarrow hk_1e^{-\alpha_1 L_m}(1 + L_m - 2T_1) - (hk_1(1 + L_m)e^{-\alpha_1 L_m} +$ c1k2e−α2Lm1+log1+Lm−T11+Lm−sk3e−α3LmT− T1=0

$$\text{i.e.,}\ T_1 = \frac{(1+L_m-\log(1+L_m))(hk_1(1+L_m)e^{-\alpha_1 L_m}+c_1k_2e^{-\alpha_2 L_m})-hk_1(1+L_m)e^{-\alpha_1 L_m}+sk_3e^{-\alpha_3 L_m}T}{hk_1e^{-\alpha_1 L_m}(L_m-1)+c_1k_2e^{-\alpha_2 L_m}+sk_3e^{-\alpha_3 L_m}}$$

(13)

and $\frac{\partial z(L_m,T_1,T)}{\partial T} = 0$

$\Rightarrow \frac{1}{T^2}\{r + D[hk_1e^{-\alpha_1 L_m}(1 + L_m - 2T_1)T_1 + (1 + L_m -$ T1log1+Lm−T11+Lmhk11+Lme−α1Lm+c1k2e−α2L m− sk3e−α3Lm(T−T1)=0

$$\text{i.e.,}\ T = \frac{D(T_1-(1+L_m))\{hk_1e^{-\alpha_1 L_m}T_1+\log[\frac{1+L_m-T_1}{1+L_m}](hk_1(1+L_m)e^{-\alpha_1 L_m}+c_1k_2e^{-\alpha_2 L_m})\}}{Dsk_3e^{-\alpha_3 L_m}}$$

(14)

By Equations (12), (13) and (14) we can obtained the optimal values $L_m^* = L_m$, $T_1^* = T_1$, $T^* = T$.

By using following algorithm we can find the optimal decision variables L_m^*, T_1^* , T^* to maximize the profit $z^*(L_m^*, T_1^*, T^*)$.

Algorithm

Step 1. Input the values

Step 2. Substituting the values into equation (12) and find L_m

Step 3. Using L_m determine T_1 from equation (13)

Step 5. Using L_m, T_1 determine T from equation (14)

Step 6. Set $L_m^* = L_m, T_1^* = T_1, T^* = T$

Step 7. Using equation (10) determine $z^*(L_m^*, T_1^*, T^*)$

III. NUMERICAL EXAMPLE

In order to illustrate the model, we consider an inventory situation with following data:

$D = 100$ units, $k_1 = 2$, $k_2 = 1$, $k_3 = 3$, $r = 10$, $h = 0.2$, $s = 0.1$, $c_1 = 0.5$, $c_3 = 4$, $T = 0.2$ year, $T_1 = 0.15$ year, $\alpha_1 = 8, \alpha_2 = 4, \alpha_3 = 1, F = 0.5$

The optimal solutions are found as

$L_m^* = 0.7043, T^* = 0.6757, T_1^* = 0.2149, z^*(L_m^*, T_1^*, T^*) = 185.0366$

Sensitivity Analysis

We now study the effects of changes in the value of system parameters D, r, F, $c_3, \alpha_1, \alpha_2, \alpha_3$ on the optimal decision variables L_m^*, T^*, T_1^* the maximum profit function $z^*(L_m^*, T_1^*, T^*)$ of the above Example. The sensitivity analysis is performed by taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Table 2 and Table 3.

(1) Table 2 shows that when the optimal decision variable T_1^* is closed to optimal decision variable T^* the profit $z^*(L_m^*, T_1^*, T^*)$ will be maximized. i.e., the seller's reaches maximum profit when the shortage time is closed to total length of the cycle.

(2) An increase in value of Demand D, Market price c_3 and seller's default risk F, the profit $z^*(L_m^*, T_1^*, T^*)$ will increase.

(3) An increase in value of ordering cost r, the profit $z^*(L_m^*, T_1^*, T^*)$ will decrease.

(4) A decrease value of exponential function of holding cost $h(L_m^*)$, the profit $z^*(L_m^*, T_1^*, T^*)$ will increase.

(5) Reduction on value of exponential function of purchase cost $c_1(L_m^*)$, the profit $z^*(L_m^*, T_1^*, T^*)$ will increase.

(6) Reduction on value of exponential function of shortage cost $s(L_m^*)$ along with decrease of T_1^*, the profit $z^*(L_m^*, T_1^*, T^*)$ will increase. Our results reveal that the profit increases when the holding cost, purchase cost and shortage cost decrease. It's general fact. Therefore, our model is suitable for practical inventory problems.

IV. CONCLUSION

In this paper, an inventory model for the maximum lifetime of a deteriorating item with constant demand is developed under completely backlogged shortages. It was assumed that the deteriorating products having exponential function of holding cost, shortage cost and purchasing cost. In addition, the model assumes deteriorating items not only deteriorate continuously, and has a maximum lifetime. Some useful lemmas to characterize the optimal solutions have been obtained. Numerical examples are also provided to illustrate the proposed model. Moreover, the model used software Matlab 7.0 to study the sensitivity analysis on the optimal solution with respect to each parameter to illustrate the model and provide some managerial insights. The proposed model can be extended in several ways. First, we may extend the constant demand to a more generalized demand pattern that fluctuates with time, price or stock-dependent demand rate. Second, we could extend the model to incorporate some more realistic features, such as quantity discount, price discount also fluctuating with time. Third, we could generalize the model under trade credit period strategy.

Figures and Tables

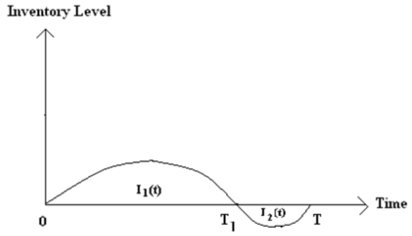

Fig 1: Graphical representation of inventory model

Table 1: Major characteristic of inventory models on selected articles

Authour's and Year	**Deterioration Rate**	**Maximum life time deterioration**	**Shortages**	**Exponential function of holding cost, shortage cost and purchasing cost**
Bakker et al.(2012)	Constant	No	No	No
Chen and Teng (2012)	Time varying	Yes	No	No
Ghosh and Chaudhuri(2006)	Time proportional	No	Yes	No
Goyal and Giri (2003)	Time varying	No	No	No
Kreng and Tan (2011)	Constant	No	No	No
Muniappan and Uthayakumar(2014)	Constant	No	Yes	No
Muniappan et al. (2014)	Constant	No	No	No
Sarkar (2012)	Time varying	No	No	No
Sarkar. B and Sarkar S. (2013)	Time varying	No	Yes	No
Sarkar et al. (2013)	Constant	No	No	No
Teng et al. (2012)	Constant	No	No	No
Wan-Chih Wang et al. (2014)	Time varying	Yes	No	No
Present Paper	Time varying	Yes	Yes	Yes

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Table 2: Effect of changes in the parameters of the inventory

Parameters		$h(L_m^*)$	$c_1(L_n^*)$	$s(L_m^*)$	L_m^*	T_1^*	T^*	$z^*(L_m^*, T_1^*)$
D	100	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	185.0366
	150	0.0014	0.0299	0.1483	0.7043	0.2149	0.4502	277.5250
	200	0.0014	0.0299	0.1483	0.7043	0.2149	0.3375	370.0149
	250	0.0014	0.0299	0.1483	0.7043	0.2149	0.2699	462.4998
r	5	0.0014	0.0299	0.1483	0.7043	0.2149	0.3386	185.0554
	6	0.0014	0.0299	0.1483	0.7043	0.2149	0.4060	185.0455
	7	0.0014	0.0299	0.1483	0.7043	0.2149	0.4734	185.0399
	8	0.0014	0.0299	0.1483	0.7043	0.2149	0.5408	185.0370
c_3	6	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	285.0366
	8	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	385.0366
	10	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	485.0366
F	0.005	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	383.0366
	0.050	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	365.0366

Table 3: Effect of changes in the parameters of the inventory

Parameters		$h(L_m^*)$	$c_1(L_n^*)$	$s(L_m^*)$	L_m^*	T_1^*	T^*	$z^*(L_m^*, T_1^*)$
α_1	8.0	0.001400	0.0299	0.1483	0.7043	0.2149	0.6757	185.0366
	8.5	0.000270	0.0161	0.1271	0.8588	0.2052	0.7886	187.2889
	9.0	0.000020	0.0061	0.0999	1.0999	0.2010	1.0033	190.0572
α_2	4.00	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	185.0366
	4.05	0.0020	0.0340	0.1544	0.6639	0.2182	0.6488	184.3699
	4.10	0.0026	0.0381	0.1601	0.6281	0.2217	0.6259	183.7450
α_3	0.52	0.0014	0.0299	0.2080	0.7045	0.2107	0.4825	179.1025
	0.54	0.0014	0.0299	0.2051	0.7045	0.2108	0.4893	179.3914
	0.56	0.0014	0.0299	0.2022	0.7045	0.2110	0.4962	179.6764

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