Nano Pre-Regular and Strongly Nano Pre-Regular Spaces.

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***Abstract:** This paper is commited to induct and investigate the characterizations of nanopre-irresolute, almost nanopre-irresolute, quasi nanopre-irresolute, nanosemi-regular, nanopre-regular, strongly nano regular, almost nanopre-regular and obtain some relationship between the existing sets.*

***Keywords:** nanopre-irresolute, almost nanopre-irresolute, quasi nanopre-irresolute, nanopre-regular, strongly nano regular and almost nanopre-regular*

I. INTRODUCTION

Sathishmohan et.al[6] defined nanopre-neighbourhoods, nanopre-interior, nanopre-limit point, nanopre-derived set, nanopre-frontier and nanopre-regular in nanotopological spaces and obtained some of its properties.

And in [7], they introduced and investigated the properties of nanosemipre-neighbourhood, nanosemipre-interior, nanosemipre-frontier, nanosemipre-exterior, nano-dense and nano-submaximal. Further, the authors[5] introduced and investigated the properties of nano-T0 space, nanosemi-T0 space, nanopre-T0 space, nano-T1 space, nanosemi-T1 space, nanopre-T1 space, nano-T2 space, nanosemi-T2 space, nanopre-T2 space and obtain some of its basic results.

In this paper, we study some additional characterizations of nanopre-regularity. Also, we introduce and study the nanopre-irresolute, almost nanopre-irresolute and quasi nanopre- irresolute, strongly nano regular and almost nanopre-regular.

II. NANOPRE-REGULARITY

Definition 2.1. A map ḟ:(Ù, τ_R (X))→(Ṿ, $\tau_R{'}$(Y)) can be said as nanopre-irresolute if the inverse of the image ḟ(A), $ḟ^{-1}$(A) of each nanopre-openset A in V is nanopre-open in Ù.

Definition 2.2. Ù can be said as nanosemi-regular, if for each nanosemi-closedset Ḟ of Ù and each point x∉Ḟ, ∃ disjoint nanosemi-opensets X and Y ∍ Ḟ ⊂ X, x∈Y.

Definition 2.3. Ù can be said as nanopreregular if for every nanopre-closedset Ḟ of Ù, every point x ∉ Ḟ, ∃ disjoint nanopre-opensets X, Y ∍ Ḟ ⊂ X and x ∈ Y.

Note 2.4. Clearly every nanoregular space is nanopreregular, but the converse of this is not true.

Example 2.5. Let Ù ={ȧ, ƀ, ċ, đ}, Ù /R={{ȧ}, {ƀ, đ}, {ċ}}, X = {ȧ, ƀ} and τ_R(X) = {Ù, φ, {ȧ}, {ȧ, ƀ, đ}, {ƀ, đ}} be a nanotopology on Ù. Let x ={ȧ, ċ, đ}, be a nanopreregular space but not nano-regular space.

Lemma 2.6. Let (Ù, τ_R(X)) is a nanotopologicalspace. Then NSPO (Ù, τ_R(X)) = NPO (Ù, τ_R(X)) iff ((Ù, τ_R(X)) is nanoextremely disconnected.

Theorem 2.7. If (Ù, $\tau_R(X)^{\alpha}$) is nanopreregular, (Ù, τ_R(X)) is nanopreregular.

Proof: Let (Ù, $\tau_R(X)^{\alpha}$) is nanopreregular, Ù is a closedset in (Ù, τR(X)) and x ∈ Ù − Ḟ. Since $\tau_R(X) \subset \tau_R(X)^{\alpha}$). Ḟ is nanoclosed set in (Ù, $\tau_R(X)^{\alpha}$) and x∉ Ḟ. Since (Ù, $\tau_R(X)^{\alpha}$) is nanopreregular, ∃ disjoint nanopre-open sets X,Y in (Ù, $\tau_R(X)^{\alpha}$) ∍ x∈Y and Ḟ ⊂Y. But by [2], X, Y ∈ NPO(Ù, τ_R(X)). Thus, ∃ disjoint nanopre-open sets X and Y in (Ù, τ_R(X)) separating Ḟ and x respectively. Therefore (Ù, τ_R(X)) is nanopreregular.

Theorem 2.8. In a nano extremely disconnected space (Ù, τ_R(X)), then following are equal.

(1)(Ù, τ_R(X)) is nanopreregular.

(2)For every nanoclosed set Ḟ and every point x∉ Ḟ ∃ disjoint nano semipre-open sets X and Y ∍ Ḟ ⊂X and x∈Y.

Proof: (1)⇒(2), since NPO (Ù, τ_R(X)) ⊂ NSPO(Ù, τ_R(X)). (2)⇒(1). Let Ḟ be a nanoclosed set, each point x∉ Ḟ. By (2) ∃ two disjoint nanosemipre-open sets Ġ and Ḥ ∍ Ḟ ⊂ Ġ and x ∈ Ḥ. But by [5] Ġ, Ḥ ∈ NPO (Ù, τ_R(X)). Therefore, (Ù, τ_R(X)) is nanopreregular.

Lemma 2.9. If Y ∈ NPO(Ù) and X ∈ NSO(Ù) then X ∪ Y ∈ NPO(X).

Theorem 2.10. If Ù is nanopreregular space and Ṿ is a nanosemi-open subset of Ù. Then the subspace Ṿ is nanopreregular.

Proof: Let Ḟ is a nanoclosed set of Ṿ, x ∉ Ḟ. Then ∃ nanoclosed set E of Ù ∍ Ḟ = E ∩ Ṿ and x ∉ E. Since Ù is nanopreregular, ∃ X_x, X_E ∈ NPO(Ù) ∍ x ∈ X_x , E ⊂ X_E with $X_x \cap X_E = \varphi$. Now, put Y_x=X_x∩ Ṿ and Y_F = X_F ∩ Ṿ. Then by Lemma 2.9, x ∈ Y_x ∈ NPO(Ṿ) and Ḟ ⊂ Y_F ∈ NPO(Ṿ) with $Y_x \cap Y_F = \varphi$. Therefore Ṿ is nanopreregular.

Definition 2.11. A function ḟ: Ù→ Ṿ can be said as almost nanopre-irresolute if for every x ∈ Ù, for every nanopre-neighbourhood Y of ḟ (x), $(ḟ^{-1}(Y))_*$ is a nanopre-neighbourhood of x.

Clearly all nanopre-irresolutemap is almost nanopre-irresolute.

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Theorem 2.12. For ḟ: Ù → Ṿ, the below statements are equal.
(1) ḟ is almostnanopre-irresolute.
(2) $ḟ^{-1}$(X) ⊂ (($ḟ^{-1}$(X))*)$_*$ for each X ∈ NPO(V).
Proof: (1)⇒(2). Let X ∈ NPO(Ṿ) and x∈ $ḟ^{-1}$(X). Since X be a nanopre-neighbourhood of ḟ (x), ($ḟ^{-1}$(X))* is a nanopreneighbourhood of x, hence ∃ Y∈NPO(x) ∍ Y⊂($ḟ^{-1}$(X))*. Therefore, we have x ∈ Y ⊂ (($ḟ^{-1}$(X))*)$_*$. This shows that $ḟ^{-1}$(X) ⊂ (($ḟ^{-1}$(X))*)$_*$.
(2)⇒(1). Let x∈ Ù and X be any nanopre-neighbourhood of ḟ (x). ∃ Ġ ∈NPO(f(x)) contained in X. Hence we obtain that x∈ $ḟ^{-1}$(Ġ)⊂(($ḟ^{-1}$(Ġ))*)$_*$⊂ ($ḟ^{-1}$(Ġ))* ⊂($ḟ^{-1}$(X))*. Therefore ($ḟ^{-1}$(X))* is a nanopre-neighbourhood of x. Thus ḟ is almost nanopre-irresolute map.
Theorem 2.13. A function ḟ: Ù → Ṿ is almost nanopre-irresolute iff ḟ (Ġ *)⊂ (ḟ(Ġ))* for each Ġ ∈ NPO(Ù).
Definition 2.14. ḟ: Ù → Ṿ is said quasi nanopre-irresolute if for every x ∈ Ù, Y ∈ NPO(ḟ(x)), ∃ X ∈ NPO(x) ∍ ḟ(X) ⊂ Y*.

Clearly, every nanopre-irresolute map is quasi nanopre-irresolute but converse is not true.
Example 2.15. Let Ù ={ ȧ, ƀ, ċ, đ }, Ù /R={{ ȧ }, { ƀ, ċ }, { đ }}, X={ ȧ, ƀ } and $τ_R$(X)={ Ù, φ, { ȧ }, { ȧ, ƀ, ċ }, { ƀ, ċ }} are nano topology on Ù. Let f(ȧ)= ƀ, f(ƀ)=ȧ, f(ċ)= đ, f(đ) = ċ. Let x={ ƀ } and Y={ ȧ, ƀ, đ } and X={ ƀ, ċ } is quasi nanopre-irresolute but not nanopre-irresolute.
Lemma 2.16. For ḟ: Ù → Ṿ the following are equal.
(1) ḟ is nanopre-continuous.
(2) For every point x in Ù and every nano open set Ġ ∈ Ṿ with ḟ(x)∈ Ġ, there is a nanopre-open set X ⊂ Ù ∍ x ∈ Ù and ḟ(Ù) ⊂ Ġ.
(3) The inverse image of each nanoclosed set of Ṿ under mapping ḟ is nanopre-closed.

However we show that every quasi nanopre-irresolute map is nanopre-continuous if the range space is nanopreregular.
Theorem 2.17. Let Ù and Ṿ be two spaces. Assume ḟ: Ù → Ṿ be a map where Y be nanopreregular. Then ḟ be nanopre-continuous whenever it is quasi nanopre-irresolute.
Theorem 2.18. ḟ: Ù → Ṿ is quasi nanopre-irresolute iff for every nanopre-open set Y of Ṿ, $ḟ^{-1}$(Y) ⊂ ($ḟ^{-1}$(Y *))$_*$.
Lemma 2.19. Let x is a point of Ù. Then x ∈ NA* iff A ∩ Y ≠ 0 for every Y ∈ NPO(x).
Theorem 2.20. If ḟ: Ù → Ṿ is quasi nanopre-irresolute then ($ḟ^{-1}$(Y))* ⊂ $ḟ^{-1}$(Y*) for every Y ∈ NPO(Ṿ).
Definition 2.21. Ù can be said as strongly nano-regular if foreach nanopre-closed set Ḟ & each point x∉ Ḟ, ∃ disjoint nanopre-open sets X, Y ∍ x ∈ X and F ⊂ Y.

Clearly, every strongly nano-regular space is nanopreregular.
Example 2.22. Let Ù ={ ȧ, ƀ, ċ, đ }, Ù /R={{ ȧ }, { ƀ, đ }, { ċ }}, let X={ ȧ, ƀ }, $τ_R$(X)={{Ù}, {φ}, { ȧ }, { ƀ, đ }, { ȧ, ƀ, đ }}, Ḟ ={ ċ, đ }, X={ ƀ }, Y={ ȧ, ċ, đ }. Then x ∈ X and Ḟ ⊂ Y. Therefore it is nanopreregular but not strongly nano regular.
Lemma 2.23. Let A, B are the subsets of Ù. Then the below holds,
(1) A is nanopre-closed in Ù iff A = NA*.
(2) NA* ⊂ NB* if A ⊂ B.
(3) (NA*)* = NA*.
(4) A is nanopre-closed set in Ù.

Theorem 2.24. For Ù, the following are equal,
a) Ù is strongly nano-regular.
b) Forall point x∈Ù and each nanopre-openset Ġ containing x ∃ nanopre-open set Ḥ ∍ x ∈ Ḥ ⊂ Ḥ * ⊂ Ġ.
c) For each nanopre-closedset Ḟ, the intersection of all the nanopre-closed nanopre-neighbourhoods of Ḟ is Ḟ.
d) For each set Ġ and a nanopre-openset Ḥ ∍ Ġ ∩ Ḥ ≠ φ, ∃ a nanopre-open set X with Ġ ∩ X ≠ φ and X* ⊂ Ḥ.
e) For each non-empty set Ġ and nanopre-closed set Ḟ ∍ Ġ ∩ Ḟ = φ, ∃ disjoint nanopre-opensets Y and W ∍ Ġ ∩ Y ≠ φ and Ḟ ⊂ W.
Theorem 2.25. If Ṿ is strongly nano-regular space, a fuction ḟ: Ù → Ṿ is quasi nanopre-irresolute iff it is nanopre-irresolute.
Proof is similar to Theorem 2.17

III. Almost Nanopre-Regular Spaces

Definition 3.1. Ù can said as almost nanopreregular, for every nano regularclosed set Ḟ and x∉ Ḟ, then ∃ disjoint nanopre-& open sets X and Y ∍ x ∈ Ù and Ḟ ⊂ Ṿ.

Therefore, every almost nanoregular is almost nanopreregular. And every nanopreregular space is almost nanopreregular.
Theorem 3.2. For Ù, the following are equal.
a) Ù is almost nanopreregular.
b) For x ∈ Ù and a nanoregular-open set *Ġ* containing *x*, ∃ a nanopre-openset X ∍ x ∈ X ⊂ X* ⊂ *Ġ*.
c) Every nano regular closed set Ḟ is the intersection of all nanopre-closed nanopre-neighbouhoods of Ḟ.
d) For each set A and a nano regular openset B ∍ A ∩ B ≠ φ, ∃ a nanopre-open set X ∍ A ∩ Ù ≠ φ and Ù *⊂ B.
e) For every non-empty set A and nano regular closedset B ∍ A ∩ B = φ, ∃ disjoint nanopre-opensets Ġ, Ḥ ∍ A ∩ Ġ ≠ φ, B ⊂ Ḥ.
Lemma 3.3. Let X_0 is a nano regular openset of Ù, A is a subset of X_0. Then A ∈ NRO(Ù) iff A ∈ NRO(X_0).
Lemma 3.4. If A ⊂ Y ⊂ X and A ∈ NPO(X), then A ∈ NPO(Y) whenever Y is nano open in X.
Lemma 3.5. If X_0 is an open subspace of X and A ⊂ X_0. Then $A^*_{X0} = X_0 \cap A^*_X$.
Theorem 3.6. If Ù is an almost nanopreregular space and X_0 is a nano regularopen set of Ù. Then subspace X_0 is almost nanopreregular.
Proof: Let x ∈ $X_{0,}$ X is a nano regularopen subset of X_0 ∍ x ∈ X. Then by lemma 3.3, X ∈ NRO(Ù) and x ∈ X. Since Ù is almost nanopreregular, ∃ a nanopre-open set Y in Ù ∍ x ∈ Y ⊂ Y* ⊂ X by Theorem 3.2.
Since Y ⊂ X_0 ⊂ Ù and Y ∈ NPO(Ù) and so by Lemma 3.4, Y ∈ NPO(Ù). Again X_0 ∈ NRO(Ù) ⊂ τ and by Lemma 3.5, x ∈ Y ⊂ X_0 ∩ $Ṿ^*_{U}$ = $Ṿ^*_{X0}$ ⊂X ∩ X_0 = X.

Therefore, it follows that the subspace X_0 is almost nanopreregular.

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IV. RESULT

In the above work we have look into the characterizations of nanopre-irresolute, almost nanopre-irresolute, quasi nanopre-irresolute, nanosemi-regular, nanopre-regular, strongly nano regular, almost nanopre-regular and obtain some relationship between the existing sets

V. CONCLUSION

The characterization of nanopre-irresolute, almost nanopre-irresolute, quasi nanopre-irresolute, nanosemi-regular, nanopre-regular, strongly nano regular, almost nanopre-regular and obtain some relationship between the existing sets.

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REFERENCES

1. Bhuvaneshwari.K and Ezhilarasi.A, Nano semi-generalized irresolute maps in nano topological spaces, *International journal of mathematical archive*, 7(3), 2016, 68-75.
2. Lellis Thivagar and Richard.C, On nano forms of weakly open sets, *International journal of mathematics and statistics invention*, 1(1), 2013, 31-37.
3. Levine.N, Generalized closed sets in topological spaces, *Rend. Circ. Math., Palermo*, (2), 1970, 89-96.
4. Pawalk.Z, Rough sets, Theoretical Aspects of Reasoning about Data, *Kluwer Academic Publishers*, Boston, 1991
5. P.Sathishmohan, V.Rajendran and P.K.Dhanasekaran, Further properties of nanopre-T_0, nanopre-T_1 and nanopre-T_2 spaces, *Malaya Journal of Mathematik*, Vol.7, No.1, 2019, 34-38
6. P.Sathishmohan, V.Rajendran, P.K.Dhanasekaran and S.Brindha, More on preneighbourhoods in nano topological spaces, *Journal of Applied Science and Computations*, 5(10), 2018, 899-907.
7. P.Sathishmohan, V.Rajendran, C.VigneshKumar and P.K.Dhanasekaran, On nano semi pre neighbourhoods on nano topological spaces, *Malaya Journal of Mathematik*, 6(1), 2018, 294-298.

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