A Few Separation Axioms on Nano Topological Spaces

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Abstract: The main goal of this work is to induct and look into the properties of $N\beta -T_0$ space, $N\beta -T_1$ space, $N\beta -T_2$ space and obtain the relation between some of the subsisting sets.

Keywords: nano $T_0$ space, nano $T_1$ space, nano $T_2$ space, nano semi-$T_0$ space, nano semi-$T_1$ space, nano semi-$T_2$ space.

I. INTRODUCTION

Lelvis Thivagar and Richard [1] established the notion of nano topology in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also make known about nano-closed sets, nano-interior, nano-closure and weak form of nano open sets namely nano semi-open sets, nano pre-open, nano $\alpha$-open and nano $\beta$-open sets. Nasef et.al.[2] make known about some of nearly open sets in nano topological spaces. Revathy and Gnamambal Illango [4] gave the idea about the nano $\beta$-open sets. Sathishmohan et.al.[6] brings up the idea about nano neighbourhoods in nano topological spaces. This motivates the author to induct and study the properties of $N\beta -T_0$ space, $N\beta -T_1$ space, $N\beta -T_2$ space in nano topological spaces.

II. PRELIMINARIES

Definition 2.1. [5] A space $U$ is called nano $T_0$ (or $NT_0$) space for $c, d \in U$ and $c \neq d$, $\exists$ a nano-open set $G$ such that $c \in G$ and $d \notin G$.

Definition 2.2. [5] A space $U$ is called nano semi-$T_0$ (or $NST_0$) space [resp. nano pre-$T_0$ (or $NPT_0$)] space for $c, d \in U$ and $c \neq d$, $\exists$ a nano semi-open [resp. nano pre-open] set $G$ such that $c \in G$ and $d \notin G$.

Definition 2.3. [5] A space $U$ is called nano $T_1$ (or $NT_1$) (resp. nano semi-$T_1$ (or $NST_1$), nano pre-$T_1$ (or $NPT_1$)) space for $c, d \in U$ and $c \neq d$, $\exists$ a nano semi-open [resp. nano semi open, nano pre open] $G$ and $H$ such that $c \in G$, $d \notin G$ and $d \in H$, $c \notin H$.

Definition 2.4. [5] A space $U$ is called nano $T_2$ (or $NT_2$) (resp. nano semi-$T_2$ (or $NST_2$), nano pre-$T_2$ (or $NPT_2$)) space for $c, d \in U$ and $c \neq d$, $\exists$ disjoint nano-open sets [resp. nano semi open, nano pre open] $G$ and $H$ such that $c \in G$ and $d \in H$.

Definition 2.5. [6] A subset $M, \subset U$ is called a $N\beta$-neighbourhood ($N\beta$-nhd) of a point $c \in U$ iff $\exists$ a $A \in N\beta\{U, C\}$ such that $c \in A \subset M$, and a point $c$ is called $N\beta$-nhd point of the set $A$.

III. $N\beta -T_0$ SPACE

Definition 3.1. A space $U$ is called nano $\alpha$-$T_0$ (or $Na-T_0$) space for $c, d \in U$ and $c \neq d$, $\exists$ a nano $\alpha$-open set $G$ such that $c \in G$ and $d \notin G$.

Definition 3.2. A space $U$ is called nano semi-$T_0$ (or $NST_0$) space for $c, d \in U$ and $c \neq d$, $\exists$ a $\beta$-open set $G$ such that $c \in G$ and $d \notin G$.

Theorem 3.3. Let $(U, t_0(X))$ be a nano topological space, then for every $N\beta$-space is $N\beta$-space but not conversely.

Example 3.4. Let $U = \{1, 2, 3, 4\}$, $t_0(X) = \{U, \emptyset, \{1\}, \{1, 2, 4\}, \{1, 4\}\}$ be a nano topology on $U$. Let $c = \{1, 3\}$ and $d = \{3\}$ then it is $N\beta$-space but not $N\beta$-space.

Theorem 3.5. Every $N\beta$-space (resp. $NP\beta$-space, $Na\beta$-space) space is $N\beta$-space but not conversely.

Proof: Same as Theorem 3.3.

Example 3.6. From the Example 3.4, Let $c = \{2\}$ and $d = \{3\}$ then it is $N\beta$-space, but not $N\beta$-space.

Example 3.7. From the Example 3.4, Let $c = \{1, 3\}$ and $d = \{3\}$ then it is $N\beta$-space but not $N\beta$-space.

Example 3.8 From the Example 3.4, Let $c = \{1, 3\}$ and $d = \{3\}$ then it is $N\beta$-space but not $Na\beta$-space.

Theorem 3.9. A nano topological space $U$ is $Ncl\{c\}$ iff $Nfcl\{c\} \neq Nfcl\{d\}$ for $c \neq d$ in $U$.

Proof: Let $c, d \in U$ and $c \neq d$ with $U$ as $N\beta$-space. We have to prove that $Nfcl\{c\} \neq Nfcl\{d\}$. Consider the set $A = U - \{c\}$, it is clear that $Ncl\{A\}$ is either $A$ or $U$. If $Ncl\{A\} = A$ then $A$ is nano-closed and hence $N\beta$-closed. Therefore $U - A = \{c\}$ is a $N\beta$-open set which contains $c$ but not $d$. So $c \notin Nfcl\{d\}$. But $c \in Nfcl\{c\}$ and hence $Nfcl\{c\} \neq Nfcl\{d\}$. If $Ncl\{A\} = U$, then $A$ is $N\beta$-open and so $U - A = \{c\}$ is $N\beta$-closed. Therefore $Nfcl\{c\} = \{c\}$. Since $d \notin \{c\}$ and $d \in Nfcl\{d\}$, it follows that $Nfcl\{c\} \neq Nfcl\{d\}$.

Conversely: For $c, d \in U$ and $c \neq d$. Let $Nfcl\{c\} \neq Nfcl\{d\}$. Therefore $\exists$ a point $z$ in $U$ such that $z \in Nfcl\{c\}$ but $z \notin Nfcl\{d\}$. If we suppose that $c \in Nfcl\{d\}$ then $Nfcl\{c\} \subset Nfcl\{d\}$ and this implies $z \in Nfcl\{d\}$ which is contradiction. Therefore our supposition is wrong, i.e., $c \notin Nfcl\{d\}$ implies $c \in U - Nfcl\{d\}$ and $Nfcl\{d\}$ is a $N\beta$-open set containing $c$ but not $d$. This implies $U$ is $N\beta$-space.
IV. \(N\beta\)-T1 SPACE

**Definition 4.1.** A space \(U\) is called nano \(\alpha\)-T1 (or \(Na\)-T1) space for \(c, d \in U\) and \(c \neq d\), \(d\) a \(Na\)-open sets \(G\) and \(H\) such that \(c \in G, \ d \notin G\) and \(d \in H, \ c \notin H\).

**Definition 4.2.** A space \(U\) is called \(N\beta\)-T1 (or \(N\beta\)-T1) space for \(c, d \in U\) and \(c \neq d\), \(\exists\) a \(N\beta\)-open sets \(G\) and \(H\) such that \(c \in G, \ d \notin G\) and \(d \in H, \ c \notin H\).

**Theorem 4.3.** Every nano-T1 (resp. \(NST_1\), \(NP_T\), \(Na\)-T1) space is \(N\beta\)-T1 space but not conversely.

**Example 4.4.** Let \(U = \{1, 2, 3, 4\}, \ \tau_g(X) = \{U, \phi, \{1, 2, 3, 4\}\}\) be a nano topology on \(U\), we have \(Let c = \{2\} and \ d = \{3\}\) then it is \(N\beta\)-T1 space but not \(NT_1\) space.

**Example 4.5.** From the Example 4.4, \(Let c = \{1\} and \ d = \{3\}\) then it is \(N\beta\)-T1 space but not \(NST_1\) space.

**Example 4.6.** From the Example 4.4, \(Let c = \{2\} and \ d = \{3\}\) then it is \(N\beta\)-T1 space but not \(Na\)-T1 space.

**Lemma 4.7.** Let \(C\) and \(D\) be the subsets of \(U\) such that \(C\) and \(D\) is \(N\beta\)-open, then \(C\) is \(N\beta\)-open subset of \(D\) iff \(C\) is \(N\beta\)-open subset of \(U\).

**Definition 4.8.** For and subset \(A\) of \(U\)

\[
1) \ Nfnint(Nfncl(A)) = Nfncl(Nfnint(A)).
2) \ Nfnint(Nfncl(A)) = Ncl(Nfnint(A)). \ (3) \ Ncl(Nfnint(A)) = Nint(Nfncl(A)).
\]

**Lemma 4.9.** If \(f : (U, \tau_g(X)) \rightarrow (V, \tau_h(Y))\) is nano-open and nano-continuous then for and subset \(A\) of \(U\) then

\[
1) \ f(Nint(A)) \subset Nnt(f(A)).
2) \ f(Ncl(A)) \subset Ncl(f(A)).
\]

**Theorem 4.10.** Let \(U\), \(\tau_g(X)\) be an nano topological space, then for each \(N\beta\)-T1 (resp. \(NST_1\), \(NP_T\)) space is \(N\beta\)-T1 space.

**Example 4.11.** Let \(U = \{1, 2, 3, 4\}, \ \tau_g(X) = \{U, \phi, \{1, 2, 3, 4\}\}\) be a nano topology on \(U\), we have

Let \(G = \{1, 4\}\) and \(H = \{2, 3\}\)

Let \(c = \{1\}\) and \(d = \{3\}\), \(c \cup H \in U\) and \(c \neq d\), then it is clear that \(c \in G\), \(d \in G\) and \(d \in H\) and \(c \notin H\). Then we can sad that it is \(N\beta\)-T1 space.

**Theorem 4.12.** For a topological space \(U\), each of the following statements are equivalent

\(a)\) \(U\) is \(N\beta\)-T1 space.

\(b)\) Each one point set is \(N\beta\)-closed in \(U\).

\(c)\) Each subset of \(U\) is the intersection of all \(N\beta\)-open sets containing it.

\(d)\) The intersection of all \(N\beta\)-open sets containing the point \(c\) in \(U\) is \(c\).

**Proof** 
\(a) \Rightarrow (b)\) \(Let c \in U\), hence for and \(d \in V, \ d \neq c \in G\) a \(N\beta\)-open set \(G\) containing \(d\) but not \(c\). Hence \(d \in G\) and \(c \notin G\). Clearly \(\{c\}^* \subset \{G\}^*\) for \(c\) being a union of \(N\beta\)-open set \(G\) is \(N\beta\)-closed.

\(b) \Rightarrow (c)\) \(Suppose each one point set is \(N\beta\)-closed. Let A \(\subset U,\ A \neq \phi \exists\ a \subset\ d\in\ A\) such that \(A \subset \{d\}^*\) and each \(N\beta\) open, \(N\beta\)-continuous and \(V\) is \(N\beta\)-T1 space then \(U\) is the intersection of all \(N\beta\)-open sets containing \(A\) is the set \(A\) itself.

\(c) \Rightarrow (d)\) \(Suppose (c)\) in \(c\) let \(A = \{c\}\) then \(d \notin \{c\}\) and \(d \in c\)

\(d) \Rightarrow (a)\) \(Suppose (d)\). Let \(c, d \in U\) and \(c \neq d\), then by hypothesis for \(c \in U, \ \{c\} = \{A \in N\beta(O(U))c \in A\}\), it follows that there must exists a nano \(N\beta\)-open set \(G\) such that \(c \in G\), and \(d \notin \).
VI. RESULT

In the above work we have compared investigated their properties of $N\beta$-$T_0$, $N\beta$-$T_1$, $N\beta$-$T_2$ spaces with some of the existing sets by proving the some theorems.

VII. CONCLUSION

We have defined few separations axioms in nano topological spaces and compared its properties with the existing spaces and proved some theorems.

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