A Few Separation Axioms on Nano Topological Spaces

P. Sathishmohan, V. Rajendran, C. Vignesh Kumar, P.K. Dhanasekaran

Abstract: The main goal of this work is to induct and look into the properties of Nβ-T_0 space, Nβ-T_1 space, Nβ-T_2 space and obtain the relation between some of the subsisting sets.

Keywords: nano-T_0 space, Nβ-T_0 space, nano-T_1 space, Nβ-T_1 space, nano-T_2 space, Nβ-T_2 space.

I. INTRODUCTION

Lellis Thivagar and Richard [1] established the notion of nano topology in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also make known about nano-closed sets, nano-interior, nano-closure and weak form of nano open sets namely nano semi-open sets, nano pre-open, nano α-open sets and Nβ-open sets. Nasef et.al.[2] make known about some of nearly open sets in nano topological spaces. Revathy and Gnanambal Illango [4] gave the idea about the nano β-open sets. Sathishmohan et.al.[6] brings up the idea about nano neighourhoods in nano topological spaces. This motivates the author to induct and study the properties of Nβ-T_0 space, Nβ-T_1 space, Nβ T_2-space in nano topological spaces.

II. PRELIMINARIES

Definition 2.1. [5] A space U is called nano-T_0 (or NT_0) space for $c, d \in U$ and $c \neq d$, ∃ a nano-open set G such that $c \in$ G and $d \notin$ G.

Definition 2.2. [5] A space U is called nano semi-T_0 (or NST_0) [resp. nano pre-T_0 (or NPT_0)] space for $c, d \in U$ and $c \neq d$, ∃ a nano semi-open [resp. nano pre-open] set G such that $c \in$ G and $d \notin$ G.

Definition 2.3. [5] A space U is called nano-T_1 (or NT_1) [resp. nano semi-T_1 (or NST_1), nano pre-T_1 (or NPT_1)] space for $c, d \in U$ and $c \neq d$ ∃ a nano-open sets [resp. nano semi open, nano pre open] G and H such that $c \in$ G, $d \notin$ G and $d \in$ H, $c \notin$ H.

Definition 2.4. [5] A space U is called nano-T_2 (or NT_2) [resp. nano semi-T_2 (or NST_2), nano pre-T_2 (or NP-T_2)] space for $c, d \in U$ and $c \neq d$,∃ disjoint nano-open sets [resp. nano semi open, nano pre open] G and H such that $c \in$ G and $d \in$ H.

Definition 2.5. [6] A subset $M_c \subset U$ is called a $N\beta$-neighbourhood ($N\beta$-nhd) of a point $c \in U$ iff ∃ a A ∈ $N\beta O(U, C)$ such that $c \in$ A ⊂ M_c and a point c is called $N\beta$-nhd point of the set A.

III. Nβ-T0 SPACE

Definition 3.1. A space U is called nano α-T_0 (or Nα-T_0) space for $c, d \in U$ and $c \neq d$, ∃ a nano α-open set G such that $c \in$ G and $d \notin$ G.

Definition 3.2. A space U is called *nano semipre*-T_0 (or $N\beta$-T_0) space for $c, d \in U$ and $c \neq d$, ∃ a $N\beta$-open set G such that $c \in$ G and $d \notin$ G.

Theorem 3.3. Let $(U, \tau_R(X))$ be a nano topological space, then for every NT_0 space is $N\beta$-T_0 space but not conversely.

Example 3.4. Let $U = \{1,2,3,4\}$, $\tau_R(X) = \{U,\varphi,\{1\},\{1, 2, 4\},\{1, 4\}\}$ be a nano topology on U. Let $c = \{1, 3\}$ and $d = \{3\}$ then it is $N\beta$-T_0 space but not NT_0 space.

Theorem 3.5. Every NST_0 (resp. NP-T_0, Nα-T_0) space is $N\beta$-T_0 space but not conversely.

Proof: Same as Theorem 3.3

Example 3.6. From the Example 3.4, Let $c = \{2\}$ and $d = \{3\}$ then it is $N\beta$-T_0 space but not NST_0 space.

Example 3.7. From the Example 3.4, Let $c = \{1, 3\}$ and $d = \{3\}$ then it is $N\beta$-T_0 space but not NP-T_0 space.

Example 3.8 From the Example 3.4, Let $c = \{1, 3\}$ and $d = \{3\}$ then it is $N\beta$-T_0 space but not Nα-T_0 space.

Theorem 3.9. A nano topological space U is Ncl$\{c\}$ iff $N\beta$cl$\{c\} \neq N\beta$cl$\{d\}$ for $c \neq d$ in U.

Proof: Let $c, d \in U$ and $c \neq d$ with U as $N\beta$-T_0 space. We have to prove that $N\beta$cl$\{c\} \neq N\beta$cl$\{d\}$. Consider the set A = $U - \{c\}$, it is clear that Ncl(A) is either A or U. If Ncl(A) = A then A is nano-closed and hence $N\beta$-closed. Therefore $U -$ A = $\{c\}$ is a $N\beta$-open set which *contains* c but not d. So $c \notin N\beta$cl$\{d\}$. But $c \in N\beta$cl$\{c\}$ and hence $N\beta$cl$\{c\} \neq N\beta$cl$\{d\}$. If Ncl(A) = U, then A is $N\beta$-open and so $U -$ A = $\{c\}$ is $N\beta$-closed. Therefore $N\beta$cl$\{c\} = \{c\}$. Since $d \notin \{c\}$ and $d \in N\beta$cl$\{d\}$, it follows that $N\beta$cl$\{c\} \neq N\beta$cl$\{d\}$.

Conversely: For $c, d \in U$ and $c \neq d$. Let $N\beta$cl$\{c\} \neq N\beta$cl$\{d\}$. Therefore ∃ a point z in U such that z ∈ $N\beta$cl$\{c\}$ but z ∈ $N\beta$cl$\{d\}$. If we suppose that $c \in N\beta$cl$\{d\}$ then $N\beta$cl$\{c\} \subset N\beta$cl$\{d\}$ and this implies z ∈ $N\beta$cl$\{d\}$ which is contradiction. Therefore our supposition is wrong, i.e., $c \notin N\beta$cl$\{d\}$ implies $c \in U - N\beta$cl$\{d\}$ and $N\beta$cl$\{d\}$ is a $N\beta$-open set containing c but not d. This implies U is $N\beta$-T_0 space.

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IV. Nβ-T1 SPACE

Definition 4.1. A space U is called nano α-T_1 (or Nα-T_1) space for $c, d \in U$ and $c \neq d$, ∃ a Nα-open sets G and H such that $c \in$ G, $d \notin$ G and $d \in$ H, $c \notin$ H.

Definition 4.2. A space U is called $N\beta$-T_1 (or $N\beta$-T_1) space for $c, d \in U$ and $c \neq d$, ∃ a $N\beta$-open sets G and H such that $c \in$ G, $d \notin$ G and $d \in$ H, $c \notin$ H.

Theorem 4.3. Every nano-T_1 (resp. NST_1, NP-T_1, Nα-T_1) space is $N\beta$-T_1 space but not conversely.

Example 4.4. Let $U = \{1, 2, 3, 4\}$, $\tau_R(X) = \{U, \varphi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$ be a nano topology on U, we have

Let $c = \{2\}$ and $d = \{3\}$ then it is $N\beta$-T_1 space but not NT_1 space.

Example 4.5. From the Example 4.4, Let $c = \{1\}$ and $d = \{3\}$ then it is $N\beta$-T_1 space but not NST_1 space.

Example 4.6. From the Example 4.4, Let $c = \{2\}$ and $d = \{3\}$ then it is $N\beta$-T_1 space but not Nα-T_1 space.

Lemma 4.7. Let C and D be the subsets of U such that $C \subset D$ and D is $N\beta$-open, then C is $N\beta$-open subset of D iff C is $N\beta$-open subset of U.

Lemma 4.8. For and subset A of U

(1) $N\beta$int($N\beta$cl(A)) = $N\beta$cl($N\beta$int (A)).

(2) Nint ($N\beta$cl(A)) = Ncl($N\beta$*int* (A)). (3) Ncl($N\beta$int (A)) = N*int*($N\beta$cl(A)).

Lemma 4.9. If f: $(U, \tau_R(X)) \to (V, \tau_R{*}(Y))$ is nano-open and nano-continuous then for and subset A of U then

(1) f(N*int* (A)) ⊂ N*int* (f(A)).

(2) f(Ncl(A)) ⊂ Ncl(f(A)).

Theorem 4.10. Let $(U, \tau_R(X))$ be an nano topological space, then for each $N\beta$-T_1 (resp. NST_1, NP-T_1) space is $N\beta$-T_0 space.

Example 4.11. Let $U = \{1, 2, 3, 4\}$, and $\tau_R(X) = \{U, \varphi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$ be a nano topology on U, we have

Let G = {1,4} and H = {2,3}.

Let $c = \{1\}$ and $d = \{3\}$, $c, d \in U$ and $c \neq d$, then it is clear that $c \in$ G, $d \notin$ G and $d \in$ H and $c \notin$ H. Then we can sad that it is $N\beta$-T_0 space.

Theorem 4.12. For a topological space U, each of the following statements are equivalent

(a) U is $N\beta$-T_1 space.

(b) Each one point set is $N\beta$-closed in U.

(c) Each subset of U is the intersection of all $N\beta$-open sets containing it.

(d) The intersection of all $N\beta$-open sets containing the point $\{c\}$ in U is $\{c\}$.

Proof (a) ⇒ (b) : Let $c \in U$, hence for and $d \in$ V , $d \neq c$ ∃ a $N\beta$-open set G_d containing d but not c. Hence $d \in G_d \subset \{c\}^c$. Clearly $\{c\}^c = {}^S\{G_d : d \in \{c\}^c\}$ so $\{c\}^c$ being a union of $N\beta$-open set is $N\beta$-open ⇒ $\{c\}$ is $N\beta$-closed.

(b) ⇒ (c): Suppose each one point set is $N\beta$-closed. Let A ⊂ U, then for each $d \in$ A ∃ a subset $\{d\}^c$ such that A ⊂ $\{d\}^c$ and each of these sets $\{d\}^c$ is $N\beta$-open. Hence A = ${}^T\{\{d\}^c : d \in A^c\}$ so that the intersection of all $N\beta$-open sets containing A is the set A itself.

(c) ⇒ (d) : Suppose (c). In c) let A = $\{c\}$ then $d \notin \{c\}$ and $U - \{d\}$ is $N\beta$-open set containing c. Therefore from (c) $\{c\} = {}^T\{U - \{d\}$ / each $U - \{d\}$ is $N\beta$-open set containing $c\}$

(d) ⇒ (a): Suppose (d). Let $c, d \in U$ and $c \neq d$, then by hypothesis for $c \in U$, $\{c\} = {}^T\{A \in N\beta O(U)/c \in A\}$, it follows that there must ecists a $N\beta$-open set G_c such that $c \in G_c$ and $d \notin G_c$. In similar manner there must exists a $N\beta$-open set G_d such that $d \in G_d$ and $c \notin G_d$. Hence U is $N\beta$-T_1.

Lemma 4.13. Let f : $(U, \tau_R(X)) \to (V, \tau_R{*}(Y))$ be nano-open and nano-continuous then for each $N\beta$-open set A of U, f(A) is $N\beta$-open subset of D .

Theorem 4.14. The property of being $N\beta$-T_1 space is a nano-topological property.

Theorem 4.15. Every open subspace of a $N\beta$-T_1 space is $N\beta$-T_1 space.

Theorem 4.16. Let U be NT_1 space and f : $(U, \tau_R(X)) \to (V, \tau_R{*}(Y))$ be $N\beta$-closed surjection then D is $N\beta$-T_1 space.

V. Nβ-T2 SPACE

Definition 5.1. A space U is called nano α-T_2 (or Nα-T_2) space for $c, d \in U$ and $c \neq d$, ∃ disjoint Nα-open sets G and H such that $c \in$ G and $d \in$ H.

Definition 5.2. A space U is called *nano semipre*-T_2 (or $N\beta$-T_2) space for $c, d \in U$ and $c \neq d$, ∃ disjoint $N\beta$-open sets G and H such that $c \in$ G and $d \in$ H.

Lemma 5.3. If A is *nano open* in U and V is $N\beta$-open in U then A∩V is $N\beta$-open in U.

Lemma 5.4. If f : $(U, \tau_R(C)) \to (V, \tau_R'(D))$ is Nα-open and $N\beta$-continuous, then *inverse image* of $N\beta$*-open set* is $N\beta$*-open*.

Theorem 5.5. Every nano-T_2 space is $N\beta$-T_2 space but not conversely.

Example 5.6. From the *Example 4.4*, Let $c = \{2\}$ and $d = \{3\}$ then it is $N\beta$-T_2 space but not NT_2 space.

Theorem 5.7. Every NST_2 space is $N\beta$-T_2 space but not conversely.

Proof: Same as Theorem 5.5

Example 5.8. From the *Example 4.4*, Let $c = \{1\}$ and $d = \{3\}$ then it is $N\beta$-T_2 space but not NST_2 space.

Theorem 5.9. Every Nα-T_2 space is $N\beta$-T_2 (resp. NST_2, NP-T_2) space but not conversely.

Proof: Same as Theorem 5.5

Example 5.10. From the *Example 4.4*, Let c = {b} and d = {c} then it is $N\beta$-T_2 space but not Nα-T_2 space.

Theorem 5.11. For the *nano topological space U* the following are equivalent

(a) U is $N\beta$-T_2 space.

(b) If $c \in U$, then for each $d \neq c$ ∃ a $N\beta$-neighbourhood G_c of c such that $d \notin N\beta$cl(G_c).

(c) For each $c \in U$, ∪{$N\beta$cl(G) : G is a $N\beta$-neighbourhood of c} = $\{c\}$.

Theorem 5.12. The property of being $N\beta$-T_2 space is nano topological a property.

Proof: Same as Theorem 4.14

Theorem 5.13. Every nano-open subspace of $N\beta$-T_2 space is $N\beta$-T_2 space.

Theorem 5.14. If f: $(U, \tau_R(C)) \to (V, \tau_R'(D))$ is injective, Nα-open, $N\beta$-continuous and V is $N\beta$-T_2 space then U is $N\beta$-T_2 space.

Theorem5.15. If f: $(U, \tau_R(C)) \to (V, \tau_R'(D))$ is injective, $N\beta$-continuous and V is nano-T_2 space then U is $N\beta$-T_2 space.

VI. RESULT

In the above work we have compared investigated their properties of $N\beta$-T_0, $N\beta$-T_1, $N\beta$-T_2 spaces with some of the existing sets by proving the some theorems.

VII. CONCLUSION

We have defined few separations axioms in nano topological spaces and compared its properties with the existing spaces and proved some theorems.

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