Lee-Carter Mortality Forecasting: Application to Mauritian Population

Woodun Dhandevi, Ho Ming Kang, Raja Rajeswary Ponnusamy

Abstract: Recent decades have perceived remarkable improvements in life expectancies which have further driven significant declines in mortality. The unremitting decrease in mortality rates and its systematic underestimation has been drawing the substantial attention of researchers because of its impending effect on population size and structure, social security systems and from an actuarial perspective, the life insurance and pension industry worldwide. The Lee-Carter model has been widely accepted by actuaries among all the projections methods. This paper applies the Lee-Carter model to forecast the mortality rates of Mauritius for the next 20 years. The empirical mortality data sets of the Mauritian population for the period of 1984 to 2016 obtained from the Statistics Department of Mauritius was considered. The index of the level of mortality for each gender, the shape and the sensitivity coefficients for the ages 0 to 85 were obtained using the mortality forecasting model. The Singular Value Decomposition (SVD) was used to forecast the general mortality index for the period from 2017 to 2036. The software R Programme was used to generate the next two decades forecasted death rates of the Mauritian population. The future death rates were assessed using the measures of errors such as Mean Square Error (MSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (AIC). Appropriate life tables, also known as mortality tables are highly important for pricing and reserving in insurance and pension industries. The main objective of this research was to develop unabridged life tables for the Mauritian population using the projected death rates from the Lee-Carter model. With the aid of the forecasted mortality rates, mortality tables for male, female and total population were obtained from R software. Finally, the results indicate that the Lee-Carter model fitted the Mauritian mortality data reasonably well, with a percentage variation explained by Lee-Carter of 80.7%. Forecasted death rates from 2017 to 2036 had low values of MSE, AIC and BIC, showing high accuracy in the results. However, reliability tests were not conducted on the generated mortality tables, owing to time-constraint.

Keywords: Lee Carter model, SVD, life expectancies

I. INTRODUCTION

Mauritius is regarded as one of the developing countries that has experienced drastic improvement in the health and living condition of its population since its independence in 1968. According to the Statistics Department of Mauritius (SDM) in 2016, the life expectancy for females and males at birth in 2015 are 78 years and 71 years respectively, compared to 59 years and 71 years respectively in 1962.

The drastic decline in the mortality rates from1962 to 2015 has deduced that the older population in Mauritius is growing. Although the ageing population has portrayed a great improvement in the health status, it has become a huge challenge for the pension systems in Mauritius, which has found to give substantial consequences on the stability of public finances and future economic growth. To gradually empower the insurance policy pricing and better pension schemes to be provided to the population, it is essential to develop an appropriate model to forecast mortality rates in Mauritius. Particularly, the mortality projections are extensive for both private and public sector financial institutions such as provident funds, social security plans and insurance companies which provide long-term financial products such as retirement, pensions, annuities, life and health insurance policies (Hanna, 2007; Pablo, 2012; Wasana, 2014). In addition, the risk of mortality variations has a significant effect in evaluating and modelling the liabilities relating to such long-term financial products.The modeling of mortality has acknowledged extensiveconsiderations over the years due to the necessity to measure and forecast the death rates. Numerous methods have been developed (Stoeldraijer, 2013), but the most dominant model to forecast mortality rates is the Lee-Carter model (Steven 2005;Booth et. al., 2008; Marie and Arnold, 2010; Lucia and Maria, 2011; Nuraini, 2016). To estimate the parameters in the Lee-Carter model, Marie (2010) has fitted mortality data of the Nordic countries with different approaches such as Singular Value Decomposition (SVD), weighted least square (WLS) and maximum likelihood estimation (MLE). The results revealed that SVD was the best alternative to determine the mortality index in the model. Other studies such as Pengjun (2005) and Pairoteet. al. (2016) also highlighted that SVD was the best technique to estimate parameters in the Lee-Carter model.

Additionally, mortality tables contribute highly to the pricing of insurance policies and pension schemes, Mauritius does not have its own mortality table. Insurance companies and other financial institutions make use of the South-African mortality tables to value and price insurance contracts by assuming certain mortality load, since the life expectancy of Mauritius (74.6 years) is greater than that of South-Africa (62.4 years). This eventually may lead to either over-estimating or under-estimating the prices of insurance policies or pension schemes. For instance, if mortality is improved, it means that the premiums for life insurance policies will be higher as compared to the prices if mortality enhancements are not considered. Consequently, insurance companies might face insolvency in the future,

Revised Manuscript Received on January 19, 2019.

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owing to the application of inappropriate mortality tables. In response to the above problems, this proposed study aspires to explore the Lee-Carter model and apply it to the mortality data of the Mauritian population to be able to project future death rates for the next two decades. In addition, with the aid of the results, i.e. the forecasted values of mortality rate, a mortality table for the Mauritian population will be constructed to an extent whereby the latter can enhance the pricing techniques of insurance companies in Mauritius.

A. Lee-Carter Model

In this study, Lee-Carter model is applied to Mauritian mortality data to project the death rates for the forthcoming two decades. The model will be applicable to 18 different age groups such as 0, 1-4, 5-9, 10-14, 15-19 and so on of males and females deaths data from year 1984 to 2016. The forecasting period will be 20 years, i.e. from 2017 to 2036. The data to be used includes number of deaths $D_{x,t}$ and exposure to death $E_{x,t}$ for lives aged x last birthday during year t.

The central mortality rate $m_{x,t}$ is computed by dividing the number of deaths for age group *x* with the average number of people living for age group *x* in year *t*, as shown in Eq. (1).

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}} \quad (1)$$

The first and foremost application of Lee-Carter model conglomerates a rich yet parsimonious demographic model with statistical time-series methods to forecast age-specific mortality in USA for the period 1990 to 2065. Lee and Carter (1992) have used past age-specific death rates for the entire US population for the period 1933 to 1987 for model fitting. The Lee-Carter model is an extrapolative method and does not require to include awareness about medical, behavioral, or social impacts on mortality variation. The data to be used is annual age-specific death rates.

$$m_{x,t} = e^{a_x + b_x + k_t + \varepsilon_{x,t}} \quad (2)$$

where $m_{x,t}$ is the central mortality rate at age *x* and year *t*

a_x is average (over time) log-mortality at age *x*

b_x measures the response at age *x* to change in the overall level of mortality over time

k_t represents the overall level of mortality in year *t*

$\varepsilon_{x,t}$ is the residual

The Lee-Carter model cannot be fitted by ordinary regression methods since there are no given regressors, which is the reason why SVD is applied to find approximate values of the parameters. It is also assumed that the errors $\varepsilon_{x,t}$ are homoscedastic, of equal variance. Under the constraints $\sum_t k_t = 0$ and $\sum_x b_x^2 = 1$, a_x simply becomes the mean values over time of the $\ln(m_{x,t})$ values for each x as shown below where n is the number of age groups.

$$\hat{a}_x = \frac{1}{n} \sum_1^n \ln(m_{x,t}) \text{ for } x = 0, 1\text{-}4, 5\text{-}9 \ldots 85+ \quad (3)$$

An estimate of k_t is obtained by summing both sides of Eq. 8 over the ages and using $\sum_x b_x^2 = 1$ to obtain $\widehat{k_t} = \sum_x (\ln(m_{x,t}) - \widehat{a_x})$. Similarly, an estimate for b_x can be obtained by differentiating both sides of Eqn. 8 with respect to time t to derive $\widehat{b_x}$ as given by the equation below. Then, b_x captures the relative density of the logarithm of the central death rates to change in the mortality index k_t. The function b_x controls the time-dependent element k_t by age.

$$\hat{b}_x = \frac{(\delta \ln(m_{x,t})/\delta t)}{(\delta \hat{k}_t/\delta t)} \quad (4)$$

Since all the parameters on the right-hand side of the Eq.7 are observation, they cannot be estimated using the Ordinary Least Squares. Owing to this, Lee and Carter (1992) applied the SVD method in fitting the model. Lee and Carter (1992) used the SVD to estimate the parameters. It is a technique which produces matrices when the matrix Z = { $\ln(m_{x,t}) -$ ax} is applied. These matrices are obtained such as:

$$PdQ^1 = SVD(Z_{xt}) = d_1 P_{x1} Q_{t1} + \cdots + d_X P_{xX} Q_{tX} \quad (5)$$

Approximation to the first term gives the estimates $\widehat{b_x} = P_{x1}$ and $\widehat{k_t} = d_1 Q_{t1}$. The above can be illustrated as shown below:

$$Z = \ln(m_{x,t}) - \hat{a}_x$$

That is;

$$Z = \begin{pmatrix} \ln(m_{0,1984-1985}) - \widehat{a_0} & \cdots & \ln(m_{0,2015-2016}) - \widehat{a_0} \\ \vdots & \ddots & \vdots \\ \ln(m_{85+,1984-1985}) - \widehat{a_{85+}} & \cdots & \ln(m_{85+,2015-2016}) - \widehat{a_{85+}} \end{pmatrix}$$

Applying SVD to the matrix Z, the following decomposition is obtained:

$$SVD(Z) = d_1 P_{x1} Q_{t1} + d_2 P_{x2} Q_{t2} + d_3 P_{x3} Q_{t3} \ldots + d_X P_{xX} Q_{tX} \quad (6)$$

Where *X* = rank (Z), d_i (i = 1, 2… *X*) are singular values in inclining order with $P_{x,i}$ and $Q_{t,i}$ (*i* =1, 2, 3… *X*) as the conforming left and right singular vectors.

$$\widehat{b_x} = P_{x1} \text{ and } \widehat{k_t} = d_1 Q_{t1} \quad (7)$$

The Lee-Carter model is popular due to its simplicity for the parameter estimation by SVD. The proportion of variation explained by the Lee-Carter model is given below.

$$\frac{d_1^2}{\sum_{i=1}^X d_i^2} \quad (8)$$

To fit the Mauritian data to the Lee-Carter model, SVD analysis is used.

Projecting is the major aim behind the stochastic modelling. One of the remarkable properties of the Lee-Carter model is that, after it is fitted (i.e. once values of $\widehat{a_x}$, $\widehat{b_x}$ and $\widehat{k_t}$ are calculated), only the mortality index k_t over time must be forecasted for future time points. Lee and Carter (1992) had fitted the ARIMA (0, 1, 0), i.e. random walk with drift for modelling k_t for the US population and had also recommended to use the appropriate ARIMA models for diverse populations.

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Additionally, model selection criterions such AIC, BIC and MSE are used to determine the most appropriate model. It is highlighted that k_t in most cases is captured by ARIMA (0, 1, 0) model which is expressed as follows:

$$k_t = k_{t-1} + d + e_t \quad (9)$$

where d is the drift parameter and e_t is the error term. A forecast will always deviate from the actual value. A forecast error is the difference between the forecast and actual estimate of the parameter. It is usually recommended that the error is as slight as possible. To determine the forecasting errors, model selection tools such as MSE, AIC and BIC will be used in this study.

$$MSE = \frac{\sum (k_t - \hat{k}_t)^2}{n} \quad (10)$$

$$AIC = n\ln(MSE) + 2k \quad (11)$$

$$BIC = n\ln(MSE) + k\ln n \quad (12)$$

II. RESULTS AND DISCUSSION

Figure 1 compares the log mortality versus age from 0 to 85 for male, female and total population. The curves become essentially a straight line after age of 30. The steepness of the mortality curve is given by the value of k_t.Thus, it can be observed that the female mortality curve is steeper than that of the male over most of the age range, i.e. k_t is larger. From the mortality curve of female, it can be deduced that in the late 80s, females had higher mortality rate than males, mostly those of age 5 to 9. This was due to epidemics of malaria and lack of health care in Mauritius. However, year 2000 onwards, females experienced lower mortality rates than males for all ages.

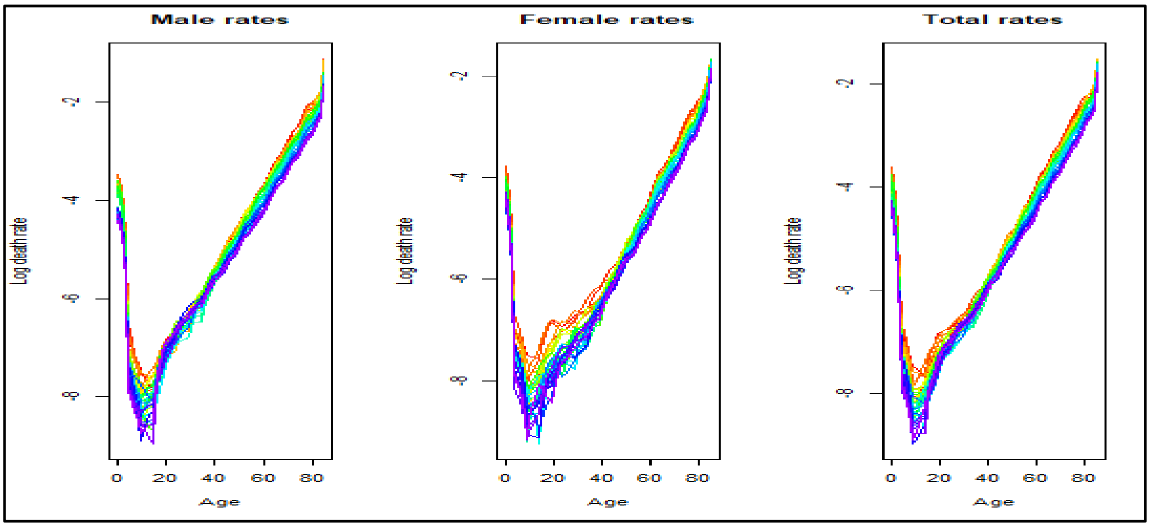

Figure 1: Log-Death rates versus Age

Figure 2illustrates that there is a decreasing trend in the mortality rates of males, females and the total population of Mauritius over the years from 1984 to 2016 for all ages. The first downward purple trend represents the mortality rate for the age 85+ with a higher value as older people are usually prone to infections or diseases. In addition to this, it is apparent that the death rate is falling at all ages with a dissimilar behavior according to different ages. Furthermore, the decrease in death rates is not regular for all ages over the 32 years.

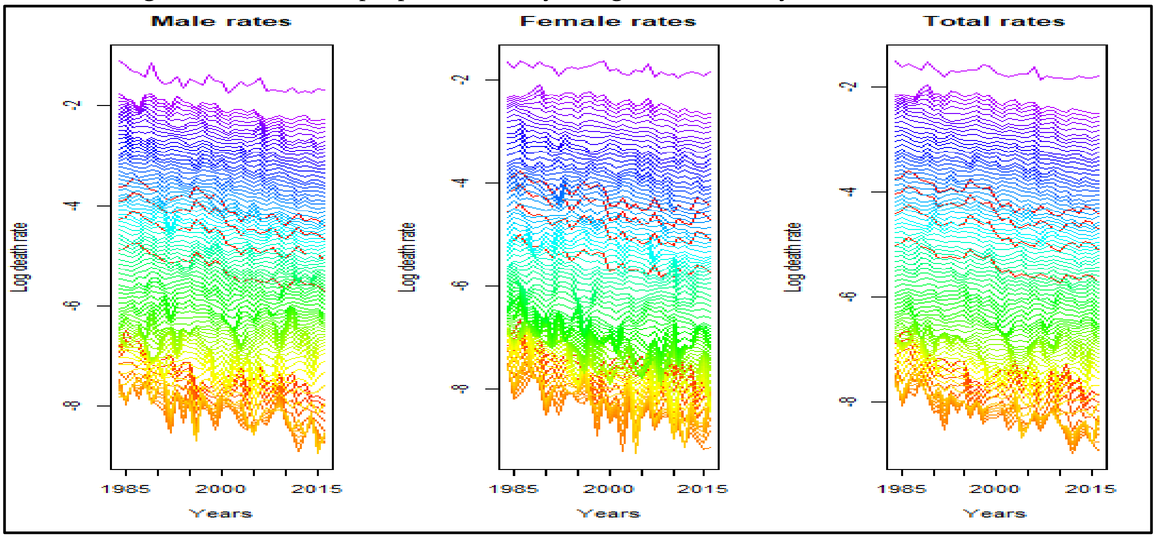

Figure 2: Log-Death rates versus Year.

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The parameter a_x describes the general shape of age-specific death rates as shown in Figure 3. It gives the average mortality, which is observed to grow as the age upsurges. The mortality hump at age 5 can be observed from the graph above, which indicates the high likelihood of youths and teenagers to die. The figure exhibits that the values of a_x for males are higher than that of females, thus it can be deduced that males are more inclined to die faster than females. Furthermore, k_t portrays the foremost trend in the level of mortality relative to the calendar year t. It can be perceived that the mortality rate has a downward trend for males, female as well as for the whole population of Mauritius. The values of k_t for males are lower than that females indicating that the mortality rate for males decreases at a slower pace as compared to females for all the different ages over the years. In addition to this, the estimates for parameter b_x designate the propensity of death at age x to change when the overall level of mortality (k_t) alters. For instance, when b_x is large for some age x, then the death rate at age x fluctuates considerably when the general level of mortality varies and vice-versa.

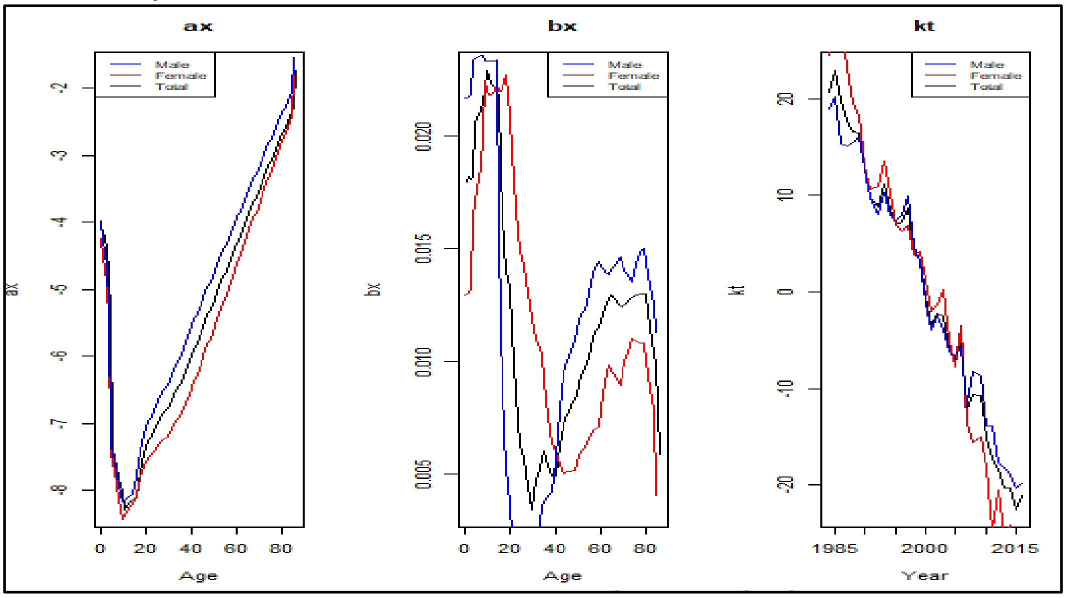

Figure 3: Trends in the parameters of the Lee-Carter model.

Since Lee Carter model is applied to the Mauritian mortality data, an ARIMA (0, 1, 0) is more appropriate in forecasting the mortality index k_t. Table 1 and Figure 4 show the projected values of k_t with the blue line representing the mean of the forecasted k_t values. The forecast is grounded on ARIMA extrapolation.

Table 1: Forecasted values of the mortality index parameter using ARIMA (0, 1, 0)

Age (x)	Forecasted k_t values		
	Male	**Female**	**Total**
2017	-1.2127	-1.5390	-1.3043
2018	-2.4255	-3.0780	-2.6087
2019	-3.6382	-4.6170	-3.9130
2020	-4.8510	-6.1560	-5.2173
2021	-6.0637	-7.6949	-6.5216
2022	-7.2764	-9.2339	-7.8260
2023	-8.4892	-10.7729	-9.1303
2024	-9.7019	-12.3119	-10.4346
2025	-10.9147	-13.8509	-11.7390
2026	-12.1274	-15.3899	-13.0433
2027	-13.3401	-16.9289	-14.3476
2028	-14.5529	-18.4679	-15.6519
2029	-15.7656	-20.0069	-16.9563
2030	-16.9784	-21.5458	-18.2606
2031	-18.1911	-23.0848	-19.5649
2032	-19.4039	-24.6238	-20.8693
2033	-20.6166	-26.1628	-22.1736
2034	-21.8293	-27.7018	-23.4779
2035	-23.0421	-29.2408	-24.7822
2036	-24.2548	-30.7798	-26.0866

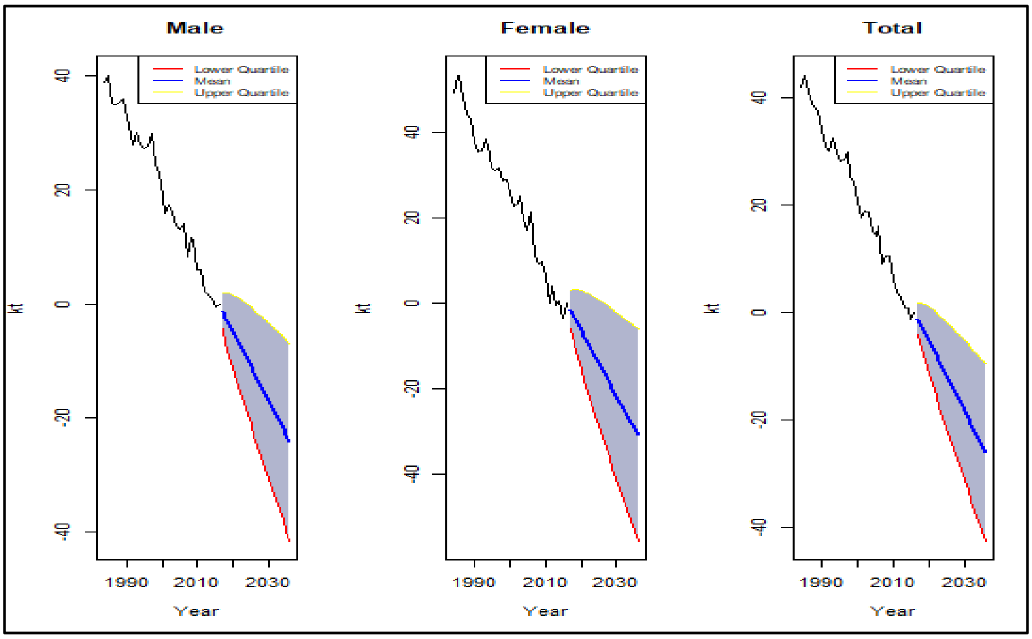

Figure 4: Forecast values of the mortality index parameter.

From the projected values of $k_t$as shown in Table 1, it can be deduced that there will be enormous improvement in the mortality rate of both male and female of Mauritius for the next 20 years. The grey shaded region in Figure 4shows the forecasted trends for all the years ahead. However, the mean of the projected values for k_t is taken into consideration, given by the blue line. In addition to this, the lower and upper quartiles of the projected values for the mortality index are represented by the red and yellow lines respectively at a 95% confidence interval. Lower future mortality rate implies that for the next 20 years pension providers as well as insurance companies will have to charge higher for pension annuities and life insurance policies, because it is predicted that the life expectancy of the Mauritian population will continue to increase.

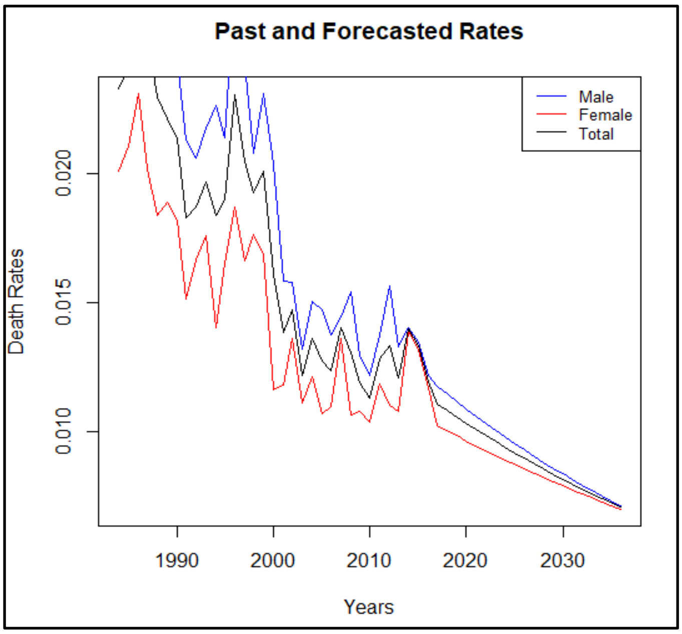

Figure 5: Comparison of past and forecasted death rates.

Figure 5 exhibits that a significant improvement in the mortality rate of the Mauritian population is expected for the years 2017 onward. It can also be observed that the death rates of female decrease more rapidly than that of male for all different ages. In the prior years, it is perceived that the death rates were fluctuating for the different ages for both male and female, i.e. for the years 1984 to 2015. As mentioned formerly, as the life expectancy of the Mauritian population will continue to enhance, pension providers as well as insurance companies offering life insurance policies will sell high premium or expensive products. This eventually leads to high level of longevity risk for both insurance institutions and pension providers. Therefore, in the aim to account for better pricing, appropriate life or mortality tables should be used.

Table 2: Measures of error to assess the forecasted death rates.

Measures of Error	Male	Female	Total
MSE	0.01132	0.01809	0.00788
AIC	-374.90	-335.05	-399.69
BIC	-367.57	-327.73	-398.36

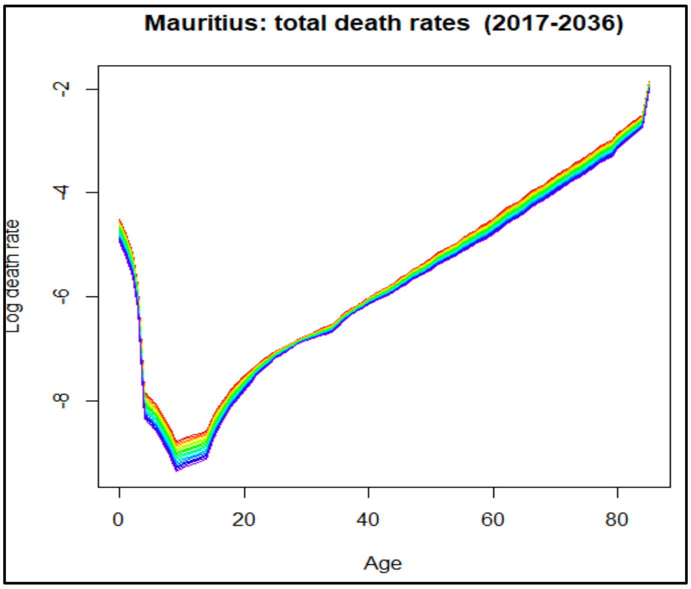

Figure 6: Trend in the forecast death rates for overall population.

Table 3: Evaluation of the forecast death rates.

ERROR MEASURES BASED ON LOG MORTALITY RATES FOR TOTAL **Averages across ages**: ME MSE MPE MAPE **0.00013 0.00788 0.00016 0.01089**
ERROR MEASURES BASED ON LOG MORTALITY RATES FOR FEMALE **Averages across ages**: ME MSE MPE MAPE **0.00037 0.01809 0.00026 0.01547**
ERROR MEASURES BASED ON LOG MORTALITY RATES FOR MALE **Averages across ages**: ME MSE MPE MAPE **0.00074 0.01132 0.00016 0.01373**

Table 3 summarizes the forecasting errors of the projected death rates for male, female and the total population of Mauritius. It can be observed that MSE values are quite low, thus indicating high accuracy in the forecasted mortality rates, whereby MSE for males is 1.132%, that of females is 1.809% and for the total population, the MSE is 0.788% which are almost close to 0. These error measures eventually denote that the Lee Carter model fitted the mortality data of Mauritius reasonably well.

III. CONCLUSION

This study focused on the application of the Lee-Carter model in the aim of forecasting the next 20 years mortality rates of the Mauritian population as well as to develop mortality tables for the country with the aid of the forecasted death rates. Owing to increase in ageing population, there was a prodigious need to estimate future mortality rates to diminish mortality risk during pricing of insurance policies and pension annuities. Thus, a mortality model that can provide accurate predictions becomes vital to sound pricing techniques. Lee-Carter was proposed in this research due to its popularity as well as the fact that it outperforms other recommended models with respect to its predicted errors. Mauritian mortality data from 1984 to 2016 was fitted to the model and the Singular Value Decomposition approach was applied to estimate the parameters. The appeal of the Lee-Carter model is the long-term linearity of its time-series component i.e. the mortality index k_t which is modelled and projected using the ARIMA (0,1,0). However, in future mortality analysis, researchers may make use of a more general class of non-linear time-series model for a more rigorous examination of the parameter k_t.

From the findings obtained, a common trend of mortality changes with age has been identified, in other words, as age increases, mortality rate also increases. It is also observed that the mortality rate of the Mauritian population is expected to continue to decrease for the projection period of 20 years i.e. from 2017 to 2036, as illustrated from the forecasted death rates. The downward trend in the mortality rate of females is faster as compared to that males. Also, from the model selection criterions such as AIC and BIC, it is seen that the Lee-Carter method fitted the Mauritian mortality data reasonably well and the forecasted death rates generated have high accuracy owing to the low values of MSE and MAPE. Furthermore, the negative values of AIC and BIC computed denote that the forecasted death rates are reliable enough to be used in the construction of the life or mortality tables for the years 2017 to 2036. In addition, the results produced show that there is a decrease in mortality rates with age and time for the Mauritian population, which eventually leads to the high likelihood of longevity risk.

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