International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878 (Online), Volume-7 Issue-5, January 2019

Blockchain and transaction Processing time using M/M/1 Queue Model

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Abstract*: The blockchain is an irrefutably a clever invention, where the digital information gets distributed across multiple nodes. The concept was hosted in bitcoin cryptocurrency systems for distribution of coins in a distributed ledger system. Introduction of smart contracts in Ethereum blockchain explored various distinct applications ranging from financial services, supply chain management, healthcare amid others. However, current set of literatures focus more on development and realization of blockchain and petite work is done on mathematical models, performance analysis and optimization of blockchain systems. In this paper, mathematical model is developed using M/M/1 queue model to evaluate transaction processing time. In M/M/1, M symbolizes Markovian arrival and departure of transactions in blocks. The arriving and departure of the blocks are denoted by symbols λ, μ respectively and operates under exponential assumptions. Three different conditions of λ, μ are taken into consideration of complete evaluation of blocks acceptance and the mathematical tactic will open a sequence of possibly favorable research in queueing theory of blockchain systems. The research founds its limitation in acceptance rate is always one unit larger than the arrivals of blocks (termed as an ergodic condition) for stable working of the complete system.

***Index Terms*: M/M/1 queue, blockchain, transaction processing time, Ergodicity.**

I. INTRODUCTION

Blockchain spreads the notion of processing transactions without any mediators [1]. The transactions are distributed by P2P nodes, which are interconnected to each other through hash numbers. Miners compute these hash numbers for the block and based on consensus, the block gets accepted in the blockchain. These blocks contain the ledger of transactions or smart contracts. This evolution of blockchain changed the perspective of internet to source of value or money [2]. World Economic Forum Report, 2017, suggested that 10% of global GDP to be stored in blockchain by 2027 [3]. Despite all the excitement, blockchain is still an immature technology and entails clarifications to unravel the issues of transactions processing time [4]. This discrepancy is because miners are captivated in picking the transactions with higher transaction fees [5], thereby, getting higher rewards, and disregarding transactions with lower transaction fees. With PayPal and Visa processing 1667 and 450 transactions/second, Ethereum and Bitcoin still does 20 and 4 transactions/second only [6],[7]. Initial research by Liu, et. al [8], Wu, et. al. [9], Farayibi [10] and Lehoczky [11] was done for e-Business systems using various traits of queuing theory and requests processing time. However, concept of blockchain queuing theory was introduced by Kasahara and Kawahara [12], [13], who proposed that transaction processing time follows continuous probability distribution. Adding to the research, Li, et. al. [14] further concluded that transaction processing time essentially depends on blockchain confirmation time which is sum of block-generation process and blockchain building process. In block generation process, the miners try to figure the block by stroking all the valid transactions and computing nonce [15], whereas, in block building process, the block after getting consensus from other nodes, gets placed in the blockchain[16]. The present study combines studies conducted by Kasahara and Kawahara [12], [13] and Li, et. al. [14], to mathematically identify the transaction processing time using M/M/1 queuing model. The arrival of transactions at block generation process λ, and departure of transactions in block for blockchain building process μ follows an exponential distribution, with three possibilities [17]:

1) $\lambda > \mu$, the number of orphaned transactions will be much higher, and thus be evaded.
2) $\lambda = \mu$, ideal condition for appraising the transaction processing time, but virtuously theoretical in nature.
3) $\lambda < \mu$, the number of blocks arriving is less than the blocks becoming the part of blockchain building process.

The study is based on condition, $\lambda < \mu$, which is also termed as Ergodicity [17]. The assumption for the proposed model is $\mu = \lambda + 1$.

Rest of the paper is divided as follows: Section 2 presents the Blockchain queue model. Section 3 offers the transaction processing time using M/M/1 queue model. Section 4 presents the outcome and conclusion of the model.

II. BLOCKCHAIN QUEUE MODEL

Evaluation of transaction processing time for blocks in a blockchain can be denoted by Figure 1 below:

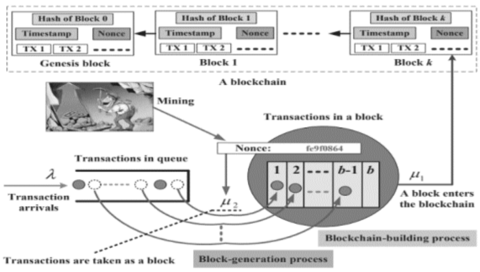

Figure 1: Blockchain Queuing Theory (Li, et. al. [14])

Revised Manuscript Received on 30 January 2019.
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Two important points to observe from Figure 1 are:

- Arrival Process – Transactions arrive at rate λ, random in nature and follows Markov distribution.
- Departure Process – As observed from Figure 1, μ_1 identifies the rate at which the blocks enters blockchain and μ_2 is the rate at which transactions enters the block. To evade any bottleneck in the queue, we considered, $\mu_1 = \mu_2 = \mu$.

III. TRANSACTION PROCESSING TIME BY M/M/1 QUEUE MODEL

The blockchain is like a queue system as given in Figure 2 given below, which makes it a typical case of M/M/1 queuing model.

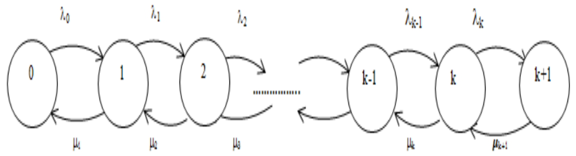

Figure 2: State diagram for M/M/1 queue model

According to [18], the balance equation for M/M/1 queue is dependent upon the linear homogenous equation of the form:

$$0 = \sum_{i \neq j} p_i q_{ij} - p_j q_j \quad (1)$$

where, p_i and p_j are the transition probabilities w.r.t. the transition rates q_{ij} and q_j.

Using equation (1), following set of equations are observed:

$$0 = -(\lambda_k + \mu_k) p_k + \lambda_{k-1} p_{k-1} + \mu_{k+1} p_{k+1} \quad (2)$$

$$0 = -\lambda_0 p_0 + \mu_1 p_1 \quad (3)$$

Rearranging equation (3),

$$\lambda_k p_k - \mu_{k+1} p_{k+1} = \lambda_{k-1} p_{k-1} - \mu_k p_k = \ldots\ldots = \lambda_0 p_0 - \mu_1 p_1$$

But from equation (4),

$$\lambda_0 p_0 - \mu_1 p_1 = 0$$

It follows that $\lambda_{k-1} p_{k-1} - \mu_k p_k = 0$.

Rearranging,

$$p_k = \frac{\lambda_{k-1}}{\mu_k} p_{k-1} \quad \forall k \geq 1$$

Thus,

$$p_k = \frac{\lambda_0 \lambda_1 \ldots\ldots\ldots \lambda_{k-1}}{\mu_1 \mu_2 \ldots\ldots\ldots \mu_k} p_0 = p_0 \prod_{i=0}^{k-1} \left(\frac{\lambda_i}{\mu_{i+1}} \right) \quad \forall k \geq 1 \quad (5)$$

Using equation (5), the steady state probabilities for the blocks in a blockchain can be evaluated as

$$p_k = p_0 \prod_{i=0}^{k-1} \left(\frac{\lambda}{(i+1)\mu} \right)$$

$$= p_0 \left(\frac{\lambda}{\mu} \right)^k \frac{1}{k!} \quad \forall k \geq m$$

(where m is the number of servers)

$$p_k = p_0 \prod_{i=0}^{m-1} \left(\frac{\lambda}{(i+1)\mu} \right) \prod_{j=m}^{k-1} \frac{\lambda}{m\mu} \quad (6)$$

$$= p_0 \left(\frac{\lambda}{\mu} \right)^k \frac{1}{m! m^{k-m}} \quad \forall k \geq m$$

Defining $\delta = \left(\frac{\lambda}{m\mu} \right)$, the condition for the stability is given by $\delta < 1$.

The expression for δ_0 is obtained using equation (6) and the fact that $\sum_{k=0}^{\infty} p_k = 1$.

$$p_0 = \left[\sum_{k=0}^{m-1} \frac{(m\delta)^k}{k!} + \frac{(m\delta)^m}{m!} \frac{1}{1-\delta} \right]^{-1} \quad (7)$$

Thus, the average number of acceptance of blocks is given by the equation (7) as stated below:

$$\sum_{k \geq 0} k p_k = m\delta + \delta \frac{(m\delta)^m}{m!} \frac{p_0}{(1-\delta)^2}$$

$$2\delta + \frac{2(2\delta)^2}{2!} \frac{p_0}{(1-\delta)^2}$$

$$\delta = \frac{\lambda}{2\mu}$$

$$p_0 = \left[1 + 2\delta + \frac{(2\delta)^2}{2!} \frac{1}{1-\delta} \right]^{-1}$$

$$= \frac{1-\delta}{(1-\delta)(1+2\delta) + 2\delta^2} = \frac{1-\delta}{1+\delta}$$

$$2\delta + 2\delta^3 \frac{1-\delta}{(1+\delta)(1-\delta)^2} = \frac{2\delta(1-\delta^2+\delta^2)}{1-\delta^2} = \frac{2\delta}{1-\delta^2}$$

$$= \frac{4\mu}{4\mu^2 - \lambda^2}$$

$$\mu = \lambda + 1$$

Using equation (7), following value of p0 is obtained:

$$p_0 = \left[1 + 2\delta + \frac{(2\delta)^2}{2!} \frac{1}{1-\delta} \right]^{-1}$$

$$= \frac{1-\delta}{(1-\delta)(1+2\delta) + 2\delta^2} = \frac{1-\delta}{1+\delta}$$

Thus, the average number of requests in blockchain becomes:

$$2\delta + 2\delta^3 \frac{1-\delta}{(1+\delta)(1-\delta)^2} = \frac{2\delta(1-\delta^2+\delta^2)}{1-\delta^2} = \frac{2\delta}{1-\delta^2}$$

The average response time using Little's formula is Average number of acceptance of blocks/$\boldsymbol{\lambda}$.

$$= \frac{4\mu}{4\mu^2 - \lambda^2} \quad (8)$$

Putting μ=λ+1, we obtain

$$\frac{4(\lambda+1)}{3\lambda^2 + 8\lambda + 4} \quad (9)$$

Equation (9) depicts close connection of queuing theory with time and its correlation with exponential distribution to evaluate the of waiting time of blocks. Figure 3 illustrates the outcomes of result based on equation (9) based on number of confirmations of blocks.

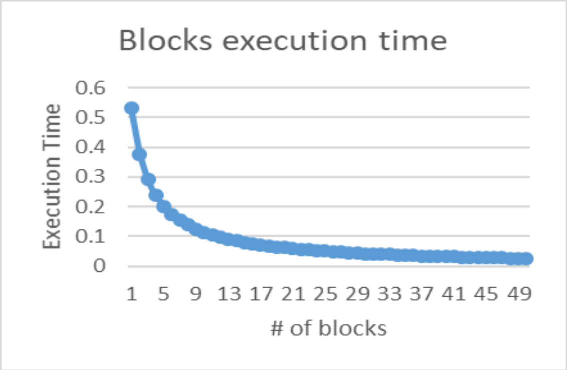

Figure 3: Blocks acceptance in Blockchain-Evaluation using M/M/1 queue

IV. CONCLUSION

There are two conclusions that needs to be emphasized from the theoretical developed model:

Conclusion 1: The result displays that for blocks waiting time is relatively higher when the number of blocks are less and less when blocks increases.

Conclusion 2: Owing to growing importance of blockchain in various business field, evaluation of blocks execution time is independent of number of transactions in the blocks.

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