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# Mechanism of The Effect of Transverse Shifts on The Stress State in The Problems of Plate and Shell Mechanics

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***Abstract*****: *The work considers a series of problems on bending of shallow and flat plates under a normal load, as well as contact problems for the given plates and a solid base. The equations of a theory refined by considering transverse shifts and reduction are taken as the input equations describing the stress-strain state in plates. A systematic effect of the transverse shifts on the stress state is observed in all problems due to the fact that the graphs of bending moments from changes in curvature and from transverse shifts are in antiphase.***

***Index Terms*****: *shallow plate, transverse shift, transverse contraction, bending moment, antiphase effect.***

## I. THE MECHANISM OF THE EFFECT OF TRANSVERSE SHIFTS RECORD ON A DE-CREASE IN THE STRESS STATE IN A SHALLOW SHELL

In order to eliminate the influence of side factors on the stress relaxation mechanism by taking into account transverse shifts, consider the problem of bending an open cylindrical shell (cylindrical panel), rectangular in plan, under the action of a normal load uniformly and distributed over a certain area, similar in shape to the region of the middle surface of the panel [1].Consider an open cylindrical panel as a model, with a length l and width (flat pattern) $R\varphi_0$under a normal load q, evenly distributed over a region $\Omega_\varepsilon$, similar to the middle surface with a coefficient ε, with a constant resultant $Q_0$ (Fig. 1.1). Suppose that the edges of the shell are hinged relative to the displacement of the along a boundary and free from the tangential load normal to the edge. This variant of the boundary conditions allows applying the method of double trigonometric series for solving the problem.

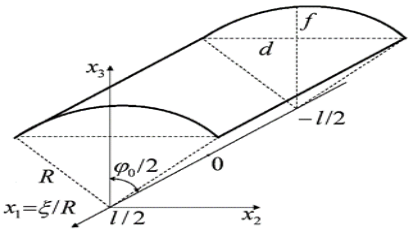


**Fig.1.1. Design of shallow shell**

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The considered shell is characterized by:

- mid-surface area

$$\Omega = \{(\xi,\varphi): |\xi| \le 1/2\xi_0, |\varphi| \le 1/2\varphi_0\}, \xi_0 = \frac{l}{R};$$

- load scope

$$\Omega_\varepsilon = \{(\xi,\varphi): |\xi| \le 1/2\varepsilon\xi_0, |\varphi| \le 1/2\varepsilon\varphi_0\},$$

- load intensity

$$q = \begin{cases} \dfrac{Q_0}{\varepsilon^2 R^2 \xi_0 \varphi_0}, (\xi,\varphi) \in \Omega_0 \\ 0, (\xi,\varphi) \notin \Omega_0 \end{cases}$$

$\xi, \phi$ – dimensionless longitudinal and circumferential coordinates.

The system of refined equations of the cylindrical shells theory [2] in the linear approximation has the following form:

$$\Delta^4 w + 2(1+\nu)\frac{\partial^4 \Delta w}{\partial \xi^4} + 4b^4\frac{\partial^4 w}{\partial \xi^4} = \frac{R^4}{d_0}\left(\Delta^2 q - \frac{kh_\psi^2}{R^2}\Delta^3 q\right),$$

$$\Delta\psi_1 - \frac{1+\nu}{2}\frac{\partial}{\partial\varphi}\left(\frac{\partial\psi_1}{\partial\varphi} - \frac{\partial\psi_2}{\partial\xi}\right) - \frac{R^2}{kh_\psi^2}\psi_1 = \frac{1}{R}\frac{\partial\Delta w}{\partial\xi},$$

$$\Delta\psi_2 - \frac{1+\nu}{2}\frac{\partial}{\partial\xi}\left(\frac{\partial\psi_2}{\partial\xi} - \frac{\partial\psi_1}{\partial\varphi}\right) - \frac{R^2}{kh_\psi^2}\psi_{21} = \frac{1}{R}\frac{\partial\Delta w}{\partial\varphi},$$

$$\Delta^2\Psi = EhR\frac{\partial^2 w}{\partial\xi^2}$$

When *k=1*, the equilibrium equations describe the plate bending according to the Tymoshenko model, and at *k=15/8* – according to the Zhuravsky model.

The boundary conditions of a hinged edge are as follows:

$$\xi = \pm\frac{1}{2}\xi_0: w = 0, \frac{\partial^2 w}{\partial\xi^2} = 0, \psi_2 = 0,$$

$$\varphi = \pm\frac{1}{2}\varphi_0: w = 0, \frac{\partial^2 w}{\partial\varphi^2} = 0, \psi_1 = 0.$$

Equilibrium equations and boundary conditions allow representing the desired functions and loads in the form of double trigonometric series

$$\begin{bmatrix} w \\ \psi_1 \\ \psi_2 \\ q \end{bmatrix} = \sum_{m,n=1,3,\dots} \begin{bmatrix} w_{mn} \cos m_*\xi \cos n_*\varphi \\ \psi_{1_{mn}} \sin m_*\xi \cos n_*\varphi \\ \psi_{2_{mn}} \cos m_*\xi \sin n_*\varphi \\ q_{mn} \cos m_*\xi \cos n_*\varphi \end{bmatrix}$$

Where

$$m_* = \frac{\pi m}{\xi_0}, n_* = \frac{\pi k}{\varphi_0}, q_{mn} = \frac{4Q_0}{R^2\ \xi_0 \varphi_0} \frac{\sin \frac{\pi m \epsilon}{2}}{\frac{\pi m \epsilon}{2}} \frac{\sin \frac{\pi n \epsilon}{2}}{\frac{\pi n \epsilon}{2}}.$$

When substituting the corresponding series in the equilibrium equations, obtain the values for the bending moments of the change in curvature

$$M_{11}^{w} = -\frac{d_0}{R^2}\left(\frac{\partial^2 w}{\partial \xi^2} + \nu \frac{\partial^2 w}{\partial \varphi^2}\right), M_{22}^{w} = -\frac{d_0}{R^2}\left(\frac{\partial^2 w}{\partial \varphi^2} + \nu \frac{\partial^2 w}{\partial \xi^2}\right)$$

and of the changes in transverse shifts

$$M_{11}^{\psi} = -\frac{d_0}{R^2}\left(\frac{\partial\ \psi_1}{\partial \xi} + \nu \frac{\partial\ \psi_2}{\partial \varphi}\right), M_{22}^{w} = -\frac{d_0}{R^2}\left(\frac{\partial\ \psi_2}{\partial \varphi} + \nu \frac{\partial\ \psi_1}{\partial\ \xi}\right).$$

Fig. 1.2 shows the results of a numerical experiment for a shallow plate with the ratio h/R=0.1. The bending moments are marked by the “Zh” index calculated according to the Zhurav-sky theory, and by the “T” index – according to the Tymoshenko theory, respectively.

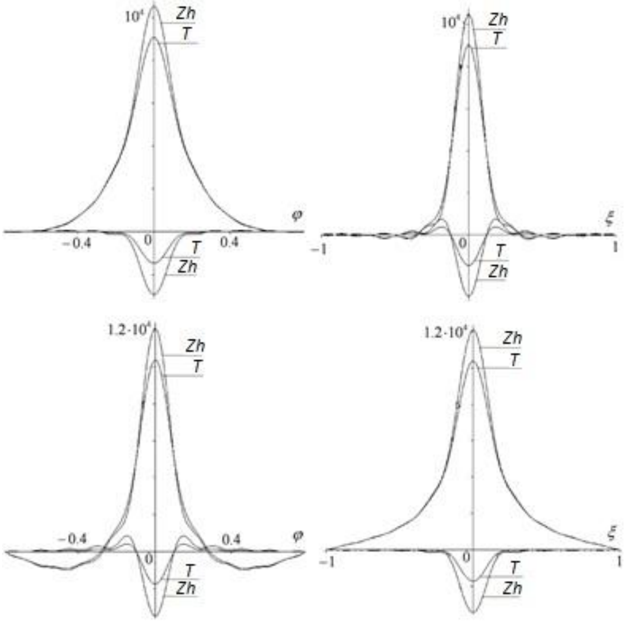


**Fig. 1.2. Moment distribution**

Analysis of the results of the numerical experiment showed that under a load close to the concentrated one:

- the graphs of bending moments of a change in the curvature of the middle surface and of a tangential change in transverse shifts in the region of their maximum modulo values are in antiphase;
- the ratio $M_{ii}^{\psi}/M_{ii}^{w}$ has a maximum value in the center of the plate and increases with increasing parameters *h/R, b/f.*

It can also be concluded that the Novozhilov-Finkelstein criterion for estimating the errors of the Kirchhoff's geometric hypothesis becomes unreliable for stressful states that have a large variability (in this case at $\varepsilon \to 0$, as well as for very shallow shells [3]. During the process of numerical experiment, the dependence of the transverse shifts on the thinness of the wall and the shallowness of the shell is violated if the latter is (relatively) thick.

## II. CONTACT INTERACTION OF CYLINDRICALLY BENT PLATE AND BASE

This section demonstrates the antiphase effect when solving the following contact problem [4]. A rigidly fixed plate of length *l* and thickness *h*, located at a distance z from an absolutely rigid ideally smooth base, is under load $q_n^+$. At the same time, the condition of rigid fastening is fulfilled on the edges of the plat *x=0* and *x=l*, and the other two edges are infinitely removed or loaded so that a cylindrical bend is realized in the plate. It is necessary to determine the plate deflection w and the resulting contact reactions *r(x).*

Using Karman-Tymoshenko's equations [2, 5], the boundary value problem with respect to the deflection of the lower front surface w can be written as follows:

$$Dw^{IV} = q_n - h_\psi^2 q_n'',$$

$$w(0) = 0, w(l) = 0, w'(0) = 0, w'(l) = 0.$$

The Green's function for the considered task is

$$G(x,\xi) = \frac{1}{6}(x-\xi)^3 H(x-\xi) - \frac{(l-\xi)^2(l+2\xi)}{6l^3} + \frac{\xi(l-\xi)^2}{2l^2}x^2.$$

Here *H* is a Heaviside function.

To solve the boundary value problem, use the method of generalized reaction [6], the iteration scheme of which has the form

$$w_k = \frac{1}{D}\int_0^l \left(q_n^+(\xi) - r_{k-1}(\xi) - h_\psi^2 \left(q_n^+(\xi) - r_{k-1}(\xi)\right)''\right) G(x,\xi) d\xi.$$

$$r_k = [r_{k-1} + \beta(w_k - z)]_+, \beta > 0,$$

Here $\phi_+ = \frac{1}{2}(\phi + |\phi|)$ – a positive cut-off function.

As an initial approximation, assume

$$r_0 = 0.$$

Using the resulting deflection and reactions, find the bending moment using the following formula [4]:

$$M = M_w + M_\psi,\ M_w = -Dw'', M_\psi = -\frac{D}{Eh}(q_n^+(x) - r(x)).$$

A numerical experiment was conducted using the proposed iterative scheme. Fig. 2 shows the distribution of moments for one contact problem; more details of the calculation results can be found in the work [4]. Here, the dots indicate the moment $M_\psi$ of shifts, the dotted line – the moment $M_w$ associated with the deflection, the solid line – the final moment $M$. The figure shows that the components of the moment are in antiphase, i.e. using transverse shifts, the maximum values of the total moment are reduced. Also in the course of a numerical experiment, it was noticed that the antiphase effect was not observed when solving conventional problems – “non-contact” or under the influence of smooth loads.

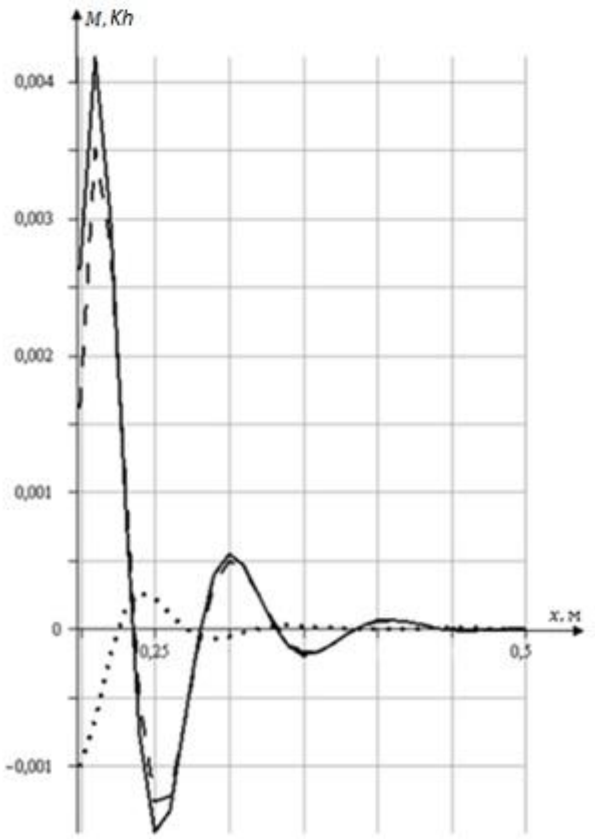


**Fig. 2. Moment distribution example**

## III. CONTACT INTERACTION OF A CIRCULAR AXISYMMETRIC PLATE AND A BASE

We authors demonstrate the "antiphase" effect on the solution of the following nonlinear problem. Consider a circular axisymmetric plate of radius *R*, thickness *h*, hinged at a distance *z* above an absolutely rigid, ideally smooth base. Under the action of a variable load $q_n^+$, the plate is lined on the base without gaps, forming a contact zone $[0, r_0]$.

In the case of axisymmetric bending, write the Karman-Tymoshenko's equations in the following form [2]:

$$D\Delta^2 w = q_n - h_\psi^2 \Delta q_n + \left(I - h_\psi^2 \Delta\right) L(\Phi, w),$$

$$\frac{1}{Eh}\Delta^2 \Phi = \frac{v}{Eh}\Delta m_n - \frac{1}{2}L(w, w),$$

$$\frac{1}{\rho}\frac{d(\rho\psi_\rho)}{d\rho} = -\frac{1}{\mu h}\left(q_n + L(\Phi, w)\right).$$

Here the deflection *w*, the function of stresses $\Phi$, transverse shifts $\psi_\rho$ are unknown functions; $\Delta$ – Laplace operator, $L(\Phi, w)$ – Karman's bilinear form; $q_n = q_n^+ - q_n^-$, $m_n = h/2(q_n^+ + q_n^-)$.

The boundary conditions of the type of hinge will take in the form

$$w(R) = 0, w''(R) = 0, \Phi(R) = \Phi'(R) = 0.$$

Then the Green's functions can be written as:

$$G(\rho, \xi) = \frac{\xi}{4}H(\rho - \xi)\left(ln\frac{\rho}{\xi}(\rho^2 + \xi^2) + \xi^2 - \rho^2\right),$$

$$G_w(\rho, \xi) = G(\rho, \xi) - \frac{\xi}{8}\left(2ln\frac{R}{\xi}(\rho^2 + \xi^2) + 3\xi^2 - 3R^2 + \rho^2 - \frac{\xi^2\rho^2}{R^2}\right),$$

$$G_\Phi(\rho, \xi) = G(\rho, \xi) + \frac{\xi}{8}\left(2R^2 - \xi^2 - 2ln\frac{R}{\xi}\right) - \frac{\xi\rho^2}{8R}\left(\frac{\xi^2}{R} + 2Rln\frac{R}{\xi}\right).$$

To solve the problem from Section 2, apply the generalized reaction method with the following iterative scheme:

$$r_k = [r_{k-1} + \beta(w_k - z)]_+, \beta > 0,$$

where

$$\Phi_{k-1} = Eh\int_0^l \left(\frac{v}{2E}\Delta(q_n^+(\xi) + r_{k-1}(\xi)) - \frac{1}{2}L(w_{k-1}, w_{k-1})\right) G_\Phi(\rho, \xi) d\xi,$$

$$w_k = \frac{1}{D}\int_0^l \left(q_n^+ - r_k(\xi) - h_\psi^2\Delta\left(q_n^+(\xi) - r_{k-1}(\xi)\right)\left(I - h_\psi^2\Delta\right)L(\Phi_{k-1}, w_{k-1})\right) G_w(\rho, \xi) d\xi,$$

$$r_0 = 0.$$

After determining the functions *w*, $\Phi$, $r$ taking into account the boundary condition $\psi_\rho(0) = 0$ one can determine the transverse shifts $\psi_\rho$ as follows:

$$\psi_\rho = -\frac{1}{\mu h \rho}\int_0^\rho (q_n + L(\Phi, w)) d\rho.$$

Fig. 3 shows an example of one calculation, where the solid line indicates the moment of bending $M_w = -D(w'' + \frac{v}{\rho}w')$, the dotted line indicates the moment from shifts $M_\psi = D(\psi_\rho' + \frac{v}{\rho}\psi_\rho)$, dots indicate the total moment $M = M_w + M_\psi$.

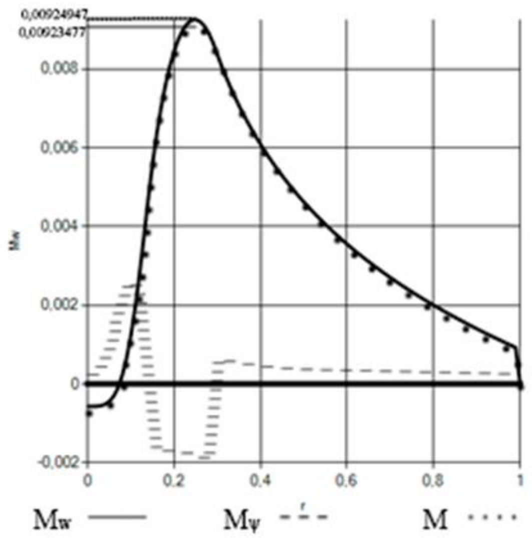


**Fig. 3. Moments**

## IV. CONCLUSIONS

As a conclusion, we would like to high-light the following:
• with the use of refined theories, the effect of "antiphase" is observed when solving such non-classical problems of shell mechanics as contact problems and bending under the load close to concentrated;•using the example of bending of a shallow shell, the unreliability of Novozhilov-Finkelstein criterion for stress states with high variability is shown.

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