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Effect of Seven Steps Approach on Simplex Method to Optimize the Mathematical Manipulation

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Abstract*: the Simplex method is the most popular and successful method for solving linear programs. The objective function of linear programming problem (LPP) involves in the maximization and minimization problem with the set of linear equalities and inequalities constraints. There are different methods to solve LPP, such as simplex, dual-simplex, Big-M and two phase method. In this paper, an approach is presented to solve LPP with new seven steps process by choosing "key element rule" which is still widely used and remains important in practice. We propose a new technique i.e. seven step process in LPP for the simplex, dual-simplex, Big-M and two phase methods to get the solution with complexity reduction. The complexity reduction is done by eliminating the number of elementary row transformation operation in simplex tableau of identity matrix. By the proposed technique elementary transformation of operation has completely avoided and we can achieve the results in considerable duration.

Index Terms*: Linear programming problem (LPP), Key element (KE), Key column (KC), Key row (KR), Profit per unit (PPU), Random variables (RV), Linear Gaussian Random variables (LGRV), standard deviation (SD), Probability Density Function (PDF)

I. INTRODUCTION

To solve a LPP, simplex method is the popular and widely used method. Simplex model in Simulink for ease in visualization and simulation in System Generator are used to achieve a fast implementation. It is efficient tableau based representation and the clock frequency achieved by such design is compared with that in general purpose software [26]. There are some certain steps are needed to solve LPP using Simplex method to implement in Standard form and it is necessary to linear programs before solving the optimal solution. There are some important requirements for the solving of LPP and to optimize: (1) If the objective function is in minization form, it has to change to maximization (2) All linear constraints must be ≤ inequality, (3) All variables must be non-negative. The above three requirements must be satisfied by converting given linear program by using basic algebra and substitution then determine slack variables i.e. To change inequality constraints to equality constraints there are some additional variables have been introduced into the linear constraints of linear program. The slack variables always have a +1 coefficient when the model is in standard form. For optimality the slack variables are introduced. By using the slack variables concept, the coefficient of objective function can be modified. According to the nature of problem it can be easily implemented on computer and these methods overcome the limitations of graphical method and the unnecessary iterations in the search can be skipped. We proposed an approach of seven steps process which resolved certain complication encountered in the application of the simplex method and feasible solution is obtained in iteration first and optimal solution can be obtained. For the efficient solution of large scale LPP some techniques and computational results have been introduced to predict permanent basic and non-random basic variables to implement in linear programming codes to save the computation time by eliminating the number of elementary row transformation operation. In implementation part(V), our proposed approach have introduced in detail with our seven steps process, by using last two steps in simplex tableau, the elementary row transformation operation are completely eliminated.

The general form of a linear program is Max (Z) $=C^TX$
Subject to $AX \leq b$, $X \geq 0$
Where A is a mxn matrix, C is a n-vector and b is an m-vector.
$(A)_{mxn}\, X_i \leq b_{mx1}$, X_i; i=1,2,….,n.

$$\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}_{mxn} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}_{nx1} + S_m = \begin{pmatrix} 8 \\ 12 \end{pmatrix}_{mx1}$$

We get linear system with 5 variables and 3 equations.
Initial simplex tableau:

	X_1	X_2	S_1	S_2	P		
S_1	2(KE)	1	1	0	0	8	KR
S_2	2	3	0	1	0	12	
P	-3	-1	0	0	1		
	KC						

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$$\frac{1}{2r}R_1 \rightarrow R_1 \quad \begin{array}{c|c} & \begin{matrix} X_1 & X_2 & S_1 & S_2 & P & \end{matrix} \\ \begin{matrix} X_1 \\ S_2 \\ \\ P \end{matrix} & \left(\begin{array}{ccccc|c} 1 & 1/2 & 1/2 & 0 & 0 & 4 \\ 2 & 3 & 0 & 1 & 0 & 12 \\ & & & & & \\ -3 & -1 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

$$\begin{matrix} -2R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \end{matrix} \quad \begin{array}{c|c} & \begin{matrix} X_1 & X_2 & S_1 & S_2 & P & \end{matrix} \\ \begin{matrix} X_1 \\ S_2 \\ P \end{matrix} & \left(\begin{array}{ccccc|c} 1 & 1/2 & 1/2 & 0 & 0 & 4 \\ 0 & 2 & -1 & 1 & 0 & 4 \\ 0 & 1/2 & 3/2 & 0 & 1 & 12 \end{array}\right) \end{array}$$

There is no negative indicator in P , so stop..!!

$X_1=4$, $X_2=0$ & Max (Z) =12

In simplex tableau, the column of their initial basic variables, should see permutation of column of identity matrix, for which converting the key element to unity by dividing its row by the leading element itself and all other elements in the column to zero.

In our proposed approach, seven steps process of last two steps, row entry in next simplex tableau with respect to replaced random variables and enters other than key row. The elementary row transformation operation is completely eliminated, which have been proved in "part V "implementation. So, the proposed approach will reduce the complexity of maintaining unit matrix by using elementary row transformation process.

II. BACKGROUND

Solutions of LPP, there are many types of Simplex methods that have been developed which offer some improvement such as reduction in the number of simplex iterations and the number of computations in each iteration. There are several techniques have been proposed. For example, Maximizing of a Linear Function of Variables Subject to Linear Inequalities and Computational Algorithm of the Simplex Method are proposed by Dantzig, G.B [21, 22], as well as by Dr. R.G. Kedia [5] and Lemke, C. E [6] given a concept of A New Variant of Simplex Method and The dual simplex method of solving the LPP through new introduced search technique. Karmarkar [30] developed a polynomial projection approach that is developed version of the ellipsoid algorithm [29], first established LPP can be solved in polynomial time but it performs poorly in practice. However, the simplex method is the popular to solve LPP models and it is very efficient in practice, Borgwardt [31] proved that its expected number of iterations is polynomial when it is applied for practical problems. In optimization problems, the Linear programming concept is applied to a large variety of scientific and industrial computing applications, there are several method have been introduced to solve LPP, out of which some methods have disadvantages and to sort out these issues new method have been introduced, the seven steps simplex algorithm is applicable to all the introduce Simplex method, dual simplex method, Big-M method, Two phase method and for some of the other method also, Big-M method have some disadvantages, Whenever we compare M with any other numeric value, M always will be greater, when number of random variables(RV) and constraints will be more then manually cannot be solve, so we need the help of computer to solve it. It is difficult to determine the value of M that has been resolved through two phase method.

III. LITERATURE REVIEW

In literature, Benichou, M., J. M. Gauthier, G. Hentges, G. Ribi`ere [7] and Hoffman, A., A. Mannos, D. Sokolowsky, D. Wiegmann [11] have introduced the algorithmic techniques and computational experiences to the efficient solution of large scale LPP and Van Roy, T. J., L. A., Wolsey[13] shows automatic reformulation techniques to solve mix LPP and how this reformulation techniques are important for manufacturing firm by applying the profit Preference Scheduling in LPP's have been implemented by Charnes, A., Cooper, W.W.[20], the proposed seven steps algorithm solves constraints integer programs introduced by Achterberg, T[8] and the process to find key element(KE), it is the KE selection methods of LPP introduced by Harris, P. J. J[10] and to optimize the linear programming problem by using polynomial-time algorithm to reduce the complexity by Dimitris Bertsimas and John Tsitsiklis[12,15] , An Alternative Method for LPP, Beale, E.M.L [19] and have been consider a mathematical computational techniques to implement over LPP to reduce the polynomial time complexity and improves functionality of LPP's.

Spielman and Teng's JACM Paper "Smoothed analysis of algorithm: why the simplex algorithm usually takes polynomial time" [32]. The application of smoothed analysis algorithm is a simplex method, which found the complexity with certain condition of standard deviation (σ) of Max $Z^T X$, Subject to; $(A+\sigma G)x \leq y$, where $\sigma \rightarrow 0$ for worst case complexity and σ is ao large that of swamps out A. Based on inputs and its domains, authors have found C-complexity worst case, average case and smoothed C-complexity have found through the concept of Gaussian random variable (GRV) on which mean=0 and variance=1. Linear combination of linear GRV (LGRV's) of $X_1, X_2, \ldots, X_L$ IS $X=a_1X_1+a_1X_1+\ldots+a_LX_L$. So, mean of GRV ($\mu$) = E(X) = $\sum_i a_i \mu_i$ and E $\{(X-\mu)^2\} = \sigma^2$, $\sigma = \sqrt{variance}$, where the range of standard deviation(SD), have considered $0 \leq \sigma \leq 1$, one criticsm of smoothed complexity, under the relative permutation, an input is mapped to constant multiple of itself, $x \rightarrow x(1+\sigma g)$ where g is the GRV of $\mu = 0$ and $\sigma = 1$

So, the linear combination of GRV have an important properties in the content of communication system. When noise is modeled as GRV which is a function of time and in the context of wireless communication channel is modeled as a complex GRV. GRV property have used to obtain the worst case and average case complexity of simplex algorithm on running time with input of form Max $Z^T X$, subject to $(A+\sigma G)X \leq Y$. To obtain the worst case analysis, simplex algorithm, $\sigma \rightarrow 0$ and to obtain average case complexity 'σ' be so large that σG Swamps out A, So a smoothed analysis of algorithm is based on Probability Density Function(PDF) to reduce the complexity of LPP. In our proposed approach the concept of LPP have reduced by removing of elementary row transformation and maintaining unit matrix in simplex tableau.

There are two artificial variable technique Big M and Two Phase method that we use to find the starting basic feasible solution and solve the LPP by using simplex method. In Big M method, first we need to write the standard form of objective function and its constraints by adding some slack and artificial variables to get the initial basis and accordingly write the decision variable, under x_j we consider all the variables, like decision, slack or surplus and artificial variables to get the initial basic feasible solution, we required to make all decision variable to zero and get the result of introduced variable. There is one disadvantage exist in Big-M method, i.e. whenever we compare M with any other numeric value, M will always be greater, M is very large value i.e. near to infinity. For big M, to get the optimal solution it contains any artificial variables in the positive values if and only if the problem is not feasible, so artificial variable would not be a part of any feasible solution.

When number of RV and constraints will be more, than it is very difficult to solve manually to compute the value of M, we need the help of computer to solve it, another method i.e. two phase method have been introduced to sort out this issues.

A. Restriction to Normalize

Slack, surplus and artificial variable are added to change the inequalities to equalities for equations.

Table 1 Normalize restrictions

Inequality type	Variable that appears
≥	- Surplus + Artificial
=	+ Artificial
≤	+ Slack

The following steps of algorithm are:

i. Right hand side should be positive by multiplying inequality constraints.
ii. In case of minimization, multiply the objective by -1 to convert into maximization.
iii. For > constraints, use surplus and artificial variables.
iv. Select value M and term the objective form as -M with artificial variables.
v. In ≤ constraints, slack variables are added to equalize it.

B. Background study of Big-M and Two-Phase method

$$\text{Maximize } Z = 2x_1 + x_2 + 3x_3$$

$$\text{Subject to. } \quad x_1 + x_2 + 2x_3 \le 5$$
$$2x_1 + 3x_2 + 4x_3 = 12$$
$$x_1, x_2, x_3 \ge 0$$

To get the initial basis, we required to add artificial variable x_5, first to write in standard form.

$$\text{Maximize } Z = \quad 2x_1 + x_2 + 3x_3 + 0x_4 - Mx_5$$

$$\text{Subject to. } \quad x_1 + x_2 + 2x_3 + x_4 + 0x_5 = 5$$
$$2x_1 + 3x_2 + 4x_3 + 0x_4 + x_5 = 12$$
$$x_j \ge 0, \quad j = 1,2,3,4,5$$

Initial basic feasible solution x_4=5, x_5=12.

After applying Big M method through proposed algorithm, the solution will be x_1=3, x_2=2, x_3=0. Maximize Z= 8

Due to disadvantages in Big M method, another method has been introduced. i.e. two phase method.

Phase one of the simplex method handles the computation of an initial feasible basis, which is handed over to phase two, the simplex method as we described it so far, T. Kitahara and S. Mizuno [1,2] have introduced a bound for different basic and basic feasible solution which have been generated by simplex method with selection rule of incoming variables with KE closes an incoming variable whose reduced cost is negative at each iteration and also introduced a bound for the dual simplex method for LPP having optimal solution that have been shown some basic result when it is applied to specified LPP. T. Kitahara and S. Mizuno, Divya K. Nadar and V. Klee and G. J. Minty [3, 4, and 17] have introduced some application, properties and evaluation of computational amount of the simplex method and shown how good is the simplex algorithm? In this paper, we addressed seven steps simplex algorithm to solve LPP to reduce complexity over mathematical computation.

Two phase method:

In the initial phase, we are not taking the actual coefficients of objective function, we are creating one auxiliary objective function, whatever slack and surplus variables and the original decision variables takes.

Coefficient of objective function including slack and surplus zero, and coefficient of artificial values, we add coefficient as -1.

In phase-I, the aim to eliminate artificial variables from the basis and calculate initial basic feasible solution.

In phase-II, we apply simplex method with original coefficient of objective function and not imposing penalty with Big-M method.

Two phase implementation

i. Convert each of the constraints into equality constraints.
ii. New auxiliary objective function.

$$\text{Max } Z^* = 0x_1 + 0x_2 + 0x_3 + \ldots .0x_n - 1x_{a1} - 1x_{a2} - 1x_{a3} \ldots - 1x_{am}$$

(-) added with all artificial variables.

Max Z*=0 Zero assigned for all artificial variables

Max Z*< 0 Positive assigned at least 1 artificial variables

Apply the two phase Simplex algorithm.

Suppose $C_j - Z_j \le 0$, the phase-I ends.

a) Max Z*=, All the artificial variables disappears from the basis and we will obtain basic feasible solution.
b) Max Z*= 0, one or more artificial variable appear in the basis with zero value, will obtain basic feasible solution.
c) Max Z *< 0, one or more artificial variables appears in basis with positive value, not obtain any basic feasible solution for the problem.

At the end of phase-I, if case (a) or (b) occur, then it will move for phase-II else end of the phase-I.

Phase-II: There is no basic feasible solution for (c)

Assign actual coefficient of the variables of objective function, we are taking original objective function.

Max Z=C_1x_1+C_2x_2+C_3x_3+……

Now apply simplex algorithm to get the solution.

Examples for two phase method:

Example 1, Solution exist

$$\text{Maximize } Z = 2x_1 + x_2 + 3x_3$$

Subject to. $\quad x_1 + x_2 + 2x_3 \le 5$

$2x_1 + 3x_2 + 4x_3 = 12 \quad x_1, x_2, x_3 \ge 0$

To get the initial basis, we required adding artificial variable, x5 is only the artificial variable.

$$\text{Maximize } Z^* = 0x_1 + 0x_2 + 0x_3 + 0x_4 - x_5$$

Subject to.

$$x_1 + x_2 + 2x_3 + x_4 + 0x_5 = 5$$
$$2x_1 + 3x_2 + 4x_3 + 0x_4 + x_5 = 12$$
$$x_j \geq 0, \quad j = 1,2,3,4,5$$

After applying simplex algorithm over phase-I and phase-II.

$x_1=3$, $x_2=2$, $x_3=0$ and $Z_{max}=8$

Example 2. No solution exist

Maximize $Z = 3x_1 + 2x_2$

Subject to. $2x_1 + x_2 \leq 2$
$$3x_1 + 4x_2 \geq 12$$
$$x_1, x_2 \geq 0$$

Condition with constraints: if the constraints form $\leq$, Either of these two method use to solve LPP, Two phase or Big-M method and required to add artificial variables.

$$Maximize\ Z = 0x_1 + 0x_2 + 0x_3 + 0x_4 - 1x_5$$

Subject to. $2x_1 + x_2 + x_3 = 2$
$$3x_1 + 4x_2 - x_4 + x_5 = 12$$
$$x_j \geq 0, \quad j = 1,2,3,4,5$$

Phase-I: Apply simplex algorithm

When, C_j-$Z_j \leq 0$ stop iteration of phase-I, the basis contains one artificial variable $x_5=4$, positive value.

If the artificial variable present in the basis and its value in the basis is positive, then no solution exist.

So, No solution exist.

Example 3. Unbounded solution exists.

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to. $-3x_1 + 2x_2 + 3x_3 = 8$
$$-3x_1 + 4x_2 + 2x_3 = 17$$
$$x_1, x_2, x_3 \geq 0$$

Two artificial variables required to add with constraints.

$$Maximize\ Z^* = 0x_1 + 0x_2 + 0x_3 - 1x_4 - 1x_5$$

Subject to. $-3x_1 + 2x_2 + 3x_3 + x_4 = 8$
$$-3x_1 + 4x_2 + 2x_3 + x_5 = 7$$
$$x_j \geq 0, \quad j = 1,2,3,4,5$$

When C_j-$Z_j \leq 0$ for all j, Phase-I ends here.

Note: in phase-II, the ratios are becomes negative. Both the ratios are negative. So, unbounded solution exists.

IV. SYSTEM DESIGN

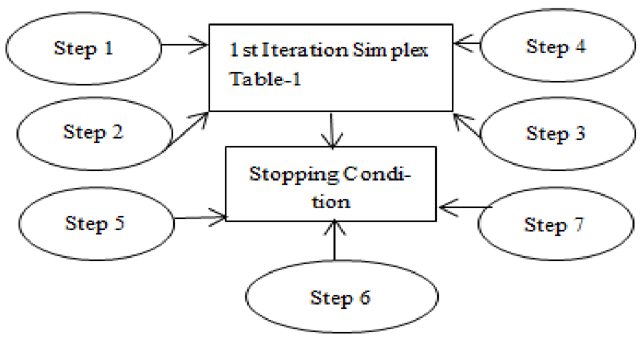

Table 1 Framework for Overall Assessment methodology based on seven steps Algorithm

Conditions to reach up to the optimality; for maximization problem; all C_j-$Z_j \leq 0$; for minimization problem; all C_j-$Z_j \geq$ Figure 1 shows our model of Simplex Solver Conditions to reach up to the optimality for maximization problem all C_j-$Z_j \leq 0$ and for minimization problem all C_j-$Z_j \geq 0$. Simplex iteratively searches for the optimal solution and checks the feasible region for its computation. At the end of the first iteration act as starting point in the next step of stopping condition, i.e. the condition to reach up to the optimality.

From step 1 to step 4, we find out the key column, i.e. the highest positive value of C_j-Z_j, key row (KR), i.e. the lowest positive value of ratio solution to the element of key column, for the first iteration, finally we get a KE, i.e. the intersection point of key column and KR, then we move to the next simplex table to reach up to the condition of optimality. In Step 5. Replace a random variable of KR with the variable of key column and update its respective C_b with the coefficient of objective function of key column and then we move for next step i.e. Step 6. For row entry in next simplex table with respect to replaced random variable(key rows entry in previous simplex table /KE of previous simplex table) and the last step to reach up to the optimality condition is Step 7, i.e. the New entries in other than KR (Previous values in other than key row-{New entries in key row*element of key column (other than element of KR} of its respective value), at the end of the algorithm we check the stopping condition, For Maximization C_j-$Z_j \leq 0$ and for Minimization C_j-$Z_j \geq 0$. So, the KE is normalized and other values of key column are cancelled.

A. Duality theory/ Dual simplex method to reduce the solution complexity

To find out the solution of LPP thorough the method of duality, first have to check either the given problem is in canonical form or not, if it is not in canonical, change in canonical the apply the Simplex algorithm to solve it.

Initial basic feasible solution, where the feasibility condition is always satisfied whenever forming initial simplex table.

Here, we check either optimality condition is satisfied or not, if optimality condition is not satisfied then we change the basis and going to next iteration, here in each iteration feasibility condition is maintained. Dual Simplex method is just the opposite of simplex methods, i.e. we are starting with initial optimal condition, i.e. optimality satisfied but feasibility may not be satisfied, so, in each iteration , we are changing to basis and trying to check either feasibility condition is satisfied or not. In each iteration optimality must be satisfied. If any LPP which has n variables and m constraints, then in dual we have just opposite on, m variables and n constraints. In dual problem, we have to minimize the consumption of the resources in subject to the condition and subject to the profit maximization constraints. Minimize the consumption of the resources subject to the maximization of the constraints.

We use dual method to reduce the solution complexity. Features:

i. Feasible solution of dual model provides a bound on the objective of original primal problem.
ii. Optimal solution of a dual is equal to optimal solution of the primal problem.
iii. Dual of dual model becomes original primal problem.

B. Primal and its dual

1. Primal and it's dual with ≥ sign.

Primal

$\text{Min } C^T X$

Subject to. $AX \geq b$

$X \geq 0$

Dual

$\text{Max } b^T Y$

Subject to. $A^T Y \leq C^T$

$Y \geq 0$

2. Primal and it's dual with ≤ sign.

Primal

$\text{Max } C^T X$

Subject to. $AX \leq b$

$X \geq 0$

Dual

$\text{Min } b^T Y$

Subject to. $A^T Y \geq C^T$

$Y \geq 0$

C. General structure to convert from primal to dual

Primal

$\text{Max } Z = C_1 x_1 + C_2 x_2 + \ldots\ldots + C_n x_n$

Subject to. $a_{i1} x_1 + a_{i2} x_2 + \ldots\ldots + a_{in} x_n \leq b_i$

$x_i \geq 0$

$i = 1, 2, 3 \ldots\ldots\ldots\ldots m$

Dual

$\text{Min } W = \sum_{i=1}^{m} b_i v_i$

Subject to. $a_{i1} v_1 + a_{i2} v_2 + \ldots\ldots + a_{im} v_m \geq C_i$

$j = 1, 2, 3 \ldots\ldots\ldots\ldots n$

$v_1, v_2, v_3, \ldots\ldots v_m \geq 0$

Convert:

$\text{Min } W = \sum_{j=1}^{n} a_{ij} x_j \leq b_i \Rightarrow \sum_{j=1}^{n} a'_{ij} x_j \geq b'_i$

$a'_{ii} = -a_{ii.}$ $\quad b'_i = -b_i$

If

$\sum_{j=1}^{n} a_{ij} x_j = b_i \Rightarrow \sum_{j=1}^{n} a_{ij} x_j \leq b_i \;\&\; \sum_{j=1}^{n} a_{ij} x_j \geq b_i$

Results in different forms of LPP

Symmetric LPP '≤' or '≥'

Un-symmetric LPP '='

Mixed LPP '≤' or '≥' or '='

Theorem1: if any constraint in primal is strict equality then corresponding dual variable is unrestricted in sign. a≥0, b≥0, C=a-b, here a, b and c are positive variables, then c either 0 or positive or negative. So, c is unrestricted in sign.

Theorem 2: if any variable of the primal model is unrestricted in sign, then the corresponding constraints of the dual will be equality.

D. If the problem is already in canonical form

Primal

$\text{Maximize } Z = 3x_1 + 2x_2$

Subject to. $3x_1 + 4x_2 \leq 22$

$3x_1 + 2x_2 \leq 16$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

Problem is in the form of Maximization.

So, it is already in canonical form.

Here no. of constraints=3, No. of variables=2

Dual

Let us assume dual variables as v_1, v_2 and v_3.

$\text{Minimize } W = 22v_1 + 16v_2 + 3v_3$

Subject to. $3v_1 + 3v_2 \geq 3$

$4v_1 + 2v_2 + v_3 \geq 2$

$v_1, v_2, v_3 \geq 0$

Here no. of constraints=2 No. of variables=3

E. If the problem is not in canonical form

Primal

$\text{Minimize } Z = x_3 + x_4 + x_5$

Subject to. $x_1 - x_3 + x_4 - x_5 = 2$

$x_2 - x_3 - x_4 + x_5 = 1$

$x_j \geq 0$

$j = 1, 2, 3, 4, 5$

Here no. of variables=5

No. of constraints=2

Here, all the constraints are in equality form. So, first it's required to change in canonical form to find its Dual.

$\text{Maximize } Z^* = -x_3 - x_4 - x_5$

Subject to. $x_1 - x_3 + x_4 - x_5 \leq 2$

$-x_1 + x_3 - x_4 + x_5 \leq -2$

$x_2 - x_3 - x_4 + x_5 \leq 1$

$-x_2 + x_3 + x_4 - x_5 \leq -1$

$x_j \geq 0$

$j = 1, 2, 3, 4, 5$

Let us assume dual variables as w_1, w_2, w_3 and w_4.

Dual

$\text{Min } D = 2w_1 - 2w_2 + w_3 - w_4$

In the given objective function, x_1 and x_2 are not present.

So, the first two constraints of dual will be ≥ 0.

Subject to. $w_1 - w_2 \geq 0$

$w_3 - w_4 \geq 0$

$-w_1 + w_2 - w_3 + w_4 \geq -1$

$w_1 - w_2 - w_3 + w_4 \geq -1$

$-w_1 + w_2 + w_3 - w_4 \geq -1$

$w_j \geq 0, \text{ for all } j$

But as per the condition of duality, the no. of variables and constraints must be 2 and 5.

So, our assumptions are w_1-w_2=v_1 and w_3-w_4=v_2, here v_1 and v_2 are two positive numbers.

So, v_1 and v_2 are unrestricted in sign.

$\text{Minimize } D = 2v_1 + v_2$

Subject to. $v_1 \geq 0$

$v_2 \geq 0$

$-v_1 - v_2 \geq -1$

$v_1 - v_2 \geq -1$

$-v_1 + v_2 \geq -1$

$v_1, v_2 \geq 0$

F. Canonical problem with unrestricted in sign

Primal

$\text{Minimize } Z = x_1 + x_2 + x_3$

Subject to. $x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \leq 3$

$2x_1 - x_3 \geq 4$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.
Let us assume $x_3 = x_3' - x_3''$
So, $x_3', x_3'' \geq 0$
Maximize $Z = -x_1 - x_2 - x_3' + x_3''$

Subject to.
$$x_1 - 3x_2 + 4(x_3' - x_3'') \leq 5$$
$$-x_1 + 3x_2 - 4(x_3' - x_3'') \leq -5$$
$$x_1 - 2x_2 \leq 3$$
$$-2x_1 + x_3' - x_3'' \leq -4$$
$$x_1, x_2, x_3', x_3'' \geq 0$$

Now it is in canonical form.
Dual
Let us assume dual variables as v_1', v_1'', v_2 and v_3.
So, its corresponding dual is
Minimize $Z^* = 5v_1' - 5v_1'' + 3v_2 - 4v_3$

Subject to.
$$v_1' - v_1'' + v_2 - 2v_3 \geq -1$$
$$-3v_1' + 3v_1'' - 2v_2 \geq -1$$
$$4v_1' - 4v_1'' + v_3 \geq -1$$
$$-4v_1' + 4v_1'' - v_3 \geq 1$$
$$v_1', v_1'', v_2, v_3 \geq 0$$

Let us assume $v_1' - v_1'' = v_1$
Minimize $Z^* = 5v_1 + 3v_2 - 4v_3$

Subject to
$$v_1 + v_2 - 2v_3 \geq -1$$
$$-3v_1 - 2v_2 \geq -1$$
$$4v_1 + v_3 = -1$$

$v_2, v_3 \geq 0$ and v_1 is unrestricted in sign.

V. RESULT BASED ON DIFFERENT ASSUMPTIONS

Primal Problem	Dual Problem	Conclusion
Feasible solution	Feasible solution	Finite optimal solution for both primal and dual.
Feasible solution	No Feasible solution	Primal objective function is unbounded
No Feasible solution	Feasible solution	Dual objective function is unbounded
No Feasible solution	No Feasible solution	Solution does not exist

Table 2 Result based on different primal-dual assumptions

Dual simplex method is the mirror image of the simplex method, like dual method is the mirror image of the primal method.
For corresponding vector 'b', AX=b, in simplex, b is always positive but in dual simplex b must be negative.
In dual simplex method we don't use any artificial variable. So, computational process and effort get reduced. In dual simplex method, initially it is not required to check feasibility condition but it is important to check optimality condition is satisfied or not, optimality condition must be satisfied.
AX=b, Z_j-C_j≥0 for all j, a_j & c_j does not depends on vector b. In initial table, optimality condition must be satisfied, but the feasibility condition may not be satisfied, if optimality condition does not exist, we cannot apply dual simplex method.

VI. STEPS OVERALL SIMPLEX METHOD

1. If the objective functions of given problem in minimization form, convert it to maximization problem.
2. Introduce the slack variables to form the basis vectors and construct the usual simplex table.
3. X_{Bi}=> initial basic solution.
4. Compute Z_j-C_j:

(i) If Z_j-C_j ≥0 for all j (optimality condition) and X_{Bi} >0 for all I (feasibility condition) both exist, then the corresponding solution is the optimal basic feasible solution.
(ii) If at least one Z_j-C_j <0 => the dual simplex method is not applicable.
(iii) If Z_j-C_j ≥0 for all j and at least one random variable X_{Bi}<0 exist, go to step 5.

5. Dual simplex method is the mirror image of the simplex method, at first we have to check which one is the departing vector, vector which has to remove from the basis.

Select the most negative value of X_{Bi}.X_{Br}= Min { X_{Bi}, X_{Bi}<0}, a_r to the departing vector.

6. Check y_{rj} (row value) for all j.

(i) If y_{rj}>0 for all j => No feasible solution exist.
(ii) If y_{rj}<0 for at least one j.
(Z_k-C_k)/ y_{rk}= Max {(Z_j-C_j/ y_{rj}; y_{rj}<0}, corresponding vector a_k inter into the basis.

VII. ARTIFICIAL CONSTRAINTS METHOD FOR INITIAL BASIC FEASIBLE SOLUTION

If $Z_j - C_j < 0$, introduce new constraints
$\sum_j x_j \leq M;\ M > 0$ and sufficiently have large
=> $\sum_j x_j + x_M = M, for\ j = p \mid Z_p - C_p \mid$ has maximum value.
$x_p = M - (x_M + \sum_{j \neq p} x_j)$,
Substitute this value in the objective function and constraints to get modified one and it will ensure that the optimality condition satisfied.

A. When $Z_j - C_j$ is satisfying the condition

Minimize $Z = x_1 + x_2$

Subject to.
$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1, x_2 \geq 0$$

Canonical form of given LPP.
Maximize $Z^* = -x_1 - x_2$

Subject to.
$$-2x_1 - x_2 + x_3 = -4$$
$$-x_1 - 7x_2 + x_4 = -7$$
$$x_1, x_2, x_3, x_4 \geq 0$$

If $Z_j - C_j \geq 0$ for all j but $X_{B1} = x_3 = -4$ (i.e negative) and $X_{B2} = x_4 = -7$ (i.e. negative), so, optimal solution is infeasible.
For leaving into the basis: Min$\{X_{Bi}, X_{Bi} < 0\} = -7$. So, x_4 will leave the basis.
For entering into the basis: Max $\{(Z_j - C_j)/y_{2j};\ y_{2j} < 0\} = -1/7$, which corresponds to the vector x_2, therefor x_2 will enter into the basis.

So, KE (intersection point of the departing and entering vectors) is -7, again $Z_j - C_j > 0$, but feasibility condition not satisfied, because one of the X_{Bi} value is negative, here, departing vector x_3 and entering vector x_1.
Again in next Simplex table.
$Z_j - C_j > 0$ For all j and X_{Bi}≥0 for all i,
The result of x_i*, x_2* and z* is to be calculated.

B. When $Z_j - C_j$ is not satisfying the condition

Maximize $Z = 3x_1 + 2x_2$

Subject to. $2x_1 + x_2 \leq 5$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

Corresponding canonical form by introducing slack variables.

Maximize $Z = 3x_1 + 2x_2 + 0x_3 + 0x_4$

Subject to. $2x_1 + x_2 + x_3 = 5$

$x_1 + x_2 + x_4 = 3$

$x_i \geq 0$ For all i.

C. If $Z_j - C_j < 0$ for all j

In initial table, optimality condition is not exist.
So, add artificial variable.
If $x_1 + x_2 \leq M, M > 0$ Add artificial variable x_M, it becomes
$x_1 + x_2 + x_M = M$

Max $\{ |Z_1 - C_1|, |Z_2 - C_2| \} = 3 =>$ This corresponds to variable x_1, so replace x_1 by $M - x_2 + x_M$.

Maximize $3M - x_2 - 3x_M$

Subject to. $-x_2 + x_3 - 2x_M = 5 - 2M$

$x_4 - x_M = 3 - M$

$x_1 + x_2 + x_M = M$

$x_1, x_2, x_M \geq 0$

Formulate initial basic simplex table and check in modified form either optimality condition exist or not.
In corresponding table.
Max {-,-} =? x_2 is entering vector, so x_2 is entering and x_3 is departing from the basis.
X_4 vector is departing & x_m vector is entering into the basis.
$Z_j - C_j \geq 0$ and $X_{Bj} \geq 0$ for all j.
So, here satisfying both optimality as well as feasibility condition. Stop here and delete the row corresponding to the artificial variable.
The result of x_i*, x_2* and z* is to be calculated.
If optimality condition not satisfied initially, in this case also using the artificial constraints.
We are reconstructing and reformulating the problem by introducing the artificial constraints and slack variables to make the artificial constraints as equality constraints and replacing one variable from here which satisfies the maximum of $|Z_j - C_j|$ criteria and reformulating the problem, and in reformulated problem, once we are constructing the initial table to find the optimality condition is satisfied.

VIII. IMPLEMENTATION

Maximize $Z = C_1x_1 + C_2x_2 + \ldots\ldots + C_jx_j$

Subject to. $a_1x_1 + a_2x_2 + \ldots\ldots \leq m_1$

$b_1x_1 + b_2x_2 + \ldots\ldots \leq m_2$

$x_1, x_2, \ldots\ldots, x_j \geq 0$

Here, , $x_j = no.of\ decision\ variables\ in\ constraints.$
Standard form:

Maximize $Z = C_1x_1 + C_2x_2 + \ldots + C_jx_j + 0s_1 + 0s_2 + \ldots + 0s_k$

Subject to. $a_1x_1 + a_2x_2 + \ldots\ldots + s_k = m_k \ (k = 1)$

$b_1x_1 + b_2x_2 + \ldots\ldots + s_k = m_k \ (k = 2)$

$x_1, x_2, \ldots\ldots, x_j, s_1, s_2 \ldots\ldots s_k \geq 0$

Here,
, $s_k = no.of\ slack\ variables\ in\ constraints.$

Max $Z = \sum_{i=1}^{j} C_jx_j + \sum_{k=1}^{k} 0s_k$

Subject to. $\sum_{j=1}^{j} a_jx_j + s_k \leq m_k (k = 1)$

$\sum_{j=1}^{j} b_jx_j + s_k \leq m_k (k = 2)$

$\sum_{j=1}^{j} x_j, \sum_{k=1}^{k} s_k \geq 0$

C_j= coefficient of objective function of standard form.(After adding the slack variables with zero coefficient)
Number of constraints=number of slack variables.
No. of decision variables in constraints=no. of decision variables in objective functions.
k=no. of constraints= no. of slack variables.

Table 1. 1st iteration

Cj (Coefficient of Objective function) =>				C_1	C_2	…	C_j	0	…	0	
BV	**PPU (C_b)**	**SV (P_0 = m_k)**	P_j =>	P_1 = x_1	P_2 = x_2	…	P_j = x_j	s_1	…	s_k	R_k
s_1	0	m_1		a_1	a_2	…	…	1	0	0	R_1
s_2	0	m_2		b_1	b_2	…	…	0	1	0	R_2
s_3	0	m_3		c_1	c_2	…	…	0	0	1	R_3
…	…	….		…	….	…	…	…	…	…	…
s_k	0	m_k		…	…	…	…	…	…	…	R_k
C_j-Z_j				C_1	C_2	…	C_j	0	…	0	

Table 3 Formal structure of simplex table (1st iteration)

In table 3, bolded PPU, BV and SV in top row of the tableau states Profit per unit (PPU), RV and solution value of the constraints, P_j and C_j represents the objective function variables of standard form and its coefficients.
The rest of rows represent linear constraint variable coefficients from LPP.
Step 1: for simplex table: 1. Z_j=0, for all j
Key column= select a column with maximum coefficient value of objective function= column of ($Max^m(C_j)$).
For next simplex table.
Before finding key column, we required to find Z_j.

$$Z_j = \sum (C_{kb})(P_{ik});$$

n=no. of variable of standard objective function.
Key column= highest positive value of C_j-Z_j.
Step 2: Ratio (R_k) = right hand side value of given constraint/coefficient of same constraints variable (variable of key column) =SV/Element of key column= m_k/ Element of key column.
$R_{k=1}$= m_k/coefficient of respective variable which have been considered for step 1(its respective key column element)

$$R_{k=2} = \frac{m_k}{\ } \quad - do -$$

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Step 3. Key row (KR) = Select a lowest positive value of R_k

(k=1, 2,…) to find a KR. i.e. lowest positive value of R_k.
Step 4. Key element (KE) =Element of key column ∩ Element of key row.

=Element of column (max^m (C_j-Z_j)) ∩ Element of row (lowest (R_k))
Now, move to next simplex table for further process.
Step 5. Replace a random variable of KR with the variable of key column and update its respective C_b with the coefficient of objective function of key column.
Step 6. Row entry in next simplex table with respect to replaced random variable=key rows entry in previous simplex table /KE of previous simplex table.
Step 7: New entries in other than key row= Previous values in other than key row-(New entries in key row*element of key column (other than element of KR) of its respective value.
End of algorithm (i.e. stopping condition):
Maximization $C_j - Z_j \le 0$ and Minimization $C_j - Z_j \ge 0$
So, the KE is normalized while the other values of the key column are cancelled.
Examples- For the special case of two variables and three constraints, it can be explicitly written as:

Maximize $Z = 3x_1 + 2$ Subject to. $2x_1 + x_2 \le 18$ $2x_1 + 3x_2 \le 4$ $3x_1 + x_2 \le 24$ $x_1, x_2 \ge 0$	Maximize $Z =$ $3x_1 + 2x_2 + 0s_1 + 0s_2 + 0s_3$ Subject to. $2x_1 + x_2 + s_1 = 18$ $2x_1 + 3x_2 + s_2 = 42$ $3x_1 + x_2 + s_3 = 24$ $x_1, x_2, s_1, s_2, s_3 \ge 0$

A new introduced seven steps simplex method concept have resolved certain complication encountered in the application of the simplex method and optimal solution can be obtained after the feasible solution which is in first iteration.

Table 1. 1st iteration

Cj (Coefficient of Objective function) =>				3	2	0	0	0	R_k
BV	PPU (C_b)	SV (P_0= m_k)	P_j =>	P_1 = x_1	P_2 = x_2	P_3 = s_1	P_4= s_2	P_5= s_3	
P_3= s_1	0	m_1= 18		2	1	1	0	0	18/2 =9
P_4= s_2	0	m_2= 42		2	3	0	1	0	42/2 =21
P_5= s_3	0	m_3= 24		3	1	0	0	1	24/3 =8
C_j-Z_j				3	2	0	0	0	

Table 4Example of 1st iteration based on table 3

After 1st iteration, the next process is to check the optimal solution of maximization LPP model.
Examples-
The problem is of maximization. So, we required to add slack variables in both the inequality equations.

$Max Z = C_1x_1 + C$ Subject to. $a_1x_1 + a_2x_2 \le m_1$ $b_1x_1 + b_2x_2 \le m_2$ $x_1, x_2 \ge 0$	$Max Z = 3000x_1 + 2000$ Subject to. $5x_1 + 2x_2 \le 180$ $3x_1 + 3x_2 \le 135$ $x_1, x_2 \ge 0$	$Max Z = 3000x_1$ $+2000x_2 + 0s_1 + 0$ Subject to. $5x_1 + 2x_2 + s_1 \le 180$ $3x_1 + 3x_2 + s_2 \le 135$ $x_1, x_2, s_1, s_2 \ge 0$

In given question, the number of coefficient of objective function is equal to the number of decision variables in constraints.

Table 1. 1st iteration

Cj (Coefficient of Objective function) =>				3000	2000	0	0	R_k
BV	PPU (C_b)	SV (P_0=m_k)	P_j =>	P_1 = x_1	P_2 = x_2	P_3 = s_1	P_4 = s_2	
P_3= s_1	0	m_1=180		5	2	1	0	180/ 5=36
P_4= s_2	0	m_2=135		3	3	0	1	135/ 3=45
C_j-Z_j				3000	2000	0	0	

Table 5 System design, 1st iteration

Step1. Key column (KC) =Highest positive value of C_j-Z_j=3000
Step2. Ratio=S.V/Element of key column= (180/5,135/3) = (36, 45)
Step3. Key row (KR) =lowest positive value of Ratio=36
Step4. Key element (KE) = Element of key column ∪ Element of key row.

= Element of column (max^m (C_j-Z_j)) ∪ Element of row (lowest (R_k)) = 5
Now, move to next simplex table for further process.

Simplex Table 2.

Cj (Coefficient of Objective function) =>				3000	2000	0	0	R_k
BV	PPU (C_b)	SV (P_0 =m_k)	P_j =>	P_1 = x_1	P_2= x_2	P_3= s_1	P_4 = s_2	
P_1= x_1	3000	m_1= 36		5/ 5= 1	2/5	1/5	0/ 5 =0	36/(2 /5)= 90
P_4= s_2	0	m_2= 27		0	9/5	-3/5	1	27/(9 /5)= 15
C_j-Z_j				0	800	-600	0	

Table 6System design, stopping condition

Step5. Replace a random variable of KR with the variable of key column and update its respective C_b with the coefficient of objective function of key column.
C_b=3000 with variable x_1
Step6. Row entry in next simplex table with respect to replaced random variable=key rows entry in previous simplex table /KE of previous simplex table.
New entries in key row = key row in old table/KE=(180/5,5/5,2/5,1/5,0/5)=(36,1,2/5,1/5,0)

Step7: New entries in other than key row= Previous values in other than key row-(New entries in key row*element of key column (other than element of key row) of its respective value={135-(36*3), 3-(1*3) ,3-(2/5*3), 0-(1/5*3), 1-(0*3)}={27,0,9/5,-3/5,1}.
To create an initial table, steps 1 to 4 are required to implement.
To create table two and it’s onwards, steps 1 to 7 are required to implement.

Simplex Table 3.

Cj (Coefficient of Objective function) =>				3000	2000	0	0	R_k
BV	PPU (C_b)	SV P_0= m_k	P_j =>	P_1= x_1	P_2= x_2	P_3= s_1	P_4= s_2	
P_1= x_1	3000	m_1= 30		1	0	1/3	-2/9	
P_2= x_2	2000	m_2= 15		0	1	-1/3	5/9	
C_j-Z_j				0	0	-1000 /3	-4000/ 9	

Conditions to reach up to the optimality: C_j-Z_j≤0 for maximization problems and C_j-Z_j≥0 for minimization problem.

Here all the values of C_j-Z_j are either 0 or ≤0, so it's reached up to the optimality.

x_1=30, x_2=15 and Max Z=1, 20,000

IX. CONCLUSION

LPP is a method of allocating resources in an optimal way and it is widely used tool in operation research as a decision making aid in almost all industries. It refers to a planning process that allocates resources in the best possible way so that costs are minimized or profits are maximized. The simplex method is the most common way to solve large LPP. We briefly presented seven steps involved in using the simplex method and these steps will give us a general overview of the procedure.

The simplex method is made of KE's, Quick review can be apply over seven steps algorithm to reduce the complexity over computation, after 1st iteration the columns for the non-basic and leaving variables change. Values should be moved for all other RV directly into the new tableau. When 0 is found in the key column, the row always will be same in the new tableau and vice-versa. In this paper simplex method was used for solving maximization problem with constraints of the form of ≤, ≥ and = constraints. We have also explored the use of LPP as an economic tool for sensitivity analysis to reduce the complexity over computation using introduced simplex algorithm.

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