International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878 (Online), Volume-7 Issue-4s2, December 2019

Separation Axioms in Ideal Minimal Spaces

M. Parimala, D. Arivuoli, S. Krithika

Abstract: ***The purpose of this article is to study few separation axioms in ideal minimal spaces. The separation axioms under mIαg-closed sets namely, mIαg-T_0-spaces and mIαg-T_1-spaces were studied. Comparison of these spaces with some existing spaces were established. Necessary and sufficient conditions of mIαg-T_0 and mIαg-T_1 spaces are also proved.2010 Mathematics Subject Classification. 54A05, 54D10,54D25,54C05***

Keywords and Phrases : m-T_0-spaces, m-T_1-spaces, mIαg-T_0 spaces, mIαg- T_1 spaces

I. INTRODUCTION

One of the classical topic of topology is the separation axioms on sets. The richness of topology could be well established through separation axioms. The separation axioms concern the separation of points, separation of points from closed sets and separation of closed sets from closed sets. Views of ideals in topological spaces was explained by Kuratowski [2]. Minimal structure spaces were studied deeply in [3]. The concept minimal ideals were discussed in [4]. mIαg-closed sets were studied in [1]. The main purpose of this article is to establish the salient features of mIαg-T_0 spaces, mIαg-T_1 spaces in ideal minimal spaces. The comparison of these spaces with some existing spaces were studied. The necessary and sufficient conditions of mIαg-T_0-spaces, mIαg-T_1-spaces were also well explained.

II. PRELIMINARIES

Definition 2.1. A collection of subsets I of a set X is defined to be an ideal [2], if the set I satisfies the conditions given below. For the subsets A, B $\subseteq$ I, (i) A ∈ I any subset B of A also belong to I (ii) Union of any two elements of I should be in I. That is if A ∈ I and B ∈ I then A ∪ B ∈ I.

Definition 2.2. [3] A set M consisting of the subsets of X is termed as minimal structure if X and φ should belong to M. The space (X, M, I) is termed as ideal minimal space. The elements of the minimal structure M are the m-open sets and their respective complements are m-closed sets. The interior and closure of m-open sets are denoted by *m-int* and *m-cl* respectively. *m-int(A)* is a set containing the union of all m-open subsets of A , *m-cl(A)* is a set containing the intersection all m-closed super sets of A.

Definition 2.3. [4] An ideal minimal space we mean, a minimal space (X, M) together with the ideal I and is denoted by (X, M, I). The power set of the ideal minimal space (X, M, I) is denoted by P (X). To define the minimal local function A_m^* it is necessary to define the mapping $(.)_m^* : P(X) \to P(X)$. The minimal local function of a set A is A_m^*, which equals the set of elements x∈ X such that $U_m \cap A \notin I$, where $U_m \in U_m(x)$.

Definition 2.4. [4] The minimal *-closure operator *m-cl*(A)* is defined as *m-cl*(A)*= $A \cup A_m^*$. The minimal structure on *m-cl** is defined as $M^*(I, M) = \{F \subset X : m\text{-}cl^*(X - F) = X - F\}$. The members of M^* are named as m*-open sets. The interior of m*-open sets is denoted by *m-int*(A)*.

Proposition 2.5. [4] Some properties of the minimal *-closure operator *m-cl** are explained below. Let P, Q, R ⊂X

(a) $m\text{-}cl^*(Q) \cup m\text{-}cl^*(R) \subset m\text{-}cl^*(Q \cup R)$.

(b) If Q ⊂ R, then $m\text{-}cl^*(Q) \subset m\text{-}cl^*(R)$.

(c) $P \subset m\text{-}cl^*(P)$.

(d) $m\text{-}cl^*(\varphi) = \varphi$ and $m\text{-}cl^*(X) = X$.

Definition 2.6. [1] Let A be a non empty of X . A is defined as a mIαg-closed set, if A_m^* subset of U whenever A subset U , U is a αm-open set.

Lemma 2.7. [1] A mapping f : (X, M, I) ⟶ (Y, N, J) is defined to be a mIαg continuous mapping, if the inverse image of V is a mIαg-closed set in (X, M, I) for each m-closed set V ⊂ (Y, N,J).

Here M, N represents the minimal structures and I, J represents the ideals.

Lemma 2.8. [5] A set which is m-closed is always a mIαg-closed set.

Definition 2.9.[6] An ideal minimal space (X, M, I) is termed as a

i) m-T_0-space if for all pair of distinct elements x, y of X, there corresponds a m-open set P, contains either of the points x, y, but not both.

ii) m-T_1-space if there exists two distinct elements x, y of X, there corresponds a m-open set P, that contains x, but not y and another m-open set Q that contains y, but not x.

Manuscript published on 30 December 2019.
*****Correspondence Author(s)

Dr. M. Parimala,Assistant Professor-Senior Grade, Department of Science and Humanities (Mathematics), Bannari Amman Institute of Technology, Sathyamangalam, Tamilnadu, India.

D. Arivuoli, Assistant Professor II, Department of Science and Humanities (Mathematics), Kumaraguru College of Technology, Coimbatore, Tamilnadu, India.

S. Krithika, Assistant Professor, Department of Electronics and Communication Engineering, Kumaraguru College of Technology, Coimbatore, Tamilnadu, India,

III. MIAG-T0-SPACES

Definition 3.1. mIαg closure of a subset A of (X, M, I) is defined as the intersection of all mIαg-closed supersets A. mIαg closure is denoted by m-$cl_{I\alpha g}$(A).

Definition 3.2. Let (X, M, I) be ideal minimal space, a subset A ⊂ X is defined as a i) m-clopen set if A is both m-open and a m-closed set. ii) mIαg-clopen set if A is both mIαg-open and a mIαg-closed set.

Definition 3.3. Consider a function f : (X, M, I) ⟶(Y, N, J), then f is referred to be i) mIαg-irresolute mapping, if for each mIαg-open set V in (Y, N, J), the inverse image $f^{-1}(V)$ is a mIαg-closed set in (X, M, I).

ii) Strongly mIαg-continuous mapping, if the inverse image $f^{-1}(V)$ is a m-closed set in (X,M, I) for each mIαg-closed set V in (Y, N, J).

iii) Perfectly mIαg-continuous mapping, if for each mIαg-open set V in (Y, N, J), $f^{-1}(V)$ is both m-open and a m- closed set in (X, M, I).

iv) mIαg-totally continuous mapping, if for each mIαg-open set V in (Y, N, J), $f^{-1}(V)$ is a m-clopen set in (X,M, I).

v) Totally mIαg-continuous mapping, if for each m-open set V in (Y, N,J), $f^{-1}(V)$ is a mIαg-clopen set in (X, M, I).Here M, N represents the minimal structures and I, J represents the ideals.

Definition 3.4. The space (X, M, I) is referred to be a mIαg-T_0 space, whenever the points x, y are distinct, there corresponds a mIαg-open set U which contains either of the points x, y but not both points. That is x ∈ U, y ∉ U or x ∉ U, y ∈ U.

Theorem 3.4. The necessary and sufficient condition for an ideal minimal space (X, M, I) to be a mIαg-T_0 space is that, the mIαg-closure of distinct points are all distinct whenever m-$cl_{I\alpha g}$(m-$cl_{I\alpha g}$(A)) = m-$cl_{I\alpha g}$(A)

Proof. Necessary Part.

Consider a pair of points x, y of a mIαg-T_0 space (X, M, I) and x ≠y. Since X is mIαg-T_0 space, there corresponds a mIαg-open set U such that x is the element of U and y is not an element of U . This implies that x ∉ U^c and y∈ U^c as stated in the definition. By the definition of mIαg-closure of y, y ∈ m-$cl_{I\alpha g}$(y) and since x ∉ U^c we get x ∉ m-$cl_{I\alpha g}$(y), which proves that m-$cl_{I\alpha g}$(x) ≠ m-$cl_{I\alpha g}$(y). Thus distinct points have distinct mIαg-closures in (X, M, I).

Sufficient part. Consider a pair of distinct points x, y such that m-$cl_{I\alpha g}${x}≠ m-$cl_{I\alpha g}${y}. Therefore there exits atleast one point v ∈ X satisfying v ∈ m-$cl_{I\alpha g}${x}. -------------(1) Also v ∉ m-$cl_{I\alpha g}${y}. To prove x ∉ m-$cl_{I\alpha g}${y}

Let us assume the contrary that x ∈ m-$cl_{I\alpha g}${y}. Which implies that {x} ⊂ m- $cl_{I\alpha g}${y}, so that m-$cl_{I\alpha g}${x} ⊂ m-$cl_{I\alpha g}$(m-$cl_{I\alpha g}${y}), which leads to m-$cl_{I\alpha g}${x} ⊂ m-$cl_{I\alpha g}${y}.------(2) From (1) and (2) v ∈ m-$cl_{I\alpha g}${x} ⊂ m-$cl_{I\alpha g}${y}. That is v ∈ m-$cl_{I\alpha g}${y}. Which contradicts to the hypothesis, this is because of our assumption that x∈ m-$cl_{I\alpha g}${y}. Therefore x ∉ m-$cl_{I\alpha g}${y}. This infers us that x ∈ (m-$cl_{I\alpha g}${x}$)^c$ which is a mIαg-open set. Thus we can find a mIαg-open set (m-$cl_{I\alpha g}${x}$)^c$ in (X, M, I) that contains x, but not y. Hence (X, M, I) is a mIαg-T_0-space.

Theorem 3.5. A subspace of a mIαg-T_0-space is always a mIαg-T_0-space.

Proof. Consider a mIαg-T_0-space (X, M, I) and let Y ⊂ X be a subspace of X. Consider two points x, y ∈ Y . As X is a mIαg-T_0-space, there exits a mIαg-open set U such that x ∈ U and y ∉ U. Since Y is a subspace of X there corresponds a set U ∩ Y of Y such that x ∈ U ∩ Y and y ∉ U ∩ Y . Hence Y is also a mIαg-T_0-space.

Theorem 3.6. An ideal minimal space, which is a m-T_0-space is a mIαg-T_0- space.

Proof. The proof follows obviously with reference to Lemma 2.8. The reverse part may not be true.

Theorem 3.7. Consider a mIαg-irresolute and a bijective function f : (X, M, I) ⟶ (Y, N, J). If (Y, N, J) is a mIαg-T_0-space, then (X, M, I) is also a mIαg-T_0-space.

Proof. Consider a pair of points x, y∈ X and x ≠ y. As the mapping f is 1-1 and onto mapping, there are points u, v ∈ X and u ≠ v such that x is mapped to u (that is f (x) = u) and y is mapped to v (that is f (y) = v), which implies that x = f^{-1}(u) and y = f^{-1}(v). As Y is a mIαg-T_0-space,there exists a mIαg-open set K in Y and u ∈ K, v ∉ K. As the mapping f is mIαg-irresolute f^{-1}(K) is a mIαg-open set in X. Also u ∈ K implies f^{-1}(u) ∈ f^{-1}(K), which gives x ∈ f^{-1}(u). Also v ∉ K, implies f^{-1}(v) ∉ f^{-1}(K), which gives y ∉ f^{-1}(K). That is for any pair of distinct points x, y ∈ X, there corresponds a mIαg-open set f^{-1}(K) in (X, M, I) which contains x, but not y. Therefore (X, M, I) is a mIαg-T_0-space.

Theorem 3.8. Consider a bijective mapping f : (X, M, I) ⟶ (Y, N, J), which is also a mIαg-continuous mapping. If (Y, N, J) is m-T_0-space, then (X, M, I) is a mIαg-T_0-space.

Proof. Consider a mapping f : (X, M, I) ⟶ (Y, N, J), which is bijective, mIαg-continuous and the space (Y, N, J) is m-T_0-space.

claim. (X, M, I) is a mIαg-T_0-space.

Let us consider a pair of points x,y ∈ X and x ≠ y. Since f is bijective mapping, there exists u, v ∈ Y with u ≠ v such that image of x is u (that is f(x) = u) and the image of y is v (that is f(y) = v), which implies that x = f^{-1}(u) and y = f^{-1}(v). Since (Y, N, J) is a m-T_0-space, there exists a m-open set K such that u ∈ K and v ∉ K. As the mapping f is taken to be a mIαg-continuous, we get f^{-1}(K) is a mIαg-open set in (Y, N, J). That is we get u ∈ K, which imples that f^{-1}(u) belongs to f^{-1}(K). That is x ∈f^{-1}(K). Similarly v ∉ K leads that f^{-1}(v) ∉ f^{-1}(K). That is y ∉ f^{-1}(K). Therefore for points x, y and x ≠ y, there is a mIαg-open set f^{-1}(K) satisfying x ∈ f^{-1}(K), but y ∉ f^{-1}(K). Hence (X, M, I) is a mIαg-T_0- space.

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

IV. MIAG-T1-SPACE

Definition 4.1. mIαg-T_1-space we mean, for any pair of distinct points x, y in an ideal minimal space (X, M, I), there corresponds a pair of mIαg-open sets U, V , among which one (U) contains x, but not y and the other (V) contains y, but not x. That is x ∈U, y ∉ U and x ∉ V, y ∈V .

Theorem 4.1. A subspace of mIαg-T_1-space is always a mIαg-T_1- space.

Proof. Assume that (X, M, I) is mIαg-T_1-space and Y is a subspace of X. Consider a pair of distinct points x, y of Y . As X is a mIαg-T_1-space, there corresponds a pair of mIαg-open sets K_1, K_2 satisfying x ∈ K_1, y ∉K_2 and x ∉ K_2, y∈K_2. As Y is a subspace of X, $K_1 \cap Y$ and $K_2 \cap Y$ are also mIαg-open sets of Y . Also x ∈ K_1, x ∈Y implies x ∈ $K_1 \cap Y$. Similarly y ∈$K_2 \cap Y$. So we have arrived that there are two mIαg-open subsets of Y namely $K_1 \cap$ Y and $K_2 \cap$ Y of Y and x ∈$K_1 \cap Y$, x ∉ $K_2 \cap Y$ and y ∉$K_1 \cap Y$, y ∈ $K_2 \cap Y$. Hence (Y, M, I) is a mIαg-T_1-space.

Theorem 4.2. A mIαg-T_1-space is always a mIαg-T_0-space.

Proof. Consider a mIαg-T_1-space (X, M, I), By referring the definition of mIαg-T_1-space, for each pair of points x, y such that x ≠ y, there corresponds a pair of mIαg-open sets K_1, K_2 satisfying x ∈K_1, y ∉ K_1 and x ∉ K_2, y ∈K_2. That is we have arrived a conclusion that there is a mIαg-open set, which contains one of the points of x, y, but does not contain both the points. Therefore (X, M, I) is a mIαg-T_0-space.

Theorem 4.3. In an injective mapping f : (X, M, I) ⟶ (Y, N, J), in which (Y, N, J) is a mIαg-T_1-space, if the mapping f is a mIαg-irresolute mapping, then (X, M, I) is a mIαg-T_1-space.

Proof. Let us assume that the ideal minimal space (Y, N, J) is a mIαg-T_1-space.

Claim: (X, M, I) is a mIαg-T_1-space.

Consider the pair of distinct points x, y of X. As the mapping f is a injective mapping, x ≠ y implies f (x) = f (y). Therefore there corresponds a pair of mIαg-open sets G and H in (Y, N, J) containing f (x), f (y) in such a way that f(x) ∈ G, **f** (y) ∉G and f (x) ∉ H, **f** (y) ∈H. From the above, we inferred that x ∈**f**$^{-1}$(G), y∉**f**$^{-1}$(G) and x ∉f^{-1}(H), y ∈ f^{-1}(H). As the mapping f is mIαg-irresolute mapping f^{-1}(G) and f^{-1}(H) are also mIαg-open sets in (X, M, I).That is for any two points x, y (x≠y), there corresponds a pair of mIαg-open sets G and H satisfying the definition of mIαg-T_1-space. Hence (X, M, I) is a mIαg-T_1-space.

Theorem 4.4. Consider a mIαg-totally continuous injective function f : (X, M, I) ⟶ (Y, N, J) and the ideal minimal space (Y, N, J) is a mIαg-T_1-space, then (X, M, I) is a m-clopen-T_1-space.

Proof. Assume that x, y be two distinct points of an ideal minimal space (X, M, I). As the mapping f is injective mapping, we get f (x) = f (y). As (Y, N, J) is a mIαg-T_1-space, there are mIαg-open sets U, V such that f(x) belongs to U (that is f(x) ∈U), f(y) not belong to U and f(x) not belong to V , f(y) ∈V . Hence we get x ∈ f^{-1}(U) , y∉ f^{-1}(U)and x∉ f^{-1}(V) y ∈ f^{-1}(V). As the mapping f is mIαg-totally m-continuous, we get f^{-1}(U) and f^{-1}(V) are clopen subsets of (X,M,I). Thus (X, M, I) is a m-clopen-T_1-space.

Theorem 4.5. The necessary and sufficient condition for a space (X, M, I) to be a mIαg-T_1-space is that {x} is a mIαg-closed set in X for all x ∈ X.

Proof. Necessary part

Let x, y be any two distinct points of the ideal minimal space (X, M, I) and let {x} and {y} be mIαg-closed sets of X. Therefore $\{x\}^C$, $\{y\}^C$ are mIαg-open sets of X.

Therefore we get x ∉ $\{x\}^C$, y ∈ $\{x\}^C$ and x ∈ $\{y\}^C$, y ∉ $\{y\}^C$. Therefore (X, M, I) is a mIαg-T_1-space.

Sufficient part. Consider (X, M, I) to be a mIαg-T_1-space.

claim: {x} is a mIαg-closed set for all x ∈ X. Consider a point y ∈ X in such a way that x *f*= y. As X is a mIαg-T_1- space. there corresponds a pair of mIαg-open sets G, H satisfying x ∈G, y ∉ G and x ∉ H, y ∈H. Consider neighbourhood V of the point y which does not contains x. Hence y is not an accumulation point of {x}, which infers that D({x}) = φ. Therefore m-$cl_{I\alpha g}$\{x\} ={x} ∪ D({x}) = m-$cl_{I\alpha g}$\{x\} ∪ φ = {x}. That is m-$cl_{I\alpha g}$\{x\}= {x}. Thus {x} is a mIαg-closed set.

V. CONCLUSION

In this work separation of points and sets of the space under mIαg-closed sets are discussed in ideal minimal spaces. Heredity properties Necessary conditions are taken care. Since the separation axioms and the connectedness are closely related, further we will work in mIαg -Haussdorff spaces and mIαg-connected spaces in ideal minimal spaces.

REFERENCES

1. D.Arivuoli, M.Parimala, On generalised closed sets in ideal minimal spaces, Asia
2. Life Sciences, Supplement 14. No.1 , March 2017,85-92.
3. K.Kuratowski, Topology, Vol. I. Academic Press, New york, 1996.
4. H.Maki,J.Umehara,T.Noiri. Every Topological space in pre T(1/2). Mem. Fac. Sci.
5. Kochi Univ. Ser. A Math.,1996,17:33-42.
6. O.B.Ozbakiri and E.D.Yildirim,On some closed sets in ideal minimal spaces, Acta Math. Hungar.,125(3)(2009),227-235.
7. M.Parimala, D.Arivuoli, Submaximality in term of ideal minimal spaces. (communicated)
8. Takashi NOIRI and Valeriu POPA, On m-D-Separation Axioms. Istanbul Univ. Fen. Fak. Mat. Dergirsi,61-62(2002-2003),15-28.

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)