

Development of Visualization System of Knowledge Necessary for Solving Mathematical Questions

Akira Nakamura, Tomoshige Kudo, Keita Nishioka

Abstract: *we have been developing the visualization system of mathematical knowledge structure based on the hyperlink structure of mathematical e-learning website and expanding the knowledge structure to the field of STEM. We applied this technology to mathematical exercises which are incorporated into our mathematical e-learning website and developed a new visualization system which shows knowledge necessary for solving mathematical questions by using network graphs. This system support learners who don't have enough knowledge to solve questions by indicating what knowledge is necessary. The learner can study basic knowledge by just clicking hyperlinks on the knowledge structure map. We also developed print materials combined with the new system via QR code. This visualization system is a good tool for self-adaptive learning.*

Index Terms: *E-Learning; Network Graph; Hyperlink; Knowledge Structure; Mathematical Question.*

I. INTRODUCTION

E-learning is widely used in modern education and will spread more in the future. There are many sorts of e-learnings and many ways to categorize them. S. Negash and M. V. Wilcox proposed six classifications according to presence and communication [1]. In the case of focusing on learning strategies, e-learnings were categorized into two types: instructive learning and constructive learning [2, 3]. There are many e-learnings of instructive type which are already used by many people such as Massive Open Online Courses (MOOC). Coursera, Udacity and edX are popular MOOCs. On the other hand, people tend to use online dictionary instead of a traditional paper dictionary. In addition, most of people use search engines on a daily basis to learn what they do not know when they encounter it. This action is a kind of constructive learning. We consider that learning using search engines is an essential action for learners who live in an Internet world. Therefore, Wikipedia becomes one of the most popular websites as learning materials. These two types of learning are complementary to each other. For example, when learners attend an e-learning of instructive type and encounter what they are not able to understand, they might

use search engines to understand it. We focus on the latter type of learning and started to develop mathematics e-learning website whose concept was an online dictionary on mathematics [4]. Therefore, our website is open to the public. We expand our e-learning website from the field of mathematics to the field of science, technology, engineering and mathematics (STEM). But we find demerit of dictionary type of e-learning materials. There is no instruction like syllabus which describes what kind of things learners study and shows them in what order to study them. In other word, instructive e-learning offers their learning path. On the other hand, in constructive learning, learners have to make their learning path according to their knowledge level and their interest. We consider this learning style belongs to self-adaptive learning. So we developed the visualizing system of knowledge structure of mathematics in order to support self-adaptive learning [5-8].

Exercise is very important to master mathematics. Every mathematics textbooks have exercise for learners to acquire mathematics knowledge deeply. We made exercise web pages and integrate them into our mathematical e-learning website using hyperlink connections. We incorporate our visualization system of knowledge structure into practice of mathematics exercise. We tried to combine print materials and web materials to establish a new learning style [9, 10].

Our purpose of developing STEM e-learning is to build an efficient learning environment for self-adaptive learning. In this paper, we introduce a previous research about visualization of knowledge structure of STEM and then present a new developed visualization system which shows important knowledge necessary for solving problem using network graph.

DEVELOPING STEM WEB SITE

A. E-Learning Web Site Design

We have been developing STEM website which integrates three websites which are open to the public on the Internet [11, 12]. The first is KIT Mathematics Navigation [13] which has been developed since 2004 dealing with precalculus and calculus. The acronym KIT stands for Kanazawa Institute of Technology. The second is KIT Physics Navigation [14] which has been developed since 2015, covering mainly mechanics at this moment and extending to other fields such as electricity and magnetism.

The third is KIT Engineering Navigation [15] which has been developed since 2017, dealing with Euler–Bernoulli beam theory in material mechanics as of now.

Manuscript published on 30 December 2018.

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These websites are described in Japanese. The concept of e-learning websites is an online dictionary. We designed the website on the assumption that learners will access our e-learning website through keyword search. Therefore, we expound one topic in one web page. A page title is keyword or key phrase which suitably expresses the topic of the web page. Actually about 80% of visitors come via search engines [16].

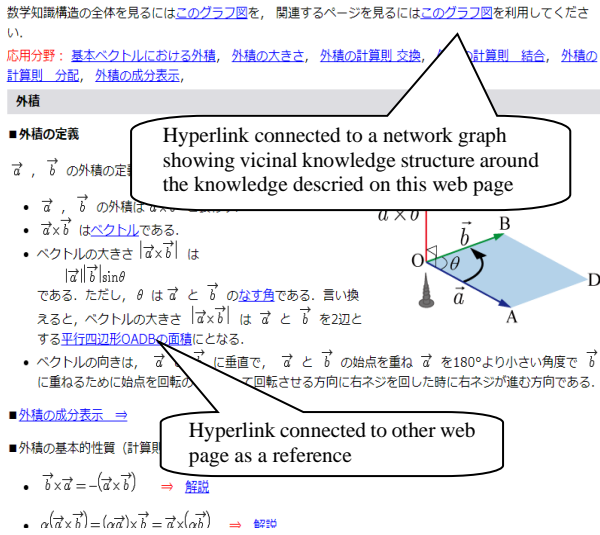


Fig. 1. Example of web pages which describing the cross product in Japanese.

We assume that a topic of own web page consists of a unit of knowledge. Key words or key phrases in description of the topic on web pages are linked to other web pages as references by hyperlinks if needed. For the time being the number of topics is about one thousand. This framework is useful to share information via social media such as Twitter, Facebook, e-mail, blogs and so on by letting someone know the address of the web page. Actually, we sometimes found that some web pages of the KIT-STEM Navigation were shared via Twitter. Physics used many mathematical knowledge in order to describe physical laws and analyze physical phenomena. Topics of physics link to not only physical web pages, but also mathematical web pages. Engineering is based on mathematics and physics. Topics of engineering link to not only engineering web pages, but also mathematical and physical web pages. We integrated our three web sites, i.e. KIT Mathematics Navigation, KIT Physics Navigation and KIT Engineering Navigation, into KIT-STEM Navigation. Fig. 1 shows an example of web pages which describes the cross product in the mathematical field. We embed some hyperlinks on keywords to open other web pages as references.

B. Knowledge Structure of STEM Fields

Each and every unit of knowledge allocates a web page and is linked to other units of knowledge by hyperlinks. Direction of hyperlink is essentially from advanced knowledge to basic knowledge as hyperlinks are created to guide basic knowledge as references in our website. Our STEM web site has huge and complicated hyperlink structure. We consider the hyperlink structure is similar to the knowledge structure of STEM fields. We tried to visualize the knowledge structure of STEM fields by using network

graph [8]. Fig. 2 shows a vicinal knowledge structure of STEM fields around the knowledge of the cross product. The network graph is generated by clicking the hyperlink on top of the web page in Fig. 1. In the network graph, a node indicated by an ellipse represents a web page in which a topic, i.e. a unit of knowledge is described and an edge indicated by an arrow represents a hyperlink between web pages. A direction of arrow coincides with a direction of hyperlink. A label of a node is a title of the web page. You can understand what knowledge a node relates to from the label. The red node represents the web page you accessed just before opening the network graph. Red arrows represent outbound hyperlinks from the web page. In default condition, nodes within three graph distances from the red node are displayed. Light blue nodes represent mathematics knowledge. Green nodes represent physics knowledge. Other nodes represent engineering knowledge. Yellow-green nodes which are not displayed in Fig. 2 represent knowledge of high school physics. Direction of arrow is towards basic knowledge. The opposite direction of arrows is towards advanced knowledge. You can understand what kind of basic knowledge the knowledge you are learning now is composed of and what kind of advanced knowledge the knowledge you are learning now is applied to.

C. How to Make a Network Graph

To build a knowledge structure, we needed to extract hyperlinks which are only related to the knowledge structure from all hyperlinks [8]. We made a set of CGI (Common Gateway Interface) programs to produce data for network graph. Vis.js (<http://visjs.org/>) is incorporated into the CGI program to generate web page of network graph. It is a dynamic, browser based visualization JavaScript library for graph drawing and dual licensed under both Apache 2.0 and MIT.

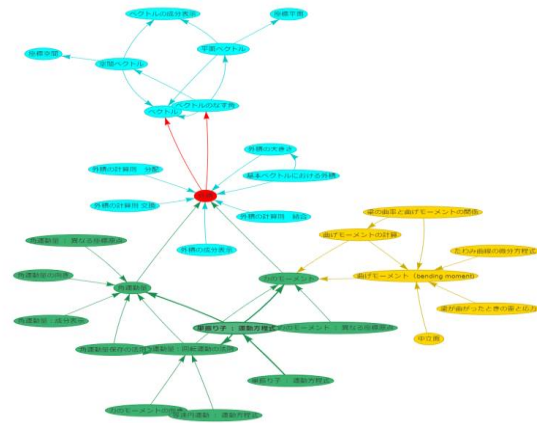


Fig. 1. Vicinal knowledge structure of STEM fields around the knowledge of cross product.

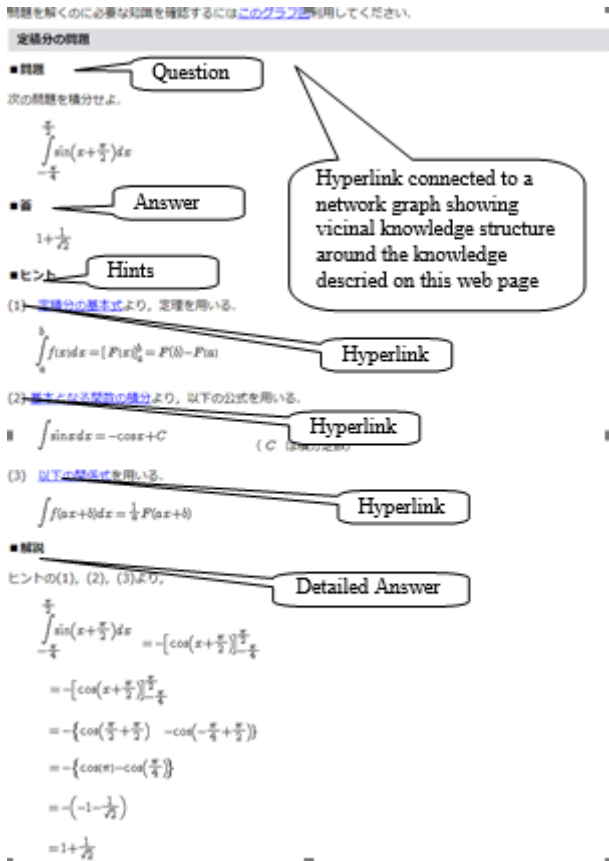


Fig. 2. Example of answer web pages.

EXERCISES IN MATHEMATICAL E-LEARNING WEB SITE

Mathematical textbooks consist of two parts. One is explanation of mathematics for learners to understand mathematics knowledge. The other is exercises of mathematics for learners to better understand mathematical knowledge. To acquire mathematical knowledge sufficiently, learners need to solve many questions and problems.

We incorporated exercises into KIT Mathematics Navigation. We created web pages which listed mathematics questions and then allocated a detailed answer for each question in one web page. Answer web pages consist of 4 parts: question, answer, hints and detailed answer. Learners who don't have enough knowledge about the question need hints. Even if some hints are given, learners cannot solve the question. So we provide a detailed answer. Learners who have little knowledge and little concept of what they are about to solve the question need a detail answer due to lack of some key knowledge. In order to support them, we create hyperlinks on important knowledge in description of hint and detailed answer. If they judge that they need to learn the knowledge, they will click the link and learn basic knowledge related to the question. This action is a beginning of making their own learning path in self-adaptive learning.

D. Visualization of Basic Knowledge related to Questions

It is very important for learners to grasp necessary basic knowledge when solving questions. It is possible to list the

knowledge related to the question directory. Knowledge level of learners who access our site is various. Learners who have much knowledge about the problem do not need to click to check references. Learners who do not have a little knowledge about the question need to check reference. Linked references is not sufficient for some learners to understand detailed answer. They need more information described on the answer web page. To navigate those learners, we developed network graph of basic knowledge about the question by modifying previous system [8]. When you click the hyperlink located at the top of the web page in Fig. 3 a new web page with network graph in Fig. 4 opens. Red node indicates the answer web page of the question. Three red arrows indicate outbound hyperlinks from the answer web page. This network graph has nodes within three graph distance from the node of the answer web page toward basic knowledge. This network graph shows knowledge which is necessary to solve the question. This is a useful tool for learners to confirm their knowledge level to the problem. If they do not know the basic knowledge, they just click the node to learn the knowledge.

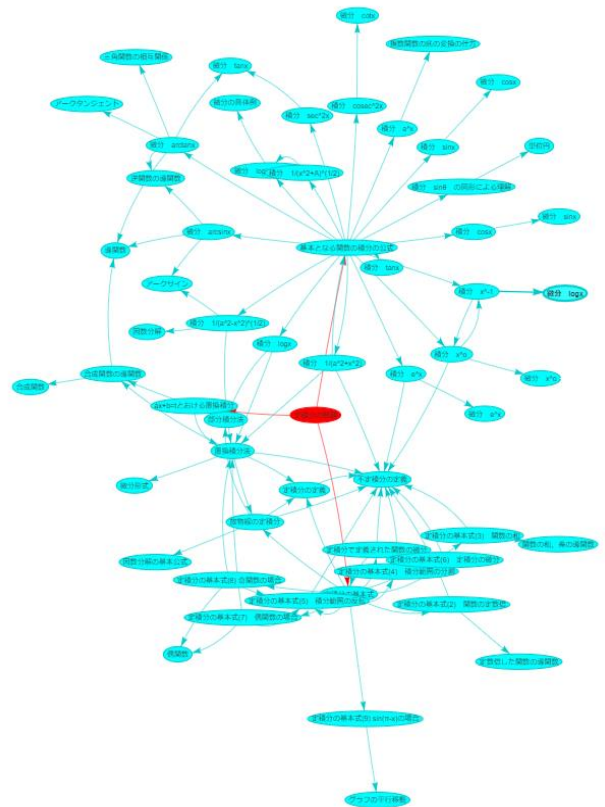


Fig. 3. Network graph which shows basic knowledge necessary for solving the question described in Fig.3..

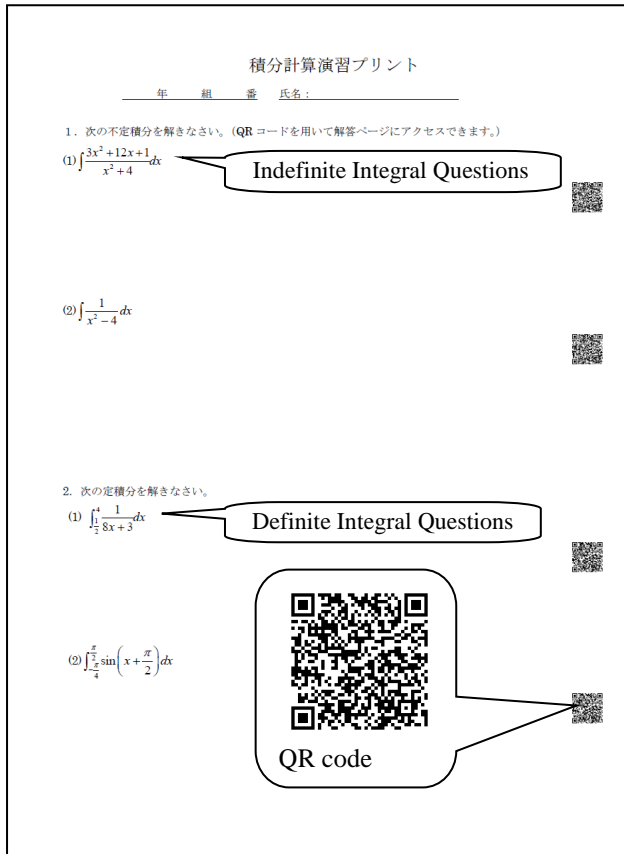


Fig. 4. Example of web pages which describing cross product in Japanese.

COMBINATION OF PRINT MATERIAL AND WEB MATERIAL

In learning mathematics, it is very important for learners to be writing mathematical expressions during the question solving process. Many researchers proposed e-learning systems for mathematics, such as web based training type, having an automatic scoring function. Those systems require learners to select answer from the multiple choice options or to input mathematical expression by using keyboard. Input of mathematical into the computer takes time as compared with handwriting due to the complexity of its input method. That is why this kind of system does not spread widely. This automatic scoring systems are beneficial for teachers and education institutes as it makes assessments very efficient and reduces the cost of assessments. Automatic evaluation systems in mathematics e-learning have a few benefits to learners. We consider that traditional print exercises will remain in the future.

We reckon self-gardening as an effective learning method in self-adaptive learning framework. It has been very common over years for learners to self-grade their answer by using textbooks. But there are some demerit using textbooks. Mathematics textbooks and reference books with detailed answers become thick and heavy as more pages are needed. Learners don't want to carry such thick and heavy books. Furthermore, in the self-grading process, learners who don't have enough knowledge about the questions need more information except detailed answers for their adequate comprehension. We consider that e-learning web sites are more suitable for self-grading than traditional paper

textbooks because e-learning websites are able to provide a variety of information according to their knowledge level by using various functions such as searching, hyperlinks, and the visualization system we developed

That is why we proposed mathematical print materials combined with web materials via QR code [9-10]. Learners will accept this method easily as the codes onto paper is common and convenient way to access web pages by scanning using smartphones. We made print materials for mathematics learning as shown in Fig. 5 for integral exercise. A detailed answer of the last question on the print in Fig. 5 is shown in Fig. 3 and Fig. 6. To realize self-grading by using their smartphone, we are converting so many existing web pages to responsive web pages. In responsive web design for mathematics e-learning, it is difficult to deal with mathematical expression as the width of mathematical expression sometimes larger than the screen width of a smartphone. Fig. 6 shows responsive design of a web page in Fig. 3. Responsive web pages are much easier for learners to read contents as compared with non-responsive web pages.

How to use this kind of prints is mentioned as follows. Teacher hand out a print to their students in class as homework. Students write down solving process on prints at home. After finishing answer questions, students access answer web pages by scanning using their smartphone and then self-grade their answers by comparing their answers with detailed answers of web pages. If they encounter what they cannot understand in detailed answer, they can click the hyperlink to learn more basic knowledge or to check the network graph which shows the basic knowledge about the question and browses some basic knowledge to understand what they don't know. In this case, developed visualization system assists to learners since the generated network graph shows various roots of basic knowledge they should know.

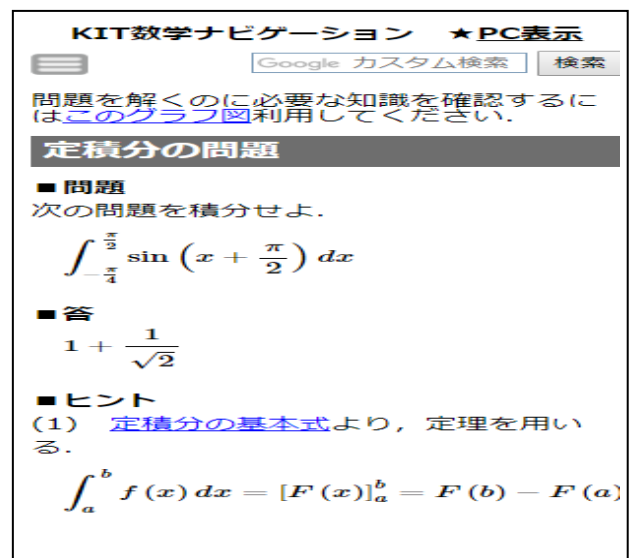


Fig. 5. Responsive design of web page in Fig.3

II. CONCLUSION

The developed visualization system of knowledge necessary for solving questions has big potential for various learners from those who have enough knowledge about the question to those who have little knowledge about the question since learners intuitively grasp indispensable knowledge from the network graph which they should know to solve the question. We notice it is important to instruct learners how to use this visualization system for learners to acquire mathematical knowledge efficiently in self-adaptive learning method. We will research how the developed visualization system affects learning efficiency and motivation in the learning of mathematics. We found merit in this system as creators. The number of nodes in network graph of the answer page indicates the quality of hints and a detailed answer. So we check the existing answer web pages and then we add some hints and knowledge in answer web pages if we encounter low quality of answer web pages while checking network graphs of answer web pages.

III. ACKNOWLEDGMENT

This work was supported by JSPS KAKENHI Grant Number JP16K011370.

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