Existence of Solution of Forest Cross-Diffusion Model

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Abstract: ***Homotopy perturbation method is attempted to evaluate the cross-diffusion forest boundary dynamics. The effects of exothermic and endothermic parameters are are discussed. AMS Subject Classiftcation: 37N25, 35Q92, 92D40, 92D25, 35B20, 37N25.***

Key Words and Phrases: Homotopy perturbation method, forest boundary dynamics.

I. INTRODUCTION

The deforestation leads to the phenomenon of climate change and it severely impacts the rise of global warming. The proper scientific methods are to be adopted in order to utilize these forest resources for the benefit of mankind as a source for fuel, building materials, paper, fiber, etc., but these industries depend only upon the forest resources. The scientific way to protect the avail- able forest resources becomes a priority sector for forest managers. This work carried out to approximate the solution age structure dynamics by reducing to cross-diffusion model [11].

II. MATHEMATICAL MODELING

A mono species dynamic has been attempted by Antonwisky and Korzukhin [4] and qualitatively described by Antonwisky et. al. [1]. The interaction of pests upon this model has been examined by Antonvsky et. al. [2] and [3]. Seed dynamics has been instigated by Kuznetsov et. al. [11] and takes $\gamma(v)$ as $a(v-b)^2+c$ transformed their mathematical model into a cross diffusion model

$$u_t = \rho v - (v-1)^2 - su + v_{xx}, \quad (1)$$

$$v_t = u - hv\,. \quad (2)$$

and qualitatively analyzed. Wu [13] describe the asymptotic stability and Yagi et. al. [7] discus the asymptotic behaviours of [11]. Forest kinematic model was studied by Gianluca Mola et. al. [9] with memory. L. Quanqua the global attractor of the model [12]. L. H. Chuan et. al. [7] discussed the stability and instability of homogeneous stationary solutions and nonexistence of inhomogeneous stationary solutions. Homotopy Perturbation Method has been attempted to prove the existence of solutions of the cross-diffusion model (1)-(2).

III. Analytical Approximation

The HPM was introduced by Mathematician J.H. He [10]. It is attempted in fin type problems [8, 5] and heat transfer equation [6]. In this section, HPM is applied to find approximate solution for the system (1) -(2) with initial conditions $u(x,0)=e^{c_1 x}$, $v(x,0)=e^{c_2 x}$.

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The following homotopy has been constructed $v(s, r) : \Omega[0,1] \rightarrow R$, $u(s,r) : \Omega[0,1] \rightarrow R$, $w(s,r) : \Omega[0,1] \rightarrow R$, $p(s,r) : \Omega[0, 1] \rightarrow R$, which satisfies

$$H(u,r) = (1-r)u_t + r[u_t - \rho v + (v-1)^2 + su - v_{xx}] = 0, \; s \in \Omega,$$

$$H(v, r) = (1-r)v_t + r[v_t - u + hv] = 0, \; s \in \Omega.$$

Solutions for (1) - (2) has been a series of powers of r

$$u(x,t) = u_0(x,t) + ru_1(x,t) + r^2u_2(x,t) + \cdots \qquad (3)$$

$$v(x,t) = v_0(x,t) + rv_1(x,t) + r^2v_2(x,t) + \cdots \qquad (4)$$

Substituting (3) - (4) in (1) - (2) and arranging the coefficient of powers of r,

$$r^0: \frac{\partial u_0}{\partial t} = -su_0 \; ; \quad \frac{\partial v_0}{\partial t} = -\mathrm{h}v_0 \quad ,$$

$$r^1: \; \frac{\partial u_1}{\partial t} = \rho v_0 - u_0(v_0-1)^2 - su_1 + \frac{\partial^2 v_0}{\partial x^2} \quad ,$$
$$\frac{\partial v_1}{\partial t} = u_0 - hv_1,$$

$$r^2: \; \frac{\partial u_2}{\partial t} = \rho v_1 - u_1(v_1-1)^2 - su_2 + \frac{\partial^2 v_1}{\partial x^2},$$
$$\frac{\partial v_2}{\partial t} = u_1 - hv_1$$

Solving the above equations

$$v_0(x,t) = e^{c_2x-ht} \qquad (5)$$
$$v_1(x,t) = \tau_5(x)e^{-st} + \tau_6(x)e^{-ht} \qquad (6)$$
$$v_2(x,t) = t\tau_1(x)e^{-ht} + \chi_1(x,t) - \frac{\tau_2(x)}{h+s}e^{-(2h+s)t} - \frac{\tau_3(x)}{s}e^{-(h+s)t} \qquad (7)$$

$$u_0(x,t) = e^{c_1x-st} \qquad (8)$$
$$u_1(x,t) = \tau_1(x)e^{-ht} + \tau_2(x)e^{-(2h+s)t} + +\tau_3(x)e^{-(h+s)t} - te^{c_1x-st} + \tau_4(x)e^{-st} \qquad (9)$$
$$u_2(x,t) = \chi_2(x,t) + \chi_3(x,t) + \chi_4(x,t) + \chi_5(x,t) + \chi_6(x,t) + \chi_7(x,t) \qquad (10)$$

Where

$$\chi_1(x,t) = \frac{\tau_4(x)e^{-st}}{h-s} - \frac{te^{c_1x-st}}{h-s} - \frac{e^{c_1x-st}}{(h-s)^2} + \tau_7(x)e^{-ht} + \tau_{17}(x)e^{-st}$$
$$\chi_2(x,t) = \tau_{10}(x)e^{-2st} - \tau_{11}(x)e^{-(h+s)t} - \tau_{12}(x)e^{-(h+2s)t} + \tau_{13}(x)e^{-3st}$$
$$\chi_3(x,t) = -\tau_{14}(x)e^{-(2h+s)t} + \tau_{15}(x)e^{-(3h+s)t} + \frac{\tau_2(x)\tau^2{}_6(x)e^{-(4h+s)t}}{4h}$$

$$\chi_4(x,t) = -\frac{\tau_7(x)\tau^2{}_6(x)e^{-3ht}}{s-3h} + \frac{\tau_1(x)\tau_6(x)e^{-2ht}}{s-h} - \frac{\tau_2(x)\tau^2{}_5(x)e^{-(2h+3s)t}}{2(s+h)}$$
$$\chi_5(x,t) = \frac{\tau_3(x)\tau^2{}_5(x)e^{-(h+2s)t}}{h+2s} - \frac{2\tau_2(x)\tau_5(x)\tau_6(x)e^{-(2s+3h)t}}{s+3h} - \frac{t^2e^{c_1x-st}}{2}$$
$$\chi_6(x,t) = te^{c_1x}\left(\frac{2\tau_6(x)e^{-(s+h)t}}{h}\right) + \frac{2\tau_5(x)}{h} + \frac{\tau_5(x)\tau_6(x)e^{-(2s+h)t}}{s+h} + e^{c_1x}\left(-te^{-st} - \frac{\tau^2{}_6(x)e^{-(s+2h)t}}{2h} - -\frac{\tau^2{}_5(x)e^{-3st}}{2s}\right)$$

$$\chi_7(x,t) = \tau_9(x)e^{-ht} + \tau_8(x)te^{-st} - \tau_{16}(x)e^{-2(h+s)t}$$

$$\tau_1(x) = \frac{(\rho + c_2{}^2)}{s-h}e^{c_2x},$$
$$\tau_2(x) = \frac{e^{(c_1+2c_2)x}}{2h},$$
$$\tau_3(x) = -\frac{e^{(c_1+c_2)x}}{h},$$
$$\tau_4(x) = e^{c_1x} - \tau_1(x) - \tau_2(x) - \tau_3(x),$$

$\tau_5(x) = \frac{e^{c_1 x}}{h-s},$
$\tau_6(x) = e^{c_2 x} - \frac{e^{c_1 x}}{h-s},$
$\tau_7(x) = e^{c_2 x} + \frac{\tau_2(x)}{h+s} + \frac{\tau_3(x)}{s} - \frac{\tau_4(x)}{h-s} - \frac{e^{c_1 x}}{(h-s)^2}$
$\tau_8(x) = \tau_4(x) + \rho\tau_5(x) + \frac{{c_1}^2}{h-s}\tau_1(x)$
$\tau_9(x) = \tau_1(x) + \rho\tau_6(x) - \frac{{c_1}^2 e^{c_1 x}}{h-s}\tau_1(x) + {c_1}^2 e^{c_2 x},$
$\tau_{10}(x) = \frac{2\tau_5(x)e^{c_1 x}}{s^2} - \frac{2\tau_4(x)\tau_5(x)}{3s},$
$\tau_{11}(x) = \frac{1}{h}\big(\tau_3(x) - 2\tau_4(x)\tau_6(x) + 2\tau_1(x)\tau_5(x)\big) + \frac{2\tau_6(x)e^{c_1 x}}{h^2},$
$\tau_{12}(x) = \frac{1}{s+h}\big(2\tau_3(x)\tau_5(x) + 2\tau_4(x)\tau_5(x)\tau_6(x) - \tau_1(x)\tau^2{}_5(x)\big) + \frac{2\tau_5(x)\tau_6(x)}{(s+h)^2},$

$\tau_{13}(x) = \frac{\tau_4(x)\tau_5(x)}{2s} - \frac{\tau^2{}_5(x)}{4s^2},$
$\tau_{14}(x) = \frac{1}{2h}\big(\tau_2(x) - \tau_4(x)\tau^2{}_6(x) - 2\tau_3(x)\tau_6(x) + 2\tau_1(x)\tau_5(x)\tau_6(x)\big) + \frac{2\tau^2{}_5(x)e^{c_1 x}}{4h^2},$
$\tau_{15}(x) = \frac{1}{3h}\big(\tau^2{}_3(x)\tau^2{}_6(x) + 2\tau_2(x)\tau_6(x)\big),$
$\tau_{16}(x) = \frac{-2}{s+2h}\big(2\tau_2(x)\tau_5(x) + 2\tau_3(x)\tau_5(x)\tau_6(x)\big),$
$\tau_{17}(x) = e^{c_1 x} - u_2(x,0).$

The following equation are tailored from (3)-(4)

$$u(x,t) = \lim_{r\to 1} u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + .., \quad (11)$$

$$v(x,t) = \lim_{r\to 1} v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + \cdots \quad (12)$$

Substituting (5)-(10) to (10)-(11) we could get the solution for the system (1)-(2).

IV. RESULTS AND DISCUSSION

In order to have physical significant points of the systems (1) to (2), numerical calculations are attempted with different values of coefficients of aging tree, coefficients of mortality of old tree, *s* and *ρ* by taking diffusion coefficient D=1.

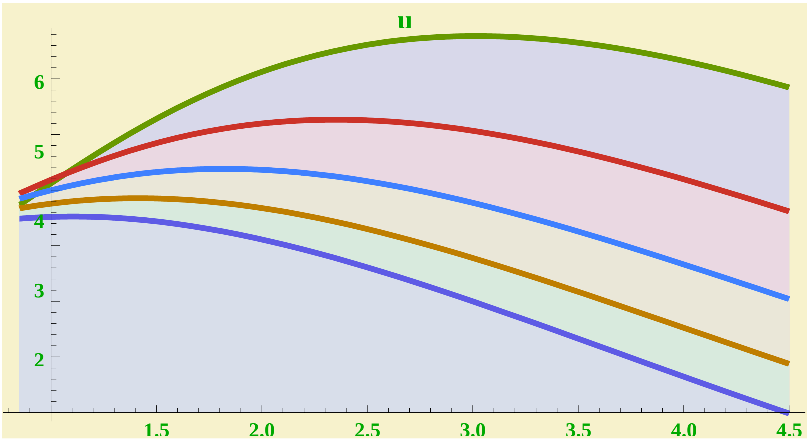

Figure 1: Young tree density when *s* = 0.5, *x* = 1, c_1 = 1, c_2 = 1, c_3 = 1, *ρ* = 1 and *h* varies from 3 to 7.

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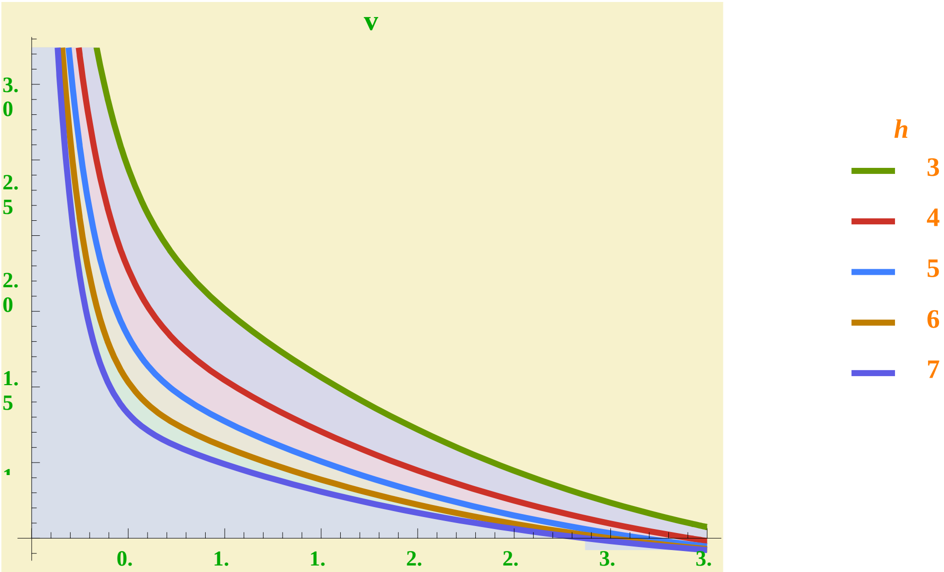

Figure 2: Old tree density when s = 0.5, x = 1, c_1 = 1, c_2 = 1, c_3 = 1, ρ = 1 and h varies from 3 to 7.

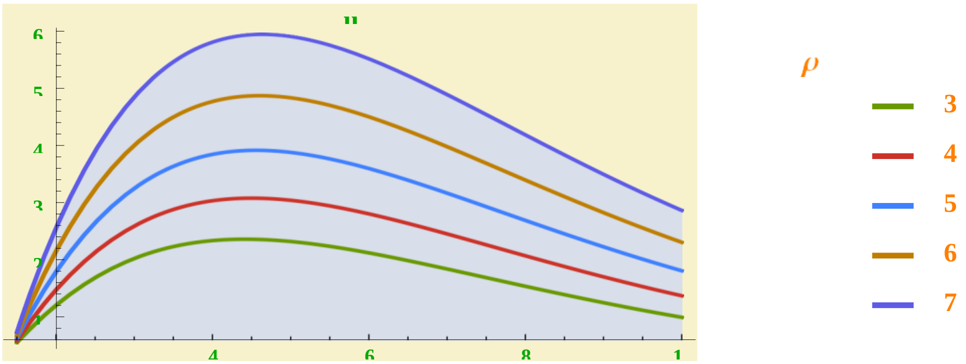

Figure 3: Young tree density when s = 0.3, h = 3, c_1 = 1, c_2 = 1, c_3 = 1, x = 1, and ρ varies from 3 to 7.

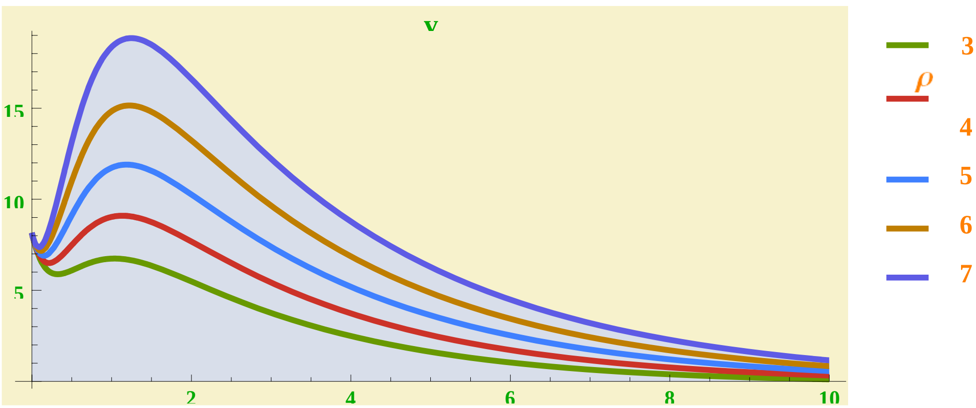

Figure 4: Old tree density when s = 0.3, h = 3, c_1 = 1, c_2 = 1, c_3 = 1, , x = 1, nd ρ varies from 3 to 7.

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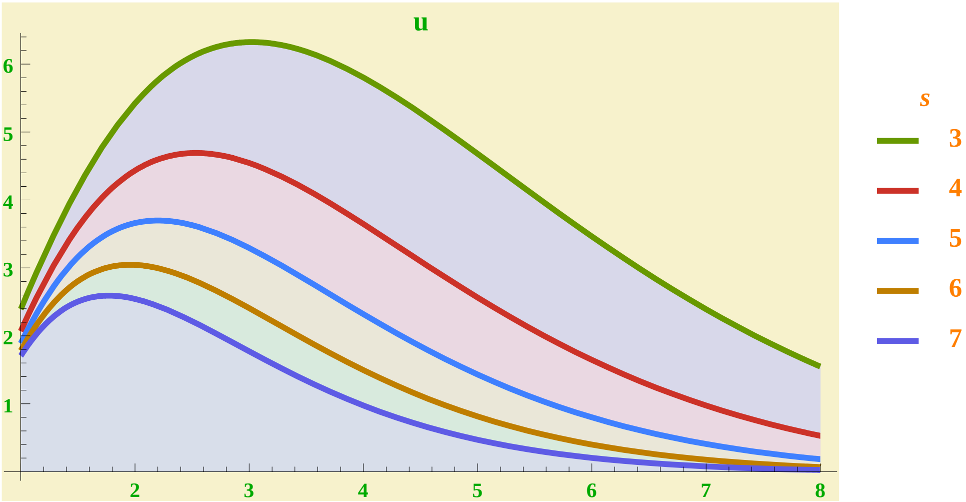

Figure 5: Young tree density when s = 0.3, h = 3, c_1 = 1, c_2 = 1, c_3 = 1, x = 1, and s varies from 3 to 7.

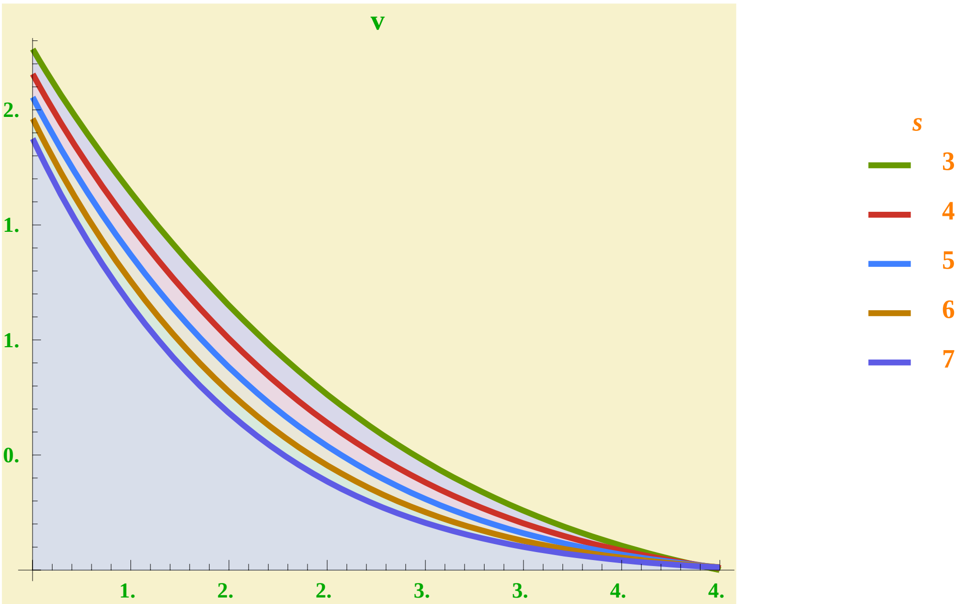

Figure 6: The density of old trees when c_1 = 1, c_2 = 1, c_3 = 1, s = 0.3, h = 3, x = 1, and s varies from 3 to 7.

From the two dimensional graphs in Fig.1 and Fig.2 it is revealed that if h is increased, u and v are decreased. From the Fig.3 and Fig.4, it has been observed that ρ is increasing with respect to the increasing density of young and old trees. Fig.5 and Fig.6 obviously indicate that the density of young and old trees are decreased when s is increased.

V. CONCLUSION

The environmental modeling study of the forest ecosystem by giving importance to the temporal effects has been carried over. This study reveals that the forest ecosystem model based on cross- diffusion model is influenced by exogenous and endogenous parameters namely, mortality Efficient of old trees h, ρ and s. The eco- tone boundary surfaces are elaborately discussed.

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