Composition of Functions under Niαg Continuous and Niαg-Irresolute Functions in Nano Ideal Topological Spaces

M.Parimala, D.Arivuoli, R.Jeevitha

Abstract: This research paper targets nIαg-continuous, nIαg-irresolute functions in nano ideal topological spaces. Comparison and Characterisation of these functions are explored. Preservation of composition of functions under nIαg-continuous, nIαg- irresolute functions are also proved. 2010 Mathematics Subject Classification.. 54A05, 54A10, 54B05.

Keywords: nIαg-continuous function, nIαg-irresolute function,Composition of functions.

I. INTRODUCTION

The initialisation of ideals in topological spaces was done by Kuratowski [1]. Elaborate studies about local function A^*, Kuratowski's closure operator cl^* and int^* in *-topology was done by Jankovic [2]. The introduction of *I*-open sets was by Jankovi et.al] [3]. Nano topology and its characterisation were studied by Lellis Thivakar [4]. He defined the nano topology, by defining the approximations and boundary regions of a subset in the universal set. Familiarity on nano generalised closed sets was given by Bhuvaneswari et.al [5]. Idealisation of nano topology was initiated by Parimala et.al. Investigation of nano local function was made by Parimala et.al [7], [8], [9]. *nIg*-closed sets, *nIαg*-closed sets were introduced by Parimala et.al [10], [11]. *nα*-closed sets were studied by ThangaNachiar [12]. This works leads to study on *nIαg*-continuous and irresolute functions in nano ideal topology. Preservation of composition of functions under *nIαg*-continuous and irresolute functions are also proved in this paper.

II. PRELIMINARIES

In the following sequel $(U, \mathbb{N}, \mathfrak{I})$ denotes the nano ideal topological space.

Definition 2.1. [1] Ideal $\mathfrak{I}$ is a set satisfies i) P∈ $\mathfrak{I}$,if Q subset of P then Q ∈ $\mathfrak{I}$ ii) Union of any two subsets of the ideal $\mathfrak{I}$ belong to $\mathfrak{I}$.

Definition 2.2. [2] In a topological space S , the local function P^* of te set P mean

$P^*=\{s \in S : U \cap P \notin \mathfrak{I}$ for every $U \in \tau$. Here $\tau(s) = \{U \in \tau : s \in U\}$.

Definition 2.3. [4]

i) $\mathfrak{R}_l(S)$ denotes the lower approximation of the set S with respect to the equivalence relation $\mathfrak{R}$ which is defined as the union of elements of $\mathfrak{R}_{(S)}$which are contained in S. That is, $\mathfrak{R}_l(S) = \cup \{\mathfrak{R}_{(S)}: \mathfrak{R}_{(S)} \subseteq S, s \in U\}$ $\mathfrak{R}_u(S)$ denotes the upper approximation of the set S with respect to the equivalence relation $\mathfrak{R}$ which is defined as the $\mathfrak{R}_u(S) = \cup \{R(S) : R(S) \cap S \neq \varphi, s \in U\}$.

ii) $\mathfrak{R}_b(S)$ denotes the boundary region and it is defined as $\mathfrak{R}_b(S) = \mathfrak{R}_u(S) - \mathfrak{R}_l(S)$.

Here U is the universal set.

Definition 2.4. [6] Salient features of $\mathfrak{R}_u$ and $\mathfrak{R}_l$ are given below.

Properties of the upper and lower approximations are as follows. Consider the approximation space (U, R) with the equivalence relation R and $X, Y \subseteq U$

i) $\mathfrak{R}_l(X) \subseteq X \subseteq \mathfrak{R}_u(X)$.

ii) $\mathfrak{R}_l(\varphi) = \mathfrak{R}_u(\varphi) = \varphi$.

iii) $\mathfrak{R}_l(U) = \mathfrak{R}_u(U) = U$.

iv) $\mathfrak{R}_u(S \cup T) = \mathfrak{R}_u(S) \cup \mathfrak{R}_u(T)$.

v) $\mathfrak{R}_u(S \cap T) \subseteq \mathfrak{R}_u(S) \cap \mathfrak{R}_u(T)$.

vi) $\mathfrak{R}_l(S \cup T) \supseteq \mathfrak{R}_l(S) \cup \mathfrak{R}_l(T)$.

vii) $\mathfrak{R}_l(S \cap T) = \mathfrak{R}_l(S) \cap \mathfrak{R}_l(T)$.

viii) $\mathfrak{R}_l(S) \subseteq \mathfrak{R}_l(T)$ and $\mathfrak{R}_u(S) \subseteq \mathfrak{R}_u(T)$ when $S \subseteq T$.

Revised Version Manuscript Received on 25 November, 2018.

M.P arimala, Department of Mathematics, Bannari amman Institute of Technology, Sathyamangalam, Tamil nadu, India

D. Arivuoli, Department of Mathematics, Kumara guru College of Technology, Coimbatore, Tamil nadu, India

R. Jeevitha, Department of Mathematics, Dr. N.G.P Institute of Technology, Coimbatore, Tamil nadu, India

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

ix) $\Re_u(S^c) = [\Re_l(S)]^c$ and $\Re_l(S^c) = [\Re_u(S)]^c$.

x) $\Re_u[\Re_u R(S)] = \Re_l[\Re_u(S)] = \Re_l(S)$.

xi) $[\Re_l(S)] = \Re_u[\Re_l(S)] = \Re_l(S)$.

Definition 2.5. [6] In an Universe U, define an equivalence relation $\Re$, and a set $\tau\Re(S) = \{U, \varphi, \Re_l(S), \Re_u(S), \Re_b(S)\}$, $S \subseteq U$. The axioms of the topology $\tau\Re(S)$ are

i) *Universal set* and *the empty set* should belong to $\tau\Re(S)$.

ii) Arbitrary union elements of any sub-collection of $\tau\Re(S)$ is in $\tau\Re(S)$.

iii) The finite intersection of the elements of $\tau\Re(S)$ is in $\tau\Re(S)$.

Here $\tau\Re(S)$ denotes the nano topology. $(U, \tau\Re(S))$ is the nano topological space. Members of $(U, \tau\Re(S))$ are the nano open sets and the complement of nano open sets are the nano closed sets.

Remark 2.6. [6] The basis of the nano topological space $(U, \tau\Re(S))$ is the set

$B = \{U, \Re_l(S), \Re_u(S)\}$.

Definition 2.7. [8] Let $(U, N, \mathfrak{I})$ be a nano topological space with respect to the ideal $\mathfrak{I}$ and $(.)^*_n$ denotes the operator from the power set $\rho(U)$ to $\rho(U)$. When $P \subseteq U$, $P_n^*(I,N) = \{s \in U : G_n \cap P \notin \mathfrak{I}$, for every $G_n \in G_n(x)\}$ is the local function of P with respect to $\mathfrak{I}$ and N in the nano topology.

Definition 2.8. [8] Let (U, N) be a nano topological space, $\mathfrak{I}, \mathfrak{I}'$ are the ideals on U and P, Q be subsets of U. The salient features of P_n^* are

i) $P \subseteq Q \Rightarrow P_n^* \subseteq Q_n^*$

ii) $\mathfrak{I} \subseteq \mathfrak{I}' \Rightarrow P_n^*(\mathfrak{I}') \subseteq P_n^*(\mathfrak{I})$

iii) $P_n^* = n\text{-}cl(P_n^*) \subseteq n\text{-}cl(P)$ (P_n^* is a nano closed subset of $n\text{-}cl(P)$)

iv) $(P_n^*)_n^* \subseteq P_n^*$

v) $P_n^* \cup Q_n^* = (P \cup Q)_n^*$

vi) $P_n^* - Q_n^* = (P - Q)_n^* - Q_n^* \subseteq (P - Q)_n^*$

vii) $V \in N$ *implies* $V \cap P_n^* = V \cap (V \cap P)_n^* \subseteq (V \cap P)_n^*$

viii) $\mathfrak{I}' \subseteq \mathfrak{I} \Rightarrow (P \cup \mathfrak{I}')_n^* = P_n^* = (P - \mathfrak{I}')$ $_n^*$.

Lemma 2.9. [8] In $(U, \mathbb{N}, \mathfrak{I})$, let $\subseteq P_n^*$, then the following equality holds. $P_n^* = cl_n(P_n^*) = cl_n(P)$.

Lemma 2.10. [8] In $(U, \mathbb{N}, \mathfrak{I})$, the operator $cl_n^*(P) = P \cup P_n^*$ for $P \subseteq S$.

Definition 2.11. [8] Properties of cl_n^* are as follows. Consider the subsets

T_1 and T_2 of U, then

i) $T_1 \subseteq cl_n^*(T_1)$

ii) $cl_n^*(\varphi) = \varphi$ and $cl_n^*(U) = U$

iii) If $T_1 \subseteq T_2$, then $cl_n^*(T_1) \subseteq cl_n^*(T_2)$

iv) $cl_n^*(T_1) \cup cl_n^*(T_2) = cl_n^*(T_1 \cup T_2)$

v) $cl_n^*(cl_n^*(T_1)) = cl_n^*(T_1)$.

Definition 2.12.[8] In $(U, \mathbb{N}, \mathfrak{I})$, the subset P is defined to be a n^*- dense in itself set (resp. n^*-perfect set) when $P \subseteq P_n^*$ (resp. $P = P_n^*$).

Definition 2.13. [8] In $(U, \mathbb{N}, \mathfrak{I})$ when the subset P is n^*-dense in itself, then the following equality holds. $P_n^* = cl_n(P_n^*) = cl_n(P) = cl_n^*(P)$.

Definition 2.14. Consider P as a subset of (U, N, I) is

i) *ng*-closed set, if $cl_n(P) \subseteq \mho$, $P \subseteq \mho$, $\mho$ is a nano open set.[5]

ii) $n\alpha$-open set, if $P \subseteq int_n(cl_n(int_n(P)))$ [4].

iii) nIg-closed, if $P_n^* \subseteq \mho$, $P \subseteq \mho$ and $\mho$ is n-open [10].

iv) $nI\alpha g$-closed set, if $P_n^* \subseteq \mho$, $P \subseteq \mho$ and $\mho$ is a nano α-open set. [11]

Theorem 2.15. [11] Consider a $(U, \mathbb{N}, \mathfrak{I})$, then the implications are true and reverse implications are may not be true.

i) A nano closed is always a $nI\alpha g$-closed set.
ii) A n^*-closed is always a $nI\alpha g$-closed set.
iii) A ng-closed is always a $nI\alpha g$-closed set.
iv) A $nI\alpha g$-closed is always a nIg-closed set.
v) A *nano* open set is always a n^*-open set. [10]3

Definition 2.16. [4] A function defined between two nano topological spaces $F : (U, \tau\mathcal{R}(S)) \rightarrow (\mho, \tau\mathcal{R}(T))$ is a nano continuous function if the inverse image $F^{-1}(P)$ is n-open in $(U, \tau\mathcal{R}(S))$ for every n- open set P in $(\mho, \tau\mathcal{R}(T))$.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

III. NIAG-CONTINUOUS AND NIAG-IRRESOLUTE FUNCTIONS.

Definition 3.1. Consider two nano ideal topological spaces $(U,\mathbb{N},\mathfrak{I})$ and $(\mho,\mathcal{M},\jmath)$ and define a function $F: (U,\mathbb{N},\mathfrak{I}) \rightarrow (\mho,\mathcal{M},\jmath)$. The function F is defined to be a *nIαg*-continuous function, if $F^{-1}(V)$ of each n-open set $\mho \subseteq (\mho,\mathcal{M},\jmath)$ is a *nIαg*-open set in $(U,\mathbb{N},\mathfrak{I})$.

Example 3.2. Consider the universal set $U = \{t1, t2, t3, t4\}$, the approximation space $U/\mathcal{R} = \{\{t1\}, \{t3\}, \{t2, t4\}\}$, $S = \{t1, t2\} \subseteq U$ with the ideal $\mathfrak{I} = \{\varphi, \{t3\}, \{t2\}, \{t2, t3\}\}$. The nano topology defined by U is $\tau\mathcal{R}(S) = \{U, \varphi, \{t1\}, \{t2, t4\}, \{t1, t2, t4\}\}$ and *nIαg*-closed sets are $\{\{t2\}, \{t3\}, \{t1, t3\}, \{t2,t3\}, \{t3, t4\}, \{t1, t2, t3\}, \{t1, t3, t4\}, \{t2, t3, t4\}, U, \varphi\}$. Let $\mho = \{t1,t2,t3,t4\}$, the approximation space $U/\mathcal{R} = \{\{t2\},\{t4\},\{t3,t4\}\}$, $S = \{t1, t4\} \subseteq \mho$ with the ideal $\mathfrak{I} = \{\varphi, \{t4\}\}$. The nano topology defined by V is $\tau\mathcal{R}(S) = \{V, \varphi, \{t4\}, \{t1, t3\}, \{t1, t3, t4\}\}$ and *nIαg*-closed sets are $\{\{t2\}, \{t4\}, \{t1, t4\}, \{t2, t3\}, \{t2, t4\}, \{t1, t2, t3\}, \{t1, t2, t4\}, \{t2, t3, t4\}, U, \varphi\}$. The function $F: (U,\mathbb{N},\mathfrak{I}) \rightarrow (\mho,\mathcal{M},\jmath)$ is defined as $F(t1) = t1$, $F(t2)\ t3$, $F(t3) = t2$, $F(t4) = t4$ is a *nIαg*-continuous function.

Definition 3.3. Consider two nano ideal topological spaces $(U,\mathbb{N},\mathfrak{I})$ and $(\mho,\mathcal{M},\jmath)$ and define a function $F: (U,\mathbb{N},\mathfrak{I}) \rightarrow (\mho,\mathcal{M},\jmath)$. The function F is defined to be a *nIαg*-irresolute function, if the inverse image $f^{-1}(\mho)$ is a *nIαg*- closed set in $(U,\mathbb{N},\mathfrak{I})$ for every *nIαg*-closed set $\mho$ in $(\mho,\mathcal{M},\jmath)$.

Example 3.4. In Example 3.2, the function defined as $F(t1) = t1$, $F(t2) = t3$, $F(t3) = t2$, $F(t4) = t4$ is a *nIαg*-irresolute function.

Theorem 3.5. Consider a function $F: (U,\mathbb{N},\mathfrak{I}) \rightarrow (\mho,\mathcal{M},\jmath)$ in nano ideal topological space. Then F is a *nIαg*-continuous function iff $F^{-1}(P)$ is a *nIαg*-closed set in $(U,\mathbb{N},\mathfrak{I})$ for every *n*-closed set $P \subseteq (\mho,\mathcal{M},\jmath)$.

Proof. Necessary part Consider a *n*-closed set $P \subseteq (\mho,\mathcal{M},\jmath)$ and a *nIαg*- continuous function $F: (U,\mathbb{N},\mathfrak{I}) \rightarrow (\mho,\mathcal{M},\jmath)$, then P^{c} is a *n*-open set in $(\mho,\mathcal{M},\jmath)$ and hence $F^{-1}(P^{c})$ is a *nIαg*-closed set in $\mho$. As $F^{-1}(P^{c}) = (F^{-1}(P))^{c}$, we get $F^{-1}(P)$ is *nIαg*-open in U.

Sufficient part Let F be a *n*-open set in $\mho$. then F^{c} is a *n*-closed set. Therefore by hypothesis $F^{-1}(F^{c})$ is *nIαg*-closed in U. As $F^{-1}(F^{c}) = (F^{-1}(F))^{c}$, we get $F^{-1}(F)$ is *nIαg*-open in U. Hence F is a *nIαg*-continuous function.

Theorem 3.6. In a nano topological space, whenever the function $F: (U,\mathbb{N},\mathfrak{I}) \rightarrow (\mho,\mathcal{M},\jmath)$ is a nano continuous function then it will be a *nIαg*-continuous function.

Proof. As F is nano continuous function, for each *n*-open set P of $(\mho,\mathcal{M},\jmath)$ $F^{-1}(P)$ is a *n*-open set in $(U,\mathbb{N},\mathfrak{I})$. Referring Theorem 2.15 (i) $F^{-1}(P)$ is a *nIαg*- open set and so F is a *nIαg*-continuous function.

Remark 3.7 A *nIαg*-continuous function may not be a nano continuous function. The following Example explains this Remark.

Example 3.8 Refer Example 3.2. The function defined is a *nIαg*-continuous function, but since $F^{-1}(\{t4\}) = \{t4\}$ is not a *n*-open set, F is not a nano continuous function.

Definition 3.9 $F: (U, \tau\mathcal{R}(S)) \rightarrow (\mho, \tau\mathcal{R}(T))$ *be a* 2function, defined to be a n^{*}-continuous(*nano generalised*-continuous) function if the the inverse image $F^{-1}(P)$ is a n^{*}-open set(*ng*-open set) in $(U, \tau\mathcal{R}(S))$ for every *n*-open set P in $(\mho, \tau\mathcal{R}(T))$.

Theorem 3.10. In $(U,\mathbb{N},\mathfrak{I})$, whenever the $F: (U,\mathbb{N},\mathfrak{I}) \rightarrow (\mho,\mathcal{M},\jmath)$ seems to be a n^{*}-continuous function then it will be a *nIαg*-continuous function.

Remark 3.11. A *nIαg*-continuous function may not be a n^{*}-continuous function. The following Example explains this Remark.

Example 3.12. Refer Example 3.2. The function defined is a *nIαg*-continuous function, but since $F^{-1}(\{t1, t3\}) = \{t1, t2\}$ is not a n^{*}-open set, F is not a n^{*}-continuous function.

Definition 3.13. A function defined between two nano topological spaces $F: (U, \tau\mathcal{R}(S)) \rightarrow (\mho, \tau\mathcal{R}(T))$ is defined to be a *nIg*-continuous function if the the inverse image $F^{-1}(P)$ is a *nIg*-open set in $(U, \tau\mathcal{R}(S))$ for every *n*-open set P in $(\mho, \tau\mathcal{R}(T))$.

Theorem 3.14. In a nano topological space, whenever the function $F: (U,\mathbb{N},\mathfrak{I}) \rightarrow (\mho,\mathcal{M},\jmath)$ seems to be a *nIαg*-continuous function then it will be a *nIg*-continuous function.

Proof. As F is a *nIαg*-continuous function, for each *n*-open set P of $(\mho,\mathcal{M},\jmath)$ $F^{-1}(P)$ is a *nIαg*-open set in $(U,\mathbb{N},\mathfrak{I})$. Referring Theorem 2.15 (iv) $F^{-1}(P)$ is a *nIg*-open set and hence F is a *nIg*-continuous function.

Remark 3.15. A *nIg*-continuous function may not be a *nIαg*-continuous function.

IV. COMPOSITION OF FUNCTIONS UNDER NIAG-CONTINUOUS AND NIAG-IRRESOLUTE FUNCTIONS

Theorem 4.1. In a ideal topological space with nano sets , the set operation composition, of two *nIαg*-continuous functions may not be a *nIαg*-continuous function.

Proof. The proof can be explained by an example.
Let $U = \{t1, t2, t3, t4\}$, the approximation space $U/\mathcal{R} = \{\{t3\},\{t4\},\{t1,t2\}\}$, $S = \{t2, t4\} \subseteq U$ with the ideal $\mathfrak{I} = \{\varphi, \{t1\}\}$. The nano topology defined by U is

$\tau\mathcal{R}(S) = \{U, \varphi, \{t4\}, \{t1, t2\}, \{t1, t2, t4\}\}$ and *nIαg*-open sets are $\{U, \varphi, \{t2, t3, t4\}, \{t1, t2, t4\},$

$\{t2, t4\}, \{t1, t2\}, \{t4\}, \{t2\}\}$.
$V = \{t1, t2, t3, t4\}$, the approximation space $U/\mathcal{R} = \{\{t2\}, \{t4\}, \{t1, t3\}\}$, $S =$

$\{t1, t4\} \subseteq V$ with the ideal $I = \{\varphi, \{t4\}\}$. The nano topology defined by V is

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$\tau\mathcal{R}(S) = \{V, \varphi, \{t4\}, \{t1, t3\}, \{t1, t3, t4\}\}$ and $nI\alpha g$-open sets are $\{U, \varphi, \{t1, t3, t4\}, \{t1, t2, t3\},$

$\{t3, t4\}, \{t1, t4\}, \{t1, t3\}, \{t4\}, \{t3\}, \{t1\}\}$.

$W = \{a, b, c, d\}$, the approximation space $U/\mathcal{R} = \{\{t1\}, \{t3\}, \{t2, t4\}\}$, S =

$\{t1, t2\} \subseteq U$ with the ideal $\mathfrak{J} = \{\varphi, \{t3\}, \{t2\}, \{t2, t3\}\}$. The nano topology defined by U

is $\tau\mathcal{R}(S) = \{U, \varphi, \{t1\}, \{t2, t4\}, \{t1, t2, t4\}\}$ and $nI\alpha g$-open sets are $\{U, \varphi, \{t1, t3, t4\}, \{t1, t2, t4\},$

$\{t2, t4\}, \{t1, t4\}, \{t1, t4\}, \{t4\}, \{t2\}, \{t1\}\}$.

Let the functions $\mathcal{F}$ and $\mathcal{G}$ can be defined as $\mathcal{F} : U \rightarrow \mho$ defined by $\mathcal{F}(t1) = t3$; $\mathcal{F}(t2) = t1$; $\mathcal{F}(t3) = t2$; $\mathcal{F}(t4) = t4$ and $\mathcal{G} : \mho \rightarrow W$ is defined by $\mathcal{G}(t1) = t2$; $\mathcal{G}(t2) = t3$; $\mathcal{G}(t3) = t4$; $\mathcal{G}(t4) = t4$. Here the functions f and $\mathcal{G}$ are $nI\alpha g$-continuous functions, but $\mathcal{G} \circ f$ is not a $nI\alpha g$-continuous function. Since $(\mathcal{G} \circ \mathcal{F})^{-1}(\{t2, t4\}) = \mathcal{F}^{-1}[\mathcal{G}^{-1}(\{t2, t4\})] = \mathcal{F}^{-1}(\{t3, t4\}) = \{t1, t4\}$ and here the $\{t1, t4\}$ is not a $nI\alpha g$-open set in U.

Theorem 4.2. In a nano ideal topological space, the composition of two $nI\alpha g$- irresolute functions is also a $nI\alpha g$-irresolute function.

Proof. Consider two $nI\alpha g$-irresolute functions $\mathcal{F} : (U, \mathbb{N}, \mathfrak{J}) \rightarrow (\mho, \mathcal{M}, \mathcal{J})$ and $\mathcal{G} : (\mho, \mathcal{M}, \mathcal{J}) \rightarrow (W, L, K)$.

Claim. $(\mathcal{G} \circ \mathcal{F}) : (U, \mathbb{N}, \mathfrak{J}) \rightarrow (W, L, K)$ is a $nI\alpha g$-irresolute function.

As g is considered to be a $nI\alpha g$-irresolute function, by definition for every $nI\alpha g$-open set $w \subseteq (W, L, K)$, $g^{-1}(w) = T$ is a $nI\alpha g$-open set in (V, M, J). Again since $\mathcal{F}$ is $nI\alpha g$-irresolute $(\mathcal{G} \circ \mathcal{F})^{-1}(w) = \mathcal{F}^{-1}(\mathcal{G}^{-1}(w)) = \mathcal{F}^{-1}(T) = H$ is a $nI\alpha g$-open set in (U, N, I). So $\mathcal{G} \circ \mathcal{F}$ is a $nI\alpha g$-irresolute function.

V. PRESERVATION OF COMPOSITION OF FUNCTIONS UNDER NIAG-CONTINUOUS AND NIAG-IRRESOLUTE FUNCTIONS.

Theorem 5.1. In a nano ideal topological space, when the function $\mathcal{F} : (U, \mathbb{N}, \mathfrak{J}) \rightarrow (\mho, \mathcal{M}, \mathcal{J})$ be a $nI\alpha g$-irresolute function and the function $\mathcal{G} : (\mho, \mathcal{M}, \mathcal{J}) \rightarrow (W, L, K)$ be a nano continuous function, then $(\mathcal{G} \circ \mathcal{F}) : (U, \mathbb{N}, \mathfrak{J}) \rightarrow (W, L, K)$ will be a $nI\alpha g$-continuous function.

Proof. Let $\mathcal{G}$ be a nano continuous function from $\mho$ to W and $w \subseteq W$ be a nano open set. Therefore $\mathcal{G}^{-1}(w)$ is a nano open set in $(\mho, \mathcal{M}, \mathcal{J})$. Referring Theorem 2.15 (i) $\mathcal{G}^{-1}(w) = T$ is a $nI\alpha g$-open set in (V, M, J). Also since $\mathcal{F}$ is a $nI\alpha g$-irresolute mapping we get $(\mathcal{G} \circ \mathcal{F})^{-1}(w) = \mathcal{F}^{-1}(\mathcal{G}^{-1}(w)) = \mathcal{F}^{-1}(T) = H$ and H is a $nI\alpha g$-open set in $(U, \mathbb{N}, \mathfrak{J})$ .Therefore $(\mathcal{G} \circ \mathcal{F})$ is a $nI\alpha g$-continuous function.

Theorem 5.2. When there is a $nI\alpha g$-continuous function $\mathcal{F}: (U, \mathbb{N}, \mathfrak{J}) \rightarrow (\mho, \mathcal{M}, \mathcal{J})$ and a nano continuous function $\mathcal{G} : (\mho, \mathcal{M}, \mathcal{J}) \rightarrow (W, L, K)$ exists, then the composition of function $(\mathcal{G} \circ \mathcal{F}) : (U, \mathbb{N}, \mathfrak{J}) \rightarrow (W, L, K)$ will be a $nI\alpha g$- continuous function.

Proof. Since $\mathcal{G}$ is a nano continuous function from V to W, for any n-open set

w as a subset of W, we get $\mathcal{G}^{-1}(w) = T$ is a n-open set in V. As $\mathcal{F}$ is a $nI\alpha g$-continuous function, we get $(\mathcal{G} \circ \mathcal{F})^{-1}(w) = \mathcal{F})^{-1}(\mathcal{G}^{-1}(w)) = \mathcal{F}^{-1}(T) = H$ and H is a $nI\alpha g$-open set in U. Hence $(\mathcal{G} \circ \mathcal{F})$ is a $nI\alpha g$-continuous function.

Theorem 5.3. Let $\mathcal{F}: (U, \mathbb{N}, \mathfrak{J}) \rightarrow (\mho, \mathcal{M}, \mathcal{J})$ be a $nI\alpha g$-irresolute function and $\mathcal{G} : (\mho, \mathcal{M}, \mathcal{J}) \rightarrow (W, L, K)$.be a n*-continuous ($ng$-continuous) function, then composition $(\mathcal{G} \circ \mathcal{F})$ is a $nI\alpha g$-continuous function.

Proof. Let $\mathcal{G}$ be a n^*-continuous (ng-continuous) function from V to W and w subset of W be a nano open set. Therefore $\mathcal{G}^{-1}(w)$ is a n*-open set(ng-open set) in (V, M, J). Referring Theorem 2.15 (ii) and (iii) $\mathcal{G}^{-1}(w) = T$ is a $nI\alpha g$-open set in (V, M, J). Also since $\mathcal{F}$ is a $nI\alpha g$-irresolute mapping we get $(\mathcal{G} \circ \mathcal{F})^{-1}(w) = \mathcal{F}^{-1}(\mathcal{G}^{-1}(w)) = \mathcal{F}^{-1}(T) = H$ and H is a $nI\alpha g$-open set in $(U, \mathbb{N}, \mathfrak{J})$. Therefore $(\mathcal{G} \circ \mathcal{F})$ is a $nI\alpha g$-continuous function.

REFERENCES

1. K.Kuratowski, Topology, Vol. I. Academic Press, New York, 1996.
2. D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer.Math. Monthly, 97(4) (1990), 295 - 310.
3. .D. Jankovi and T. R. Hamlett, Compatible extensions of ideals, Boll. Un.Mat. Ital. (7), 6-B (1992), 453-465.
4. M.Lellis Thivagar and Carmel Richard, On nano continuity, International Journal of Math ematics and statistics invention,Volume 1,Issue 1, August 2013, PP-31-37.
5. K. Bhuvaneswari and K.Mythili Gnanapriya, Nano generalized closed sets in nano topo logical spaces, International Journal of Scientic and Research Publications, Volume 4, Issue 5, May 2014, 1-3.
6. M.Lellis Thivagar and Carmel Richard, On nano forms of weakly open sets, Mathematical Theory and Modeling ISSN 2224-5804 (Paper) ISSN 2225- 0522 (Online), Vol.3, No.7, 2013, PP-32-37.
7. .M. Parimala and S. Jafari, Decomposition of continuity in nano ideal topo- logical spaces. - communicated.
8. M. Parimala, T. Noiri and S. Jafari, New types of nano topological spaces via nano ideals - communicated.
9. .M. Parimala and R. Perumal, Weaker form of open sets in nano ideal topo- logical spaces, Global Journal of Pure and Applied Mathematics (GJPAM), Volume 12, Number 1 (2016), 302-305.
10. M. Parimala, S. Jafari and S. Murali, Nano Ideal Generalised Closed Sets in Nano Ideal Topological Spaces, Annales Univ. Sci. Budapest., 60 (2017), 3-11.
11. .M.Parimala, D.Arivuoli, nIg-closed sets and Normality via nIg-closed sets in Nano Ideal Topological Spaces.
12. .R.Thanga Nachiyar, K.Bhuvaneswari, On nano generalised α-closed sets and nano α-generalised closed sets in nano topological spaces, International Journal of Engineering Trends and Technology .