Stabilization of Linear/Nonlinear Autonomous Systems using Lyapunov Functions

S. Sudhahar, D. Sharmila

Abstract: This paper investigates the Lyapunov function construction of the linear/nonlinear autonomous systems for stability. The control Lyapunov functions are used to stabilize the system without sacrificing the transient state performance and trade-off between stability and performance of the system because safety of operation is very important then the performances of the system. The linear quadratic optimal control problems are solved based on the control Lyapunov functions for the tracking and disturbance rejection of both SISO and MIMO systems. The effectiveness of the proposed control Lyapunov functions for the system stability and performances shown through the numerically simulated examples.

Index Terms: Lyapunov functions, Linear/Non linear Autonomous System, LQR, Safety Margins.

I. INTRODUCTION

The investigation of stability is a vital role for the controller design and analysis of the performances of the dynamic system. The stability analysis of continuous time system can be investigated through the location of the closed loop system poles in the ***s*** plane. If the all the poles of the closed system lie on left half of the ***s*** plane then the system is said to be stable system. Therefore, closed loop poles in the right half of the ***s*** plane then the system is said to be unstable system. That is not permissible in linear control system design. [1] The stable system does not guarantee for the satisfactory transient response characteristics. The Relative stability of the system and transient specifications of a closed loop control system can be easily analyzed and correlated to the pole zero configurations in the ***s*** plane. From the Routh's stability criterion, only the pole location of the closed loop system can be determined. In the root locus method poles and zeros of the dynamic closed loop system are plotted for all the gain values of a system. Further, the performance of a system is improved by using suitable compensation either time or frequency domain methods. Many of the publications and works have been dedicated to this matter.

In 1892, Lyapunov published the first results related to the theory of motion stability; with differentiate two types of problems [2]. (1) Absolute stability is analyzed in terms of parameters admissible for the model, and in the nonlinear and time-variant cases. Some authors proposed solutions in the state-space, others in the frequency domain. (2) If the convergence is not ensured for all the initial conditions, then the problem of local stability arises. This second problem is Therefore solved in terms of admissible initial conditions, and is often used to express a problem of robustness in relation to state (or input) perturbations.

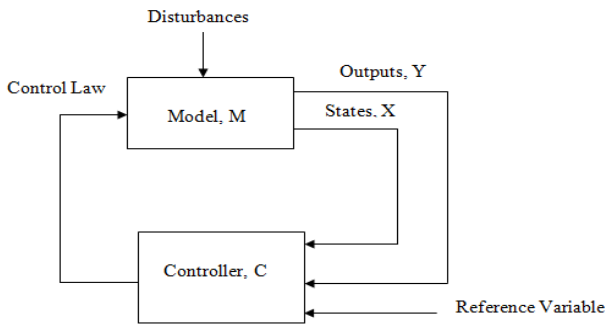

Figure: 1. Stabilization of Control Law using CLF

The issue of researching robust stability for continuous linear system moved a lot of consideration for quite a long time. Most of the examination in this field depends on the Lyapunov theory, and it is outstanding that the least difficult powerful strength portrayal depends on the idea of quadratic stability. But, for a quadratic Lyapunov function to fulfill the whole uncertainty causes moderate outcomes.

The safety of the process is a major role in control system design for the cyber physical and networked control systems. In autonomous vehicles, chemical reactors and robots are required critically stable control systems. Consequently the feedback controller are designed to meet desired performance with safe stability margins and state constraints to avoid unsafe states and very strict to input constraints [3-5]. There has been several control design methods proposed in the literature stabilization and controller design with linear / nonlinear constraints for linear/nonlinear systems. For example, Model based controller, Model Predictive Control and intelligent controllers are proposed to stabilization and trajectory tracking with its input output constraints to meet optimal performance. Due to control system hierarchy high level controllers generates guided commands to the low level controllers, so that some constraints are violated. To avoid the violation of the constraints secondary controllers are designed. Here the control design is separated into two parts. Firstly, stabilization of the system and secondly safety control.

In this paper, stability of the nonlinear system investigated using Lyapunov Function, and also proposed the Lyapunov Function based LQR nonlinear control method. It stabilizes the closed loop systems and guarantees the safety margin of the systems.

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II. LYAPUNOV FUNCTIONS

The state equation for a nonlinear system,

$$\dot{x} = g\left(x(t), u(t), t\right); x(t_0) \equiv x^0 . \qquad (1)$$

Where x is the n x 1 state vector, u is the p x 1 input vector, and $g(.) = \left[g_1(.) \quad g_2(.) \quad \ldots \quad g_n(.)\right]^T$ is the n x 1 function vector. In an unforced system (u = 0) with arbitrary initial conditions, the system state must tend towards the equilibrium point in state space. An unforced and time invariant system is called and autonomous system. The nonlinear autonomous system denoted by the state equation

$$\dot{x} = g\left(x(t)\right); x(0) \equiv x^0; g(0) = 0 \qquad (2)$$

Assumption 1: The origin of the state space has been taken as the equilibrium state of the system. There is no loss of simplification in this assumption, since any non-zero balance state can be moved to the origin appropriate transformation. The convenient choice for time invariant system is $t_0=0$ in equation (2).

Theorem 1: For autonomous system (2), adequate states of stability are as per the following conditions: Assumption made that there exists a scalar function V(x) which, for some real number ε > 0, fulfills the following designated properties for all x in the region $\|x\| < \varepsilon$:

(1) V(x) > 0; x ≠ 0
(2) V(0) = 0
} That is V(x) is positive definite function

(3) V(x) has continuous partial derivatives with respect to all values of x. Then the equilibrium state x_e = 0 of the system (2) is (4.a) asymptotically stable if $\dot{V}(x) < 0$, x≠0. That is $\dot{V}(x)$ is a negative definite function

(4.b) asymptotically stable in the large if $\dot{V}(x) < 0$, x≠0, and in addition that V(x) → ∞ as $\|x\| \to \infty$.

Theorem 2: For autonomous system (2), adequate states of stability are as per the following conditions: Assumption made that there exists a scalar function V(x) which, for some real number ε > 0, fulfills the following designated properties for all x in the region $\|x\| \le \varepsilon$:

(1) V(x) > 0; x ≠ 0
(2) V(0) = 0
} That is V(x) is positive definite function

(3) V(x) has continuous partial derivatives with respect to all values of x. Then the equilibrium state x_e = 0 of the system (2) is (4.a) asymptotically stable if $\dot{V}(x) < 0$, x≠0. That is $\dot{V}(x)$ is a negative definite function; or if $\dot{V}(x) \le 0$, ($\dot{V}(x)$ is negative semi definite) and n trajectory can stay forever at the points or on the line other than the origin, at which $\dot{V}(x) = 0$, (4.b) asymptotically stable in the large if (iv).a condition are satisfied, and in addition that V(x) → ∞ as $\|x\| \to \infty$. (4.c) stable in the sense of Lyapunov if $\dot{V}(x)$ is identically zero along trajectory.

Theorem 3: For autonomous system (2), adequate states stability are as per the following conditions: Assumption made that there exists a scalar function V(x) which, for some real number ε > 0, fulfills the following designated properties for all x in the region $\|x\| < \varepsilon$:

(1) V(x) > 0; x ≠ 0
(2) V(0) = 0
} That is V(x) is positive definite function

(3) V(x) has continuous partial derivatives with respect to all values of x. Then the equilibrium state x_e = 0 of the system (2) is unstable if $\dot{V}(x) > 0$, x≠0. That is $\dot{V}(x)$ is a positive definite function.

III. CONSTRUCTION OF DYNAMIC LYAPUNOV FUNCTIONS FOR LINEAR SYSTEMS

Consider the linear time invariant autonomous system

$$\dot{x} = Ax \qquad (3)$$

The linear autonomous system is globally asymptotically stable at the origin if and only if for the symmetric positive definite matrix Q, there exists a symmetric positive definite matrix P that satisfies matrix equation. $A^TP+PA= -Q$. (4)

Theorem 4: [2] consider the LTI system (3) and the origin is asymptotically stable equilibrium point. $P = P^T > 0$ be the solution of (4) for some positive definite matrix Q. Then $V(x,\xi) = \xi^T Px + \frac{1}{2}\|x - \xi\|_R^2$ is positive definite and the time derivative of Lyapunov function along the trajectories of the system (3) with $\dot{\xi} = F\xi + Gx$ is negative definite. Where F = -kR, G = k(R - P).

IV. CONSTRUCTION OF LYAPUNOV FUNCTIONS FOR NON LINEAR CONTINUOUS – TIME AUTONOMOUS SYSTEMS

The choice of Lyapunov function V(x), which is positive definite function to determine the stability through Lyapunov's direct method centres. There is no universal method is adopted to choose the Lyapunov function, however in literatures guidelines given to chose the Lyapunov function. For the different Lyapunov functions the stable and unstable regions may be changed. In spite of all limitations, Lyapunov's direct method is the most powerful tool to analyze the stability of the non linear systems. Let discuss about the construction of Lyapunov function using Krasovskii method and variable gradient method.

A. Krasovskii Method

Lyapunov's direct method determines the global asymptotic stability of the system.

The nonlinear autonomous system, $\dot{x} = g(x); g(0) \equiv 0$ (5)

Assumption 1: f has continuous first partial derivatives. Then J(x) be the Jacobian matrix of 'f'

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$$J\left(x\right)=\frac{\partial g\left(x\right)}{\partial x}=\begin{bmatrix}\frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_n}\\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \cdots & \frac{\partial g_2}{\partial x_n}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{\partial gn}{\partial x_1} & \frac{\partial gn}{\partial x_2} & \cdots & \frac{\partial gn}{\partial x_n}\end{bmatrix} \quad (6)$$

Theorem 5: The nonlinear system (5) is asymptotically stable at the origin if there exists a constant, positive definite and symmetric matrix P such that the matrix $G\left(x\right)=J^T\left(x\right)P+PJ\left(x\right)$ is negative definite for in some neighborhood D of the origin all x and $V\left(x\right)=g^T Pg$.

B. Variable Gradient Method

The variable gradient method suggested [6] for generating the Lyapunov function, which provides considerable in selecting suitable function. For autonomous system (5), choose a V(x) be a function of Lyapunov. The time derivative of V can be expressed as

$$\dot V\left(x\right)=\frac{\partial V}{\partial x_1}\dot x_1+\frac{\partial V}{\partial x_2}\dot x_2+\cdots+\frac{\partial V}{\partial x_n}\dot x_n \quad (7)$$

$$h\left(x\right)=gradV\left(x\right)=\begin{bmatrix}\frac{\partial V}{\partial x_1}\\ \frac{\partial V}{\partial x_2}\\ \vdots\\ \frac{\partial V}{\partial x_n}\end{bmatrix}=\begin{bmatrix}h_1\left(x\right)\\ h_2\left(x\right)\\ \vdots\\ h_n\left(x\right)\end{bmatrix} \quad (8)$$

Then (6) becomes $\dot V\left(x\right)=\left(h\left(x\right)\right)^T\dot x$ (9)

The Lyapunov function can be expressed by integrating (9) on both sides

$$V\left(x\right)=\int_0^t\frac{dV\left(x\right)}{dt}.dt=\int_0^t\left(h\left(x\right)\right)^T.\frac{dx}{dt}.dt=\int_0^x\left(h\left(x\right)\right)^T.dx \quad (10)$$

$$V\left(x\right)=\int_0^{x_1}h_1\left(\theta_1,0,0,\cdots,0\right)d\theta_1+\int_0^{x_2}h_2\left(x_1,\theta_2,0,0,\cdots,0\right)d\theta_2+\cdots+\int_0^{x_n}h_2\left(x_1,x_2,\cdots,x_{n-1},\theta_n\right)d\theta_n \quad (11)$$

The scalar function V(x) obtained by integrating (10), for a continuous vector function g(x) to be gradient of scalar V(x).

Then $\frac{\partial h_i}{\partial x_j}=\frac{\partial h_j}{\partial x_i}$; i, j = 1, 2, …,n (12)

$$\frac{\partial^2 V\left(x\right)}{\partial x^2}=\begin{bmatrix}\frac{\partial^2 V}{\partial x_1{}^2} & \frac{\partial^2 V}{\partial x_1\partial x_2} & \cdots & \frac{\partial^2 V}{\partial x_1\partial x_n}\\ \frac{\partial^2 V}{\partial x_2\partial x_1} & \frac{\partial^2 V}{\partial x_2{}^2} & \cdots & \frac{\partial^2 V}{\partial x_2\partial x_n}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{\partial^2 V}{\partial x_n\partial x_1} & \frac{\partial^2 V}{\partial x_n\partial x_2} & \cdots & \frac{\partial^2 V}{\partial x_n{}^2}\end{bmatrix} \quad (13)$$

since $\frac{\partial^2 V}{\partial x_i\partial x_j}=\frac{\partial^2 V}{\partial x_j\partial x_i}$ is symmetric matrix.

$$\frac{\partial g\left(x\right)}{\partial x}=\begin{bmatrix}\frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_n}\\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \cdots & \frac{\partial g_2}{\partial x_n}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{\partial gn}{\partial x_1} & \frac{\partial gn}{\partial x_2} & \cdots & \frac{\partial gn}{\partial x_n}\end{bmatrix} \quad (14)$$

For g(x) to be equal to grad V(x), this matrix to be symmetric likes (12).

C. Stabilization with Guaranteed Safety

Control Lyapunov Function (CLF) is used to stabilize the affine nonlinear systems [7].

Consider the autonomous system $\dot x=h\left(x\right)x\left(0\right)=x_0$ (13)

With a set of dangerous state D which is open. Then

$$\left.\begin{array}{l} L_{clf}\left(x\right)>0 \quad \forall x\in D\\ V\left(x\right)L_{clf}\left(x\right)<0 \quad \forall x\in\Re^n\setminus\left(D\cup\{0\}\right)\\ u:=\left\{x\in\Re^n \mid L_{clf}\left(x\right)\le 0\right\}\ne\phi\\ \overline{\Re^n\setminus\left(D\cup u\right)}\cap\bar D=\phi\end{array}\right\} \quad (14)$$

If the constraints (14) are satisfied then the system (13) is asymptotically stable and the system is protected with $x_0=\Re^n\setminus D$.

D. Linear Quadratic Optimal Control Problems

The linear multi variable completely controllable plant [8-10],

$$\dot x=Ax+Bu \quad (15)$$

Where x is called the state vector, $\dot x$ the derivative of the state vector, u is the input or directional vector, A is the system matrix, B the input matrix and null state x=0 is the desired steady state.

The objective is to compute optimal control law that minimizes the performance index J, with the initial conditions $x(0)\equiv x^0$

$$J=\frac{1}{2}\int_0^\infty\left(x^TQx+u^TRu\right)dt \quad (16)$$

Where Q & R is the positive, so the matrices are real and symmetric.

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Since the augmentation of A and B is completely controllable, then the state feedback control law is valid.

u= - K x (17)

Therefore the K matrix is determined from so as to minimize the performance index (16) then (14) is optimal for any initial state

x(0). $K=R^{-1}B^{T}P$. (18)

The matrix P in (15) to be satisfied following Ricatti equation

$$A^{T}P + PA - PBR^{-1}B^{T}P + Q = 0 \quad (19)$$

Where K is the real, constant and unconstraint gain matrix, so that results in an asymptotically stable closed loop system.

$$\dot{x}(t) = (Ax - BKx) = (A - BK)x \quad (20)$$

This is show that there is a Lyapunov function $V = \frac{1}{2}x^{T}Px$ for the closed loop system (18). The positive definite matrix, P, the dv/dt obtained from the trajectories of the closed loop system is negative definite.

Theorem 6: If the state feedback controller (17) is such that it minimizes the function $f(u) = \frac{dv}{dt} + \frac{1}{2}\left(x^{T}Qx + u^{T}Ru\right)$ and the minimum of f(u) for some $V = \frac{1}{2}x^{T}Px$ then the controller is optimal.

V.RESULTS AND DISCUSSION

In order to expose the use of the control Lyapunov function based technique proposed in this paper, the simulation results for various frameworks performed by utilizing MATLAB [11]. Considered the simple Examples (1-3) the stability margins are investigated. Example (4) considered for Control Lyapunov based LQR design for MIMO system and Example (5) considered for Control Lyapunov based LQR design for SISO system.

5.1. Example 1: Nonlinear SMD system

The state space model of nonlinear Spring-Mass-Damper system is given by

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -0.5x_2^3 - x_1$$

Choose the positive definite Lyapunov function $V = x_1^2 + x_2^2 \quad (V \to \infty \text{ as } \|x\| \to \infty)$ then $\dot{V} = -x_2^4$. $\dot{V}$ is negative semi definite, so the system is stable for all the values of x2.

5.2. Example 2: A Simple Nonlinear System

A simple nonlinear system is given by

$$\dot{x}_1 = x_1 + 3x_2$$

$$\dot{x}_2 = 2x_1$$

Choose the positive definite Lyapunov function $V = 2x_1^2 - 3x_2^2 \quad (V \to \infty \text{ as } \|x\| \to \infty)$ then $\dot{V} = 4x^2$. $\dot{V}$ is positive definite, so the system is unstable for the all the values of x2.

5.3. Example 3: A Nonlinear Autonomous System

Consider the following nonlinear autonomous system

$$\dot{x}_1 = -x_1 + x_2^3$$

$$\dot{x}_2 = x_1^3 - x_2$$

To investigate stability of the system by the Lyapunov function, using the suitable Lyapunov function $V = x_1^2 + x_2^2$ and its derivative $\dot{V} = 2\left(x_1^2 + x_2^2\right)\left(x_1x_2 - 1\right)$. If $x_1x_2 > 1$ then the system is stable.

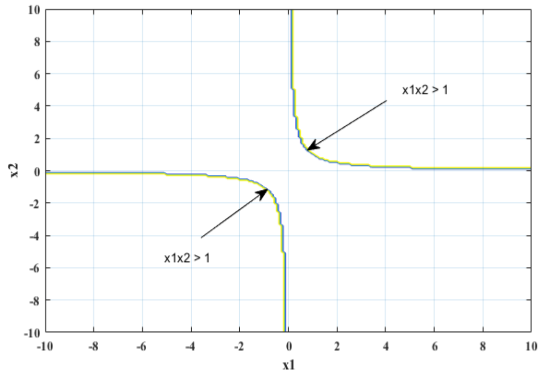

Figure: 2. The stability region of the example 3 by Lyapunov function $V = x_1^2 + x_2^2$

5.4. Example 4: Kinematic Model of 2 DoF Helicopter

Consider the kinematic model of 2 DoF helicopters [12] in this paper proposed to speed of response and minimization of objective function. Here, the Control Lyapunov Function (CLF) based stabilization with tracking performance is considered.

$$\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \ddot{\theta} \\ \ddot{\psi} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -9.2751 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.4955 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2.3667 & 0.0790 \\ 0.2410 & 0.7913 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{mp} \\ v_{my} \end{bmatrix}$$

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$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix}$$

For the given system the state feedback control law in (17), minimize the cost function in Theorem 6 and Control Lyapunov Function (CLF) in (14).

The LQR tuning parameter of Q and R and derived as

$$Q = \begin{bmatrix} 12.24 & 0 & 0 & 0 & 0 & 0 \\ 0 & 102.55 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 156.78 & 0 & 0 \\ 0 & 0 & 0 & 0 & 55.12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 200 \end{bmatrix} \&$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The Eigen values of P and Q are positive. From the results it is shown that Control Lyapunov function based LQR is better disturbance rejection with safe stability margin compared to LQR.

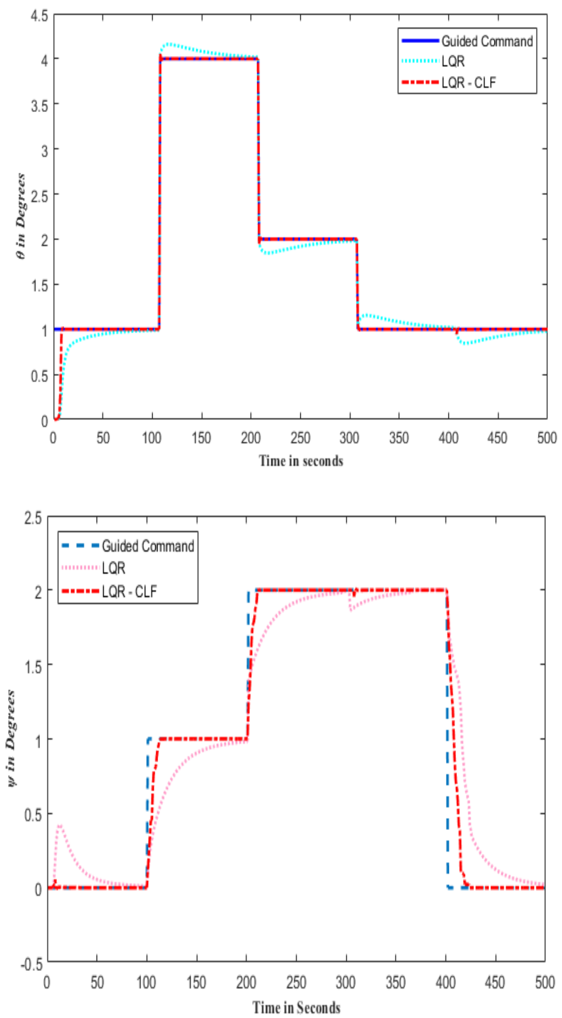

Figure: 3. Trajectory tracking and disturbance rejection response for the example 4

5.5. Example 5: Non Minimum Phase System

Consider the following third order non minimum phase state space equation

$$\dot{x}(t) = \begin{bmatrix} -4 & -5 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} x(t)$$

The LQR tuning parameter of Q and R and derived as

$$Q = \begin{bmatrix} 1.24 & 0 & 0 \\ 0 & 2.68 & 0 \\ 0 & 0 & 10.02 \end{bmatrix} \text{ \& } R=6.8.$$

The Eigen values of P and Q are positive. In this system the unit step given as disturbance at t=35 Sec. From the results it is shown that Control Lyapunov function based LQR is better disturbance rejection as well as tracking performance with safe stability margin compared to LQR.

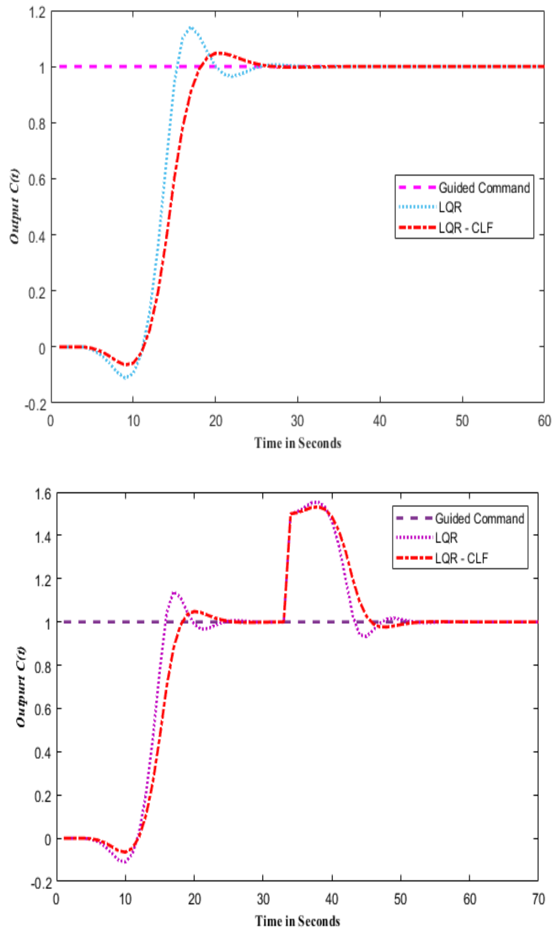

Figure: 4. Trajectory tracking and disturbance rejection response for the example 5

VI. CONCLUSION

This paper considered the control Lyapunov functions used to stabilize the system through the LQR optimal settings and stability margins are derived for the linear/nonlinear systems using Lyapunov theorems.

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The Lyapunov based LQR controller are designed and simulated with guided commands and the results are compared with conventional controller settings. It was shown that both controllers are shown the same tracking performance but the proposed give the better performance in disturbance rejection with guaranteed safety margins

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