

# Performance Index - A Metric to Analyze and Evaluate Performance of Hypercube Interconnection Networks

Karthik K, S Jena, T Venu Gopal

**Abstract:** A Multiprocessor is a computing machine which has at least two central processors, to process the tasks simultaneously. An Interconnection network that links multiple processors greatly influences the performing capability of the entire system. Various network scales to evaluate interconnection networks are: degree, speed, node coverage, reliability, scalability, diameter, connectivity, throughput, packet loss, and network cost etc. Interconnection Network like a Hypercube can be considered as an graph with undirected edges, where a vertex indicates a processor and an undirected edge denotes a communication medium among the processors. Some of the variants of Hypercube Interconnection Networks are Hypercube Network, Folded Hypercube Network, Multiple Reduced Hypercube, Multiply Twisted Cube, Recursive Circulant, Exchanged Crossed Cube Network, Half Hypercube Network etc. The vital purpose of this paper is to investigate different variants of hypercube interconnection networks and to analyze their properties to summarize the differences in their performance. It is also discussed how to analyze and evaluate the performance of Hypercube Networks and identify the changes in Performance Index based on the variations in their properties

**Index Terms:** Multiprocessor, Interconnection Network, Hypercube, Performance Index.

## I. INTRODUCTION

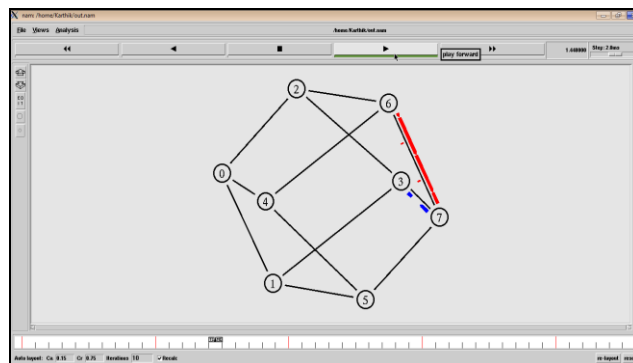
*Hypercube Network:*

The Hypercube Interconnection Network is an n-dimensional geometric figure, homologous to a cube in three dimensions and to a square in two dimensions. Diameter of an interconnection network is the least no. of steps a processor takes to transmit message packet to another processor which is farthest to it. Thus, for ex., diameter of a hypercube is two which is analogous to a square with four no. of vertices.

In a hypercube interconnection network, symmetrical to cube with eight processors with memory & processor module incident at every vertex of the cube, the diameter is 3. So, for

a system which contains  $2n$  processors which are strongly and completely connected to each other, its diameter will be  $n$  [1].

The implementation of Hypercube Interconnection Network in NS2 is as shown below:



**Fig.1: Hypercube Interconnection Network**

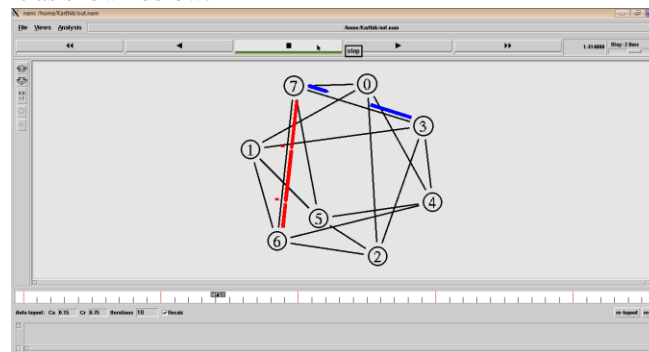
*Folded Hypercube Network:*

The Folded Hypercube Interconnection Network is a graph with directionless edges extracted from a hypercube network graph with extra perfect matching edges that connects diagonal or opposite couple vertices of the hypercube network graph.

Folded Network graph of order  $m$  which consists of  $2$  power  $m-1$  nodes can be framed by incorporating links between opposite vertex pairs in Hypercube graph of order  $m - 1$ .

Folded Hypercube may be created from a hypercube graph of  $m$  order, which has double number of vertices, by joining all opposite pair of vertices together. Folded graph of  $m$  order is  $m$  regular with two power  $m-1$  vertices and two power  $m-2$  edges [2]. Chromatic no. of  $m$  order Folded Graph is  $2$  if  $m$  is an even value &  $4$  for an odd value.

The implementation of Folded Hypercube Network in NS2 is as shown below:



**Fig.2: Folded Hypercube Network**

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\* Correspondence Author

**Karthik K\***, Associate Professor, Department of Computer Science and Engineering, BVRIT, Jawaharlal Nehru Technological University, Hyderabad (Telangana), India

**S Jena**, Associate Professor, Department of Information Technology, Gandhi Institute of Technology and Management University, Hyderabad (Telangana), India

**T. Venu Gopal**, Professor, Department of Computer Science and Engineering, Jawaharlal Nehru Technological University, Hyderabad (Telangana), India

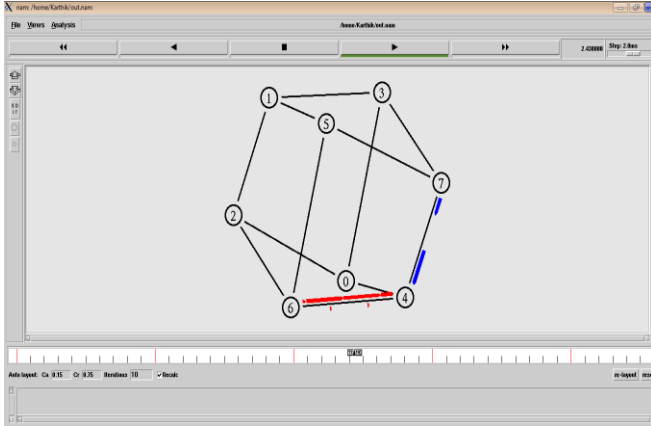
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*Multiple Reduced Hypercube Network (MRH):*

Nodes of Multiple Reduced Hypercube MRH (n) are represented as n bit strings  $S_n, S_{n-1}, \dots, S_1 \dots S_2, S_1$  comprising the bits 0s & 1s.

The edges of Multiple Reduced Hypercube are represented in 3 forms. Depending on the connection methodology, the edges are named as (i) Hypercube Edge, (ii) Exchange Edge and (iii) Complement Edge. These edges are noted as h-edge, x-edge and c-edge resp. [3].

The implementation of Multiple Reduced Hypercube Network in NS2 is as shown below:



**Fig.3: Multiple Reduced Hypercube Network**

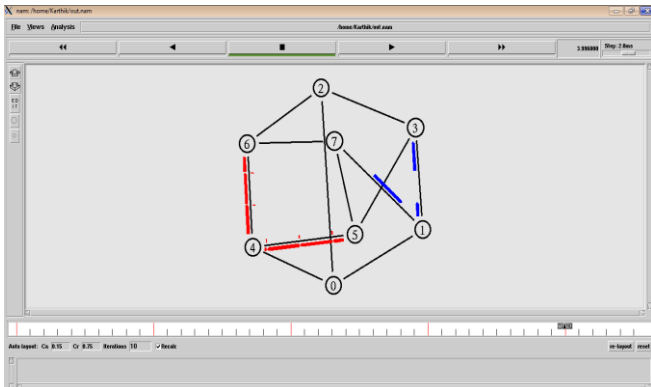
*Multiply Twisted Cube Network:*

A Multiply Twisted Cube (MTC) with p-dimensions has the same structural complexity as that of a hypercube with p dimensions.

It has the identical no. of vertices & edges, and each vertex has the same degree p. MTC has  $2^p$  vertices and diameter of  $(p+ 1)/2$  appears to be approximately 1/2 the diameter of Binary n Cube [1].

Based on the results of earlier experiments, as some of the properties are better than that of hypercube, MTC can be considered as a good architecture for developing multiprocessor systems.

The implementation of Multiple Twisted Cube Network in NS2 is as shown below:



**Fig.4: Multiple Twisted Cube Network**

*Recursive Circulant Hypercube Network:*

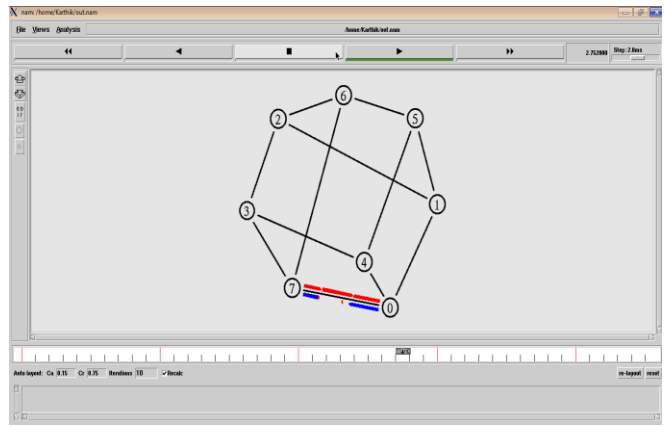
Recursive Circulant Hypercube which is identical to a Recursive Circulant Graph  $G(L, d)$  is defined as a Circulant graph which has L nodes and jumps of d powers,  $d \geq 2$ .

$G(L, d)$  is symmetric in node, has Hamiltonian cycles if not

$L \leq 2$ , and may be constructed recursively.

Definition: For any 2 positive integer values L and d, the Recursive Circulant graph represented as  $G(L, d)$  has the Vertex set  $V = \{0, 1, \dots, L - 1\}$ , a and b vertices are adjacent to each other if and only if  $a-b = \pm di \pmod L$  for some  $0 \leq i \leq [\log_d L] - 1$ . For multiprocessor systems, recursive circulant graphs family is proposed as network topology [4]. Recursive Circulant graph  $G(L, d)$  has a structure which is recursive for  $L = dm, m \geq 1$  and  $L = cd, 2 \leq c < d, m \geq 0$  [5].

The implementation of Recursive Circulant Hypercube Network in NS2 is as shown below:



**Fig.5: Recursive Circulant Hypercube Network**

Some of the properties that affect the performance of hypercube interconnection networks variants are given below.

*Properties:*

- Degree: It is association among various nodes of a network. Node connectivity implies the complexity of network. Greater number of links in the network means greater is the complexity.
- Diameter: It is defined as the maximal shortest path between any two nodes of the network. Lower diameter is desirable as the diameter puts a lesser bound on the criticality of parallel algorithms that require connection between any pair of nodes arbitrarily.
- Network Cost: It can be considered as the result of degree x diameter. This property is extensively used in evaluating the performance of a network.
- Speed: Speed can be considered as the enormity of velocity of an object. The mean speed of a packet in a time interval is the distance travelled / duration of time interval. The more speed enriches more number of packets transmission.
- Packet Loss: Packet Loss is measured as the proportion of packets lost to the packets sent over the network. The more packet loss the less reliable transmission and the less packet loss the more efficient transmission.
- Coverage: Node Coverage is the percentage of destination nodes covered out of total available nodes.

II. ANALYSIS

As a part of initial analysis and to assess the performance of considered Hypercube Interconnection Networks, the analysis of the properties such as number of nodes, degree, diameter and network cost is done as summarized below.

| Interconnection Network               | Nodes          | Degree | Diameter                  | Network Cost         |
|---------------------------------------|----------------|--------|---------------------------|----------------------|
| Hypercube Network (H(n))              | 2 <sup>n</sup> | n      | n                         | ≈ n <sup>2</sup>     |
| Folded Hypercube (FH(n))              | 2 <sup>n</sup> | n+1    | ⌈n/2⌉                     | ≈ n <sup>2</sup> /2  |
| Multiply Twisted Cube (MTC(n))        | 2 <sup>n</sup> | n      | ⌈n+1/2⌉                   | ≈ n <sup>2</sup> /2  |
| Recursive Circulant Hypercube (RC(n)) | 2 <sup>n</sup> | n      | ⌈3n/4⌉                    | ≈ 3n <sup>2</sup> /4 |
| Multiple Reduced Hypercube (MRH(n))   | 2 <sup>n</sup> | n      | ⌈n/2⌉ + ⌊(⌈n/2⌉+1)/3⌋ + 1 | ≈ n <sup>2</sup> /3  |

Degree and Diameter are analyzed to compute the approximate Network cost.

At different intervals of time units, speed, packet loss and node coverage can be calculated by implementing the various Hypercube Interconnection Networks.

For example, on implementation of Folded Hypercube Network for 30 units of time interval, Dataset can be generated and the formatted Dataset looks like:

| Event | Time    | From_Node | To_Node | pkt_type | pkt_size | fid | src_addr | des_addr | seq_num |
|-------|---------|-----------|---------|----------|----------|-----|----------|----------|---------|
| +     | 0.1     | 7         | 3       | cbr      | 1000     | 2   | 7.1      | 3.1      | 0       |
| -     | 0.1     | 7         | 3       | cbr      | 1000     | 2   | 7.1      | 3.1      | 0       |
| d     | 0.1     | 7         | 3       | cbr      | 1000     | 2   | 7.1      | 3.1      | -1      |
| +     | 0.1     | 7         | 6       | tcp      | 40       | 1   | 7        | 6        | 0       |
| -     | 0.1     | 7         | 6       | tcp      | 40       | 1   | 7        | 6        | 0       |
| d     | 0.1     | 7         | 6       | tcp      | 40       | 1   | 7        | 6        | -1      |
| +     | 0.108   | 7         | 3       | cbr      | 1000     | 2   | 7.1      | 3.1      | 1       |
| -     | 0.108   | 7         | 3       | cbr      | 1000     | 2   | 7.1      | 3.1      | 1       |
| d     | 0.108   | 7         | 3       | cbr      | 1000     | 2   | 7.1      | 3.1      | -1      |
| r     | 0.11016 | 7         | 6       | tcp      | 40       | 1   | 7        | 6        | 0       |
| +     | 0.11016 | 6         | 7       | ack      | 40       | 1   | 6        | 7        | 0       |

|   |         |   |   |     |      |   |     |     |    |
|---|---------|---|---|-----|------|---|-----|-----|----|
| - | 0.11016 | 6 | 7 | ack | 40   | 1 | 6   | 7   | 0  |
| d | 0.11016 | 6 | 7 | ack | 40   | 1 | 6   | 7   | -1 |
| r | 0.114   | 7 | 3 | cbr | 1000 | 2 | 7.1 | 3.1 | 0  |

As done with folded hypercube network, all other considered different variants of hypercube networks are implemented for different time units of 30, 60, 90, 120 units of time interval and huge raw dataset is generated.

To analyze the properties of speed, packet loss, node coverage, degree, diameter and network cost, the generated dataset is formatted in terms of time, reachability from node to node, packet type, packet size, id, source address, destination address and sequence numbers as shown above.

III. MATHEMATICAL CALCULATIONS

From the above formatted data, for Folded Hypercube Network for 30 units of time interval, the average values for the required properties (parameters) are calculated as shown below:

| Parameter Metric |              |                 |                 |                 |         |
|------------------|--------------|-----------------|-----------------|-----------------|---------|
| S.No             | Parameters   | ack             | cbr             | tcp             | Average |
| 1                | Speed        | 15.07739<br>215 | 15.049498<br>33 | 15.063048<br>32 | 15.063  |
| 2                | Packet Loss  |                 |                 |                 | 0.099   |
| 3                | Coverage     | 37.5            | 37.5            | 37.5            | 37.500  |
| 4                | Degree       | 4               | 4               | 4               | 4.000   |
| 5                | Diameter     | 2               | 2               | 2               | 2.000   |
| 6                | Network Cost | 8               | 8               | 8               | 8.000   |

Table 1: Parameter Metric

| Mapping Metric |        |
|----------------|--------|
| Parameters     | Weight |
| Speed          | W1     |
| Packet Loss    | W2     |
| Coverage       | W3     |
| Degree         | W4     |
| Diameter       | W5     |
| Network Cost   | W6     |

Thus, as shown above, the required properties of all the variants of Hypercube Networks are evaluated. A justified Weight Metric is contemplated to be mapped on the various properties to calculate a new proposed metric property Performance Index to examine the performance of various Hypercube Interconnection Networks.

Table 2: Mapping Metric

| Weight Metric |    |    |    |    |    |       |
|---------------|----|----|----|----|----|-------|
| W1            | W2 | W3 | W4 | W5 | W6 | TOTAL |
| 20            | 20 | 18 | 15 | 15 | 12 | 100   |

Table 3: Weight Metric

| Performance Index   |
|---|
| $(\text{Avg. Speed} * w1) - (\text{Avg. Packet Loss} * w2) + (\text{Avg. Coverage} * w3) + (\text{Avg. Degree} * w4) + (\text{Avg. Diameter} * w5) + (\text{Avg. Network Cost} * w6)$ |

With the justified mapping and weight metrics as shown above, the proposed metric Performance Index is calculated as shown below:

Thus, as shown above mathematically, Performance Index can be calculated for every variant of hypercube network at specified time interval.

IV. RESULTS

From the above calculation, the Performance Index for Folded Hypercube Network at 30 units of time interval is calculated as shown below:

Using the formula  $\{(Avg. Speed * w1) - (Avg. Packet Loss * w2) + (Avg. Coverage * w3) + (Avg. Degree * w4) + (Avg. Diameter * w5) + (Avg. Network Cost * w6)\}$ , Performance Index is:

|                          |
|--------------------------|
| <b>Performance Index</b> |
| <b>1160.282619</b>       |

Similarly, The Performance Index of various Hypercube Interconnection Networks are calculated at different time intervals of 30, 60, 90 and 120 units of time and summarized as shown below to identify the variations in their performance respectively.

| TYPE/TIME                         | 30          | 60          | 90          | 120         |
|-----------------------------------|-------------|-------------|-------------|-------------|
| <b>FOLDED HYPERCUBE</b>           | 1160.282619 | 1344.77261  | 1644.771473 | 1944.770371 |
| <b>HYPERCUBE</b>                  | 1056.767092 | 1356.77261  | 1656.771473 | 1956.770371 |
| <b>MULTIPLE REDUCED HYPERCUBE</b> | 1107.767092 | 1407.77261  | 1707.771473 | 2007.770371 |
| <b>MULTIPLY TWISTED CUBE</b>      | 896.981366  | 1868.967443 | 2168.973442 | 2468.975931 |
| <b>RECURSIVE CIRCULANT</b>        | 893.2912104 | 893.2912104 | 1493.288796 | 1793.280727 |

Table 4: Performance Index of hypercube variants at different time intervals

As shown above, using the chosen parameters of speed, packet loss, node coverage, degree, diameter and network cost, the Performance Index metric is calculated for

hypercube variants folded hypercube network, hypercube interconnection network, multiple reduced hypercube, multiply twisted cube and recursive circulant at 30, 60, 90, 120 units of time respectively.

The Performance of Hypercube Interconnection Networks Variants at 30, 60, 90 and 120 units of time based on Performance Index is depicted using MATLAB as shown below:

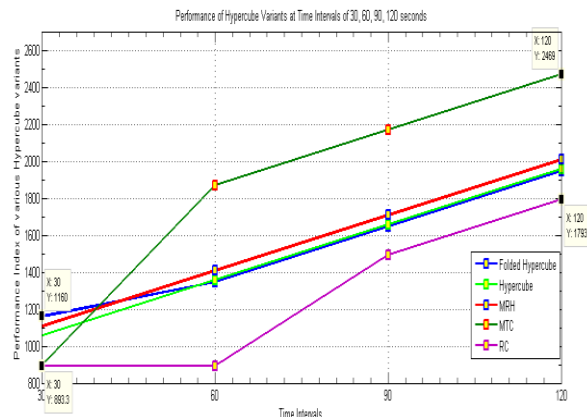


Fig.6: Comparison of Performance Index for different variants of hypercube interconnection networks at different time intervals

V. CONCLUSION

The Performance of various Hypercube Interconnection Networks is evaluated using a novel metric Performance Index. The variations in the performance of various Hypercube Interconnection Networks are observed and analyzed at different time intervals.

The Results and the Observations illustrate that out of all the five considered variants of Hypercube Networks, Multiply Twisted Cube exhibits efficient performance when compared to that of other variants in view of new property metric Performance Index at increased Time Intervals. Thus it exhibits its most suitability for massively Parallel Processing Applications.

Considering the results, the other variants can also exhibit efficient behavior by adopting variations in the properties of speed and packet loss.

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