

# Analysis of Composite Laminate for Maximum Stiffness

Sharad D. Pawar, Abhay Utpat

**Abstract**— The purpose of this study is to develop optimization procedure to maximize the stiffness and minimize the weight of composite laminate subjected to in-plane loading. The design variables for optimization problem are fiber orientation angles, thickness of lamina and number of laminas. Maximum stress failure criteria are used to determine whether load bearing capacity is exceeded for a configuration generated during optimization process. In a recent year, the application of Fiber reinforced composite material has increased with increasing need of low weight, high strength, high stiffness etc. in aerospace industry, automobile industry, sporting equipment, civil industry etc. In the case of Fiber Reinforcement Plastic composite structural design, the requirements of certain application can be achieved not only by sizing the cross sectional areas and thickness of components but by changing the material system design i.e. optimizing the material system itself such as fiber orientation angle, ply thickness, stacking sequence etc. The optimization techniques are being used to assist the designer in finding an optimized solution. Carefully designed individual composite parts at present, are about 20-30% lighter than their conventional metal parts.

**Keywords**—Composite Material, FEA, Stiffness

## I. INTRODUCTION

A composite is a structural material that consists of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibres, particles, or flakes. The matrix phase materials are generally continuous. Examples of composite systems include concrete reinforced with steel and epoxy reinforced with graphite fibres, etc. Fibre-reinforced composite materials are demanded by the industry because of their high specific stiffness/strength especially for applications where weight reduction is critical. By using composites, weight of a structure can be reduced significantly. Many studies have been done for composite laminate optimization. Mustafa Akbulut et al.[1] describes optimization procedure to minimize thickness or weight of laminated composite plates subjected to in-plane loading. The paper shows results of optimization for different combinations of in-plane loadings. G NarayanaNaik et al.[2] has presented minimization of weight of composite plates subjected to in plane loads using failure mechanism based (FMB), maximum stress and Tsai Wu failure criteria. Jacob L. et al.[3] has presented methodology for multi-objective optimization of laminated composite materials which is based on integer coded genetic algorithm.

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**Sharad D. Pawar**, Department of Mechanical Engineering, SVERI's College of Engineering, Pandharpur, India.

**Abhay Utpat**, Department of Mechanical Engineering, FTC's College of Engineering and Research, Sangola, India.

M. Walker, R.E. Smith [4] developed a methodology for using genetic algorithms with the finite element method to minimize the mass and deflection of fiber reinforced structures with several design variables is described. Further reduction is also possible by optimizing the material system itself such as fibre orientations, ply thickness, stacking sequence, etc. Rafael F. Silva, Aurea S. Holandab et al.[5] proposed methodology for minimum weight design of laminated composite tubes. Chung Hae Park a, Abdelghani Saouabet al [6] proposed an integrated optimization methodology to take into account the weight saving, the improvement of structural performance and the cost reduction of composite structures. Stiffness is defined as the resistance of a material to deflection. Sukru Karakaya, Omer Soykasapet al [7] proposed genetic algorithm and generalized pattern search algorithm are used for optimal Stacking sequence of a composite panel, which is simply supported on four sides and is subject to biaxial in-plane compressive loads. T. Rangaswamy, S. Vijayarangan et al [8] the overall objective of this paper is to design and analyze a composite drive shaft for power transmission applications. In this paper a Genetic Algorithm (GA) has been successfully applied to minimize the weight of shaft which is subjected to the constraints such as torque transmission, torsion buckling capacities and fundamental natural frequency. Avinash Ramsaroop, Krishnan Kannyet al [9] deals with the generation of MATLAB script files that assists the user in the design of a composite laminate to operate within safe conditions. In some of these studies, layer thickness and ply angles were considered to be continuous design variables. However in practice, composite laminates are fabricated with a specific thickness. Besides, fibre orientations are chosen from a finite set of angles during the design process because of the difficulty of exactly orienting fibres along a given direction. Due to these manufacturing constraints, the design variables for a fibre angle or layer thickness should take only discrete values. As opposed to a zero order search algorithm, a gradient based optimization procedure may fail to cope with the discrete nature of such problems. Moreover, in typical structural optimization problems, there may be many locally optimum configurations. With a large number of design variables, the number of local minima may increase dramatically.

## II. GOVERNING EQUATIONS

A laminate is made up of perfectly bonded layers of lamina with different fiber orientation to represent an integrated structural component. In most practical applications of composite material, the laminates are considered as thin and loaded along the plane of laminates. A thin orthotropic unidirectional lamina as depicted in Fig.2.1 has fiber orientation along the 1 direction and the direction transverse to the fiber along the 2 direction. The x-y coordinates represent the global coordinate system for the lamina.



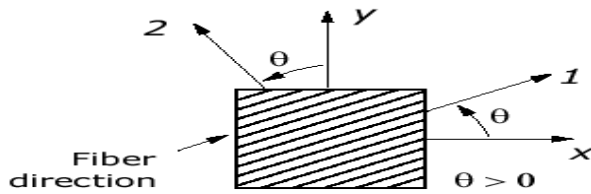


Fig. 2.1 Co-ordinate System for Laminates

**A. Stress-Strain Relationship**

The lamina is thin, the state of stress can be considered in the plane stress condition. That means [10],

$$\begin{aligned} \sigma_3 &= 0 \\ \tau_{23} &= 0 \\ \tau_{31} &= 0 \end{aligned}$$

Hence, the stress-strain relationship for thin lamina in the matrix form along the principal axis can be written as,

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \tag{1}$$

The elements in compliance matrix [S] are the functions of elastic constant of the composite lamina and can be expressed as, for glass epoxy unidirectional lamina,

$$\begin{aligned} E_1 &= 1.25 \times 10^5 \text{ N/mm}^2 \\ E_2 = E_3 &= 8600 \text{ N/mm}^2 \\ G_{12} = G_{31} &= 4700 \text{ N/mm}^2 \\ G_{23} &= 3100 \text{ N/mm}^2 \\ \nu_{12} &= 0.27 \\ \nu_{13} = \nu_{23} &= 0.27 \end{aligned}$$

Compliance Matrix Elements are,

$$\begin{aligned} S_{11} &= 1/E_1 = \frac{1}{1.25 \times 10^5} = 2 \times 10^{-12} \text{ 1/Pa} \\ S_{12} &= -\nu_{12}/E_1 = \frac{-0.27}{1.25 \times 10^5} = -2.16 \times 10^{-12} \text{ 1/Pa} \\ S_{22} &= 1/E_2 = \frac{1}{8600 \times 10^6} = 1.16 \times 10^{-10} \text{ 1/Pa} \\ S_{66} &= 1/G_{12} = \frac{1}{4700 \times 10^6} = 2.13 \times 10^{-10} \text{ 1/Pa} \end{aligned} \tag{2}$$

$$\begin{aligned} \frac{\nu_{12}}{E_1} &= \frac{\nu_{21}}{E_2} \\ \nu_{21} &= \frac{0.27 \times 8600 \times 10^6}{1.25 \times 10^5 \times 10^6} \end{aligned}$$

$$\nu_{21} = 0.0185$$

From Equation (1),

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 8 \times 10^{-12} & -2.16 \times 10^{-12} & 0 \\ -2.16 \times 10^{-12} & 1.16 \times 10^{-10} & 0 \\ 0 & 0 & 2.13 \times 10^{-10} \end{bmatrix} \begin{bmatrix} 188 \times 10^6 \\ 25.6 \times 10^6 \\ 33.53 \times 10^6 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 1.45 \times 10^{-3} \\ 2.56 \times 10^{-3} \\ 7.14 \times 10^{-3} \end{bmatrix}$$

Strains in Local Axes are,

$$\begin{aligned} \epsilon_1 &= 1.45 \times 10^{-3} \\ \epsilon_2 &= 2.56 \times 10^{-3} \\ \gamma_{12} &= 7.14 \times 10^{-3} \end{aligned} \tag{3}$$

By Inverting Equation (1), we get,

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{4}$$

Stiffness Matrix Elements are,

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{21} \times \nu_{12}} = \frac{1.25 \times 10^{11}}{1 - (0.0185 \times 0.27)} \\ &= 1.26 \times 10^{11} \text{ Pa} \\ Q_{12} &= \frac{\nu_{12} \times E_2}{1 - \nu_{21} \times \nu_{12}} = \frac{0.27 \times 8600 \times 10^6}{1 - (0.0185 \times 0.27)} \\ &= 2.33 \times 10^9 \text{ Pa} \\ Q_{22} &= \frac{E_2}{1 - \nu_{21} \times \nu_{12}} = \frac{8600 \times 10^6}{1 - (0.0185 \times 0.27)} \\ &= 8.64 \times 10^9 \text{ Pa} \\ Q_{66} = G_{12} &= 4700 \times 10^6 \text{ Pa} \end{aligned} \tag{5}$$

Since normal stresses applied in the 1-2 direction do not result in any shearing strains in the 1-2 plane because  $Q_{16} = Q_{26} = 0 = S_{16} = S_{26}$ , therefore unidirectional lamina is an orthotropic lamina. Also, the shearing stresses applied in the 1-2 plane do not result in any normal strains in the 1 and 2 directions because  $Q_{16} = Q_{26} = 0 = S_{16} = S_{26}$ . A woven composite with its weaves perpendicular to each other and short fiber composites with fibers arranged perpendicularly to each other or aligned in one direction also are especially orthotropic.

**B. Stress-Strain Transformation Matrices**

Generally, a laminate does not consist only of unidirectional lamina because of their low stiffness and strength properties in the transverse direction. Hence, in most laminates, some laminae are placed at an angle. It is thus necessary to develop the stress-strain relationship for an angle lamina. The co-ordinate system used for showing an angle lamina is as given in Fig.2.1. The axes in the 1-2 coordinate systems are called the local axes / material axes. The direction 1 is parallel to fibers and the direction 2 perpendicular to the fibers. Direction 1 is also called as longitudinal direction and direction 2 also called as transverse direction. The axes in the x-y co-ordinate system are called the global axes / off-axes. The angle between the two axes is denoted by the angle  $\theta$ . The global and local stresses in an angle lamina are related to each other through the angle of the lamina  $\theta$ .

If the co-ordinate transformation matrix as [10],

$$[T] = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & 2 \sin \theta \cos \theta \\ \sin^2 \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\ -\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix} \tag{6}$$

And

$$[T]^{-1} = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & -2 \sin \theta \cos \theta \\ \sin^2 \theta & \cos^2 \theta & 2 \sin \theta \cos \theta \\ \sin \theta \cos \theta & -\sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix} \tag{7}$$

The co-ordinate transform of plane stress can be written in the following matrix form,



$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} \quad (8)$$

Similarly, the strain transform becomes,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} \quad (9)$$

The tensor shear strain is used in the above formula. Suppose the engineering - tensor interchange matrix [R],

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad (10)$$

Then,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [R] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \quad (11)$$

The stress-strain relations for a lamina of an arbitrary orientation can therefore be derived as detailed below,

$$\begin{aligned} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} &= [T]^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = [T]^{-1}[C] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [T]^{-1}[C][R] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} \\ &= [T]^{-1}[C][R] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \\ &= [T]^{-1}[C][R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \\ &= [T]^{-1}[C][T]^{-T} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = [\bar{C}] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \end{aligned}$$

Where the stiffness matrix is defined as,

$$[\bar{C}] = [T]^{-1}[C][T]^{-T}$$

And

$$[\bar{C}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \quad (12)$$

Where,

$$\begin{aligned} \bar{Q}_{11} &= m^4 Q_{11} + n^4 Q_{22} + 2m^2 m^2 Q_{12} + 4m^2 n^2 Q_{66} \\ \bar{Q}_{22} &= n^4 Q_{11} + m^4 Q_{22} + 2m^2 m^2 Q_{12} + 4m^2 n^2 Q_{66} \\ \bar{Q}_{12} = \bar{Q}_{21} &= m^2 n^2 (Q_{11} + Q_{22} - 4Q_{66}) + (m^4 + n^4) Q_{12} \\ \bar{Q}_{16} = \bar{Q}_{61} &= m^3 n (Q_{11} - Q_{22} - 2Q_{66}) \\ &\quad + mn^3 (Q_{12} - Q_{22} + 2Q_{66}) \\ \bar{Q}_{26} = \bar{Q}_{62} &= n^3 m (Q_{11} - Q_{22} - 2Q_{66}) \\ &\quad + nm^3 (Q_{12} - Q_{22} + 2Q_{66}) \\ \bar{Q}_{66} &= m^2 n^2 (Q_{11} + Q_{22} - 2Q_{12}) + (m^2 - n^2)^2 Q_{66} \end{aligned} \quad (13)$$

The transformed reduced matrix  $[\bar{Q}]$  for 15° ply is given by,

$$[\bar{Q}] = \begin{bmatrix} 1.11 * 10^{11} & 9.3 * 10^9 & 2.7 * 10^{10} \\ 9.3 * 10^9 & 9.5 * 10^9 & 2.6 * 10^9 \\ 2.7 * 10^{10} & 2.6 * 10^9 & 1.16 * 10^{11} \end{bmatrix}$$

The transformed reduced matrix  $[\bar{Q}]$  for 30° ply is given by,

$$[\bar{Q}] = \begin{bmatrix} 7.55 * 10^{10} & 4.6 * 10^{10} & 3.73 * 10^{10} \\ 4.6 * 10^{10} & 1.72 * 10^{10} & 1.34 * 10^{10} \\ 3.73 * 10^{10} & 1.34 * 10^{10} & 2.6 * 10^{10} \end{bmatrix}$$

The transformed reduced matrix  $[\bar{Q}]$  for 45° ply is given by,

$$[\bar{Q}] = \begin{bmatrix} 3.94 * 10^{10} & 3.01 * 10^{10} & 2.99 * 10^{10} \\ 3.01 * 10^{10} & 3.95 * 10^{10} & 2.8 * 10^{10} \\ 2.99 * 10^{10} & 2.8 * 10^{10} & 3.25 * 10^{10} \end{bmatrix}$$

The transformed reduced matrix  $[\bar{Q}]$  for 60° ply is given by,

$$[\bar{Q}] = \begin{bmatrix} 1.72 * 10^{10} & 2.4 * 10^{10} & 1.34 * 10^{10} \\ 2.4 * 10^{10} & 7.5 * 10^{10} & 3.7 * 10^{10} \\ 1.34 * 10^{10} & 3.7 * 10^{10} & 3.25 * 10^{10} \end{bmatrix}$$

The transformed reduced matrix  $[\bar{Q}]$  for 75° ply is given by,

$$[\bar{Q}] = \begin{bmatrix} 9.6 * 10^9 & 9.5 * 10^9 & 2.92 * 10^9 \\ 9.5 * 10^9 & 1.11 * 10^{11} & 2.7 * 10^{10} \\ 2.92 * 10^9 & 2.7 * 10^{10} & 1.21 * 10^{10} \end{bmatrix}$$

The transformed reduced matrix  $[\bar{Q}]$  for 90° ply is given by,

$$[\bar{Q}] = \begin{bmatrix} 8.64 * 10^9 & 2.33 * 10^9 & 0 \\ 2.33 * 10^9 & 1.26 * 10^{11} & 0 \\ 0 & 0 & 4.700 * 10^9 \end{bmatrix} \quad (14)$$

The mid-plane from top and bottom of the laminate for thickness as 1 mm is shown in Fig. 2.2.

h0=z=-0.003	15	↓ z
h1=z=-0.002	30	
h2=z=-0.001	45	
h3=z=0	60	
h4=z=0.001	75	
h5=z=0.002	90	
h6=z=0.003	90	

Fig. 3.2 Mid-Plane from Top and Bottom of Laminate

Stress resultants or forces per unit length of cross section are obtained as [11],

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dz = 2 \sum_{k=1}^n n_k t_0 \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k \quad (15)$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (16)$$

The extensional stiffness matrix  $[A_{ij}]$  is given by,

$$A_{ij} = \sum [Q_{ij}] (h_k - h_{k-1}) \quad (17)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} z dz \quad (18)$$

Where,  $h_k$  is the number of plies in the  $k^{\text{th}}$  lamina  
For top layers,

$$\begin{aligned} 15^\circ & \quad z = 0.00 \\ 30^\circ & \quad z = 0.00 \\ 45^\circ & \quad z = 0.00 \end{aligned}$$

$$A_{ij} = \begin{bmatrix} 2.6314 & 1.2123 & 1.1052 \\ 1.2123 & 3.7820 & 1.0800 \\ 1.1052 & 1.0800 & 1.1290 \end{bmatrix} * 10^8$$

The stiffness coupling matrix [B] is,

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n [Q_{ij}]_k (h^2_k - h^2_{k-1}) \quad (19)$$

$$B_{ij} = \begin{bmatrix} 1.8915 & -0.2873 & 0.0768 \\ -0.2873 & 4.9720 & 0.3140 \\ 0.0768 & 0.3140 & 0.1665 \end{bmatrix} * 10^5$$

The bending stiffness matrix [D] is,

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n [Q_{ij}]_k (h^3_k - h^3_{k-1}) \quad (20)$$

$$D_{ij} = \begin{bmatrix} 983.33 & 221.19 & 279.3 \\ 221.19 & 1195.5 & 132.4 \\ 279.3 & 132.4 & 0.1665 \end{bmatrix}$$

$$M_x = M_y = M_{xy} = 0$$

$M_x, M_y$  = Bending Moment per unit length = 0

$M_{xy}$  = Twisting moment per unit length = 0

From equations (15) to (20),

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} 2.6314 & 1.2123 & 1.1052 & 1.8915 & -0.2873 & 0.0768 \\ 1.2123 & 3.7820 & 1.0800 & -0.2873 & 4.9720 & 0.3140 \\ 1.1052 & 1.0800 & 1.1290 & 0.0768 & 0.3140 & 0.1665 \\ 1.8915 & -0.2873 & 0.0768 & 983.33 & 221.19 & 279.3 \\ -0.2873 & 4.9720 & 0.3140 & 221.19 & 1195.5 & 132.4 \\ 0.0768 & 0.3140 & 0.1665 & 279.3 & 132.4 & 0.1665 \end{bmatrix} * \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \\ K_x \\ K_y \\ K_{xy} \end{bmatrix} \quad (21)$$

But

$$\begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \\ K_x \\ K_y \\ K_{xy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0015 \\ 0.0010 \\ 0.0016 \end{bmatrix}$$

Where  $\varepsilon_x^\circ, \varepsilon_y^\circ, \gamma_{xy}^\circ$  are the middle surface strains and  $K_x, K_y, K_{xy}$  are the middle surface curvatures

$$N_x = \frac{1500}{75 * 10^{-3}} = 20000 \text{ N/m}$$

$$N_y = \frac{1500}{300 * 10^{-3}} = 3000 \text{ N/m}$$

$$N_{xy} = \frac{20000 - 3000}{2} = 8500 \text{ N/m} \quad (22)$$

Middle stresses and strains are calculated for corresponding ply,

For 15<sup>o</sup> ply,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 1.6 * 10^{-3} \\ -1.72 * 10^{-4} \\ -1.18 * 10^{-3} \end{Bmatrix} m/m \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 32.65 * 10^6 \\ 12.58 * 10^6 \\ -11.17 * 10^6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} 1.776 * 10^{-3} \\ -3.5 * 10^{-4} \\ 7.03 * 10^{-5} \end{Bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 25.72 * 10^6 \\ 19.51 * 10^6 \\ -14.74 * 10^6 \end{Bmatrix}$$

For 30<sup>o</sup> ply,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 1.29 * 10^{-3} \\ -1.43 * 10^{-4} \\ 9.48 * 10^{-4} \end{Bmatrix} m/m \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 9.9 * 10^6 \\ -27.02 * 10^6 \\ 8.2 * 10^6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} 1.75 * 10^{-3} \\ -6.1 * 10^{-4} \\ -2.92 * 10^{-4} \end{Bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 7.8 * 10^6 \\ -25 * 10^6 \\ -11.9 * 10^6 \end{Bmatrix}$$

For 45<sup>o</sup> ply,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 1.03 * 10^{-3} \\ -1.143 * 10^{-4} \\ 7.2 * 10^{-4} \end{Bmatrix} m/m \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 27.4 * 10^6 \\ 44.13 * 10^6 \\ 30.7 * 10^6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} 1.2 * 10^{-3} \\ -2.62 * 10^{-4} \\ 1.14 * 10^{-3} \end{Bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 66.46 * 10^6 \\ 5.1 * 10^6 \\ 8.4 * 10^6 \end{Bmatrix}$$

For 60<sup>o</sup> ply,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 7.68 * 10^{-4} \\ -8.6 * 10^{-5} \\ 4.84 * 10^{-4} \end{Bmatrix} m/m \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 95 * 10^6 \\ 10 * 10^6 \\ 29.7 * 10^6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} 5.5 * 10^{-4} \\ 1.35 * 10^{-4} \\ -1.24 * 10^{-3} \end{Bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 57 * 10^6 \\ 48.03 * 10^6 \\ -51.7 * 10^6 \end{Bmatrix}$$

For 75<sup>o</sup> ply,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 5.04 * 10^{-4} \\ -5.7 * 10^{-5} \\ 2.52 * 10^{-4} \end{Bmatrix} m/m \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 38 * 10^6 \\ 9.5 * 10^6 \\ -12.9 * 10^6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} 1.1 * 10^{-4} \\ 3.4 * 10^{-4} \\ -7.2 * 10^{-4} \end{Bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 5 * 10^6 \\ 42.6 * 10^6 \\ 4.05 * 10^6 \end{Bmatrix}$$

For 90<sup>o</sup> ply,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 2.41 * 10^{-4} \\ -2.83 * 10^{-5} \\ 2.05 * 10^{-5} \end{Bmatrix} m/m \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 3.1 * 10^6 \\ 1.72 * 10^6 \\ 4 * 10^6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} -2.83 * 10^{-5} \\ 2.41 * 10^{-4} \\ -2.05 * 10^{-5} \end{Bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 1.72 * 10^6 \\ 3.1 * 10^6 \\ -4 * 10^6 \end{Bmatrix} \quad (23)$$

### III. METHODOLOGY AND ANALYSIS

#### A. Statement of the Problem

In this the problem is to model the composite laminate. Present work proposes a methodology to model a composite laminate with a composite material to analyse its stress, strength and stiffness using ANSYS software. In order to evaluate the effectiveness of composite, stress analysis is performed on composite using ANSYS. In this work effort is made to reduce stress levels so that advantage of ultimate weight reduction along with stresses can be obtained.

#### B. Selection of Boundary Conditions

First model analysis is done for the composite laminate and found different modes from it. In the static analysis, the one end of the laminate constrained all degrees of freedom is zero and other end of the composite laminate is applied pressure as shown in Fig.3.1. Next the material properties are considered for glass fabric/epoxy as shown in Fig.3.2.



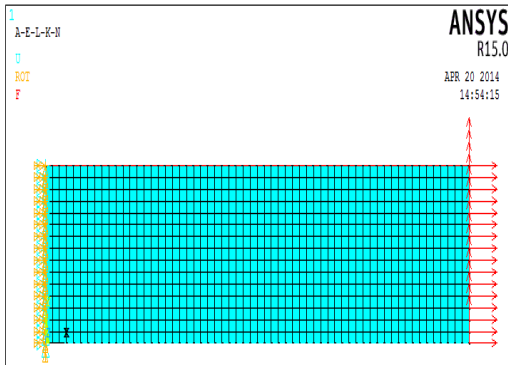


Fig. 3.1 Selection of Boundary Condition and Applying Loads

EX	1.25E+005
EY	8600
EZ	8600
PRXY	0.27
PRYZ	0.4
PRXZ	0.27
GXY	4700
GYZ	3100
GXZ	4700

Fig. 3.2 Material Properties for Glass/Epoxy

C. Simulation

Cartesian coordinate (X, Y, Z) are in back center of the composite laminate. Key points location has been given in Y-Z plane at given points/ of required dimension. Fig. 3.3 shows the areas are created as Top, Bottom, and Left Side & Right Side so as to be meshed with the different criteria's. Meshing is done using shell 181 element shows the geometry, node locations, and the element coordinate system for this element. The element is defined by shell section information and by four nodes (I, J, K, and L) as shown in Fig.3.4.

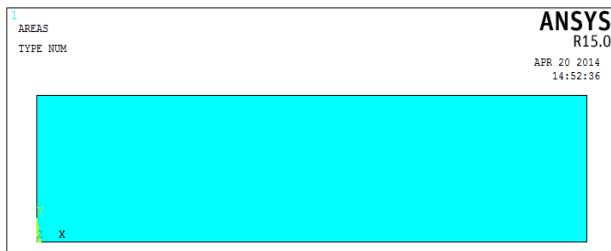


Fig.3.3 Selection areas for Laminate

A single-layer shell section definition provides flexible options. For example, we can specify the number of integration points used and the material orientation. The shell section commands allow for layered shell definition. Options are available for specifying the thickness, material, orientation, and number of integration points through the thickness of the layers.

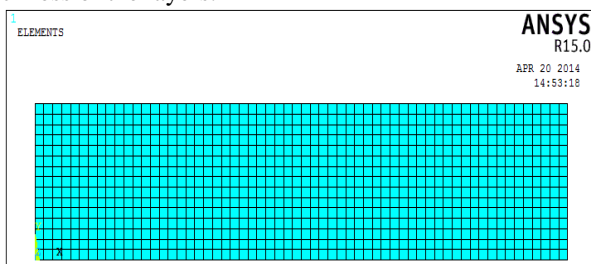


Fig.3.4 Meshing the Problem

To designate the number of integration points (1, 3, 5, 7, or 9) located through the thickness of each layer when using section input. Only one, the point is always located midway between the top and bottom surfaces. If three or more points, two points are located on the top and bottom surfaces respectively and the remaining points are distributed equal distances between the two points. The default number of integration points for each layer is three; however, when a single layer is defined and plasticity is present, the number of integration points is changed to a minimum of five during solution.

While meshing the number of layers are given of equal thickness as shown known as layers in composites using the section data tool in shell181 layer. The element has capability of multilayer so that it can store the results for each layer.

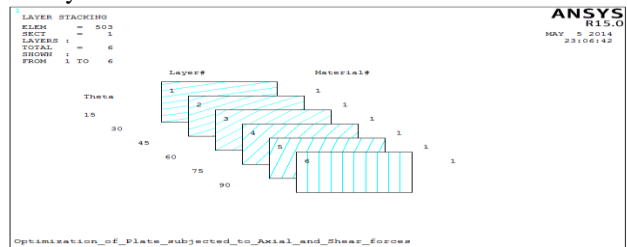


Fig. 3.5 Angle Orientation of Laminate

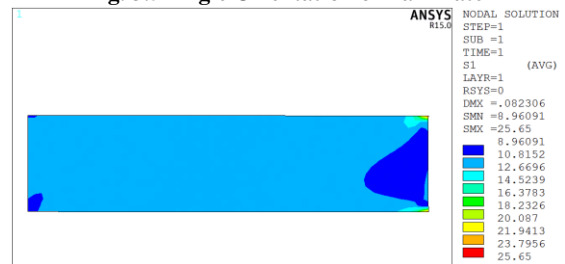


Fig. 3.6.a 1<sup>st</sup> Principal Stress for Layer 1

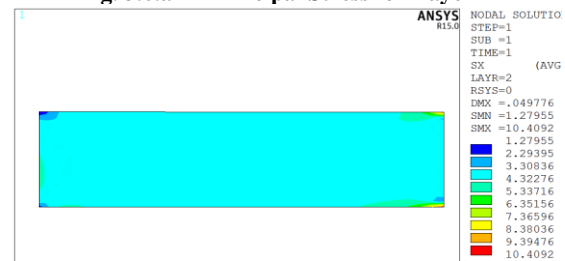


Fig. 3.6.b 1<sup>st</sup> Principal Stress for Layer 2

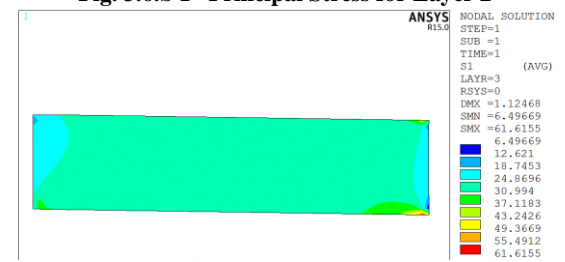


Fig. 3.6.c 1<sup>st</sup> Principal Stress for Layer 3

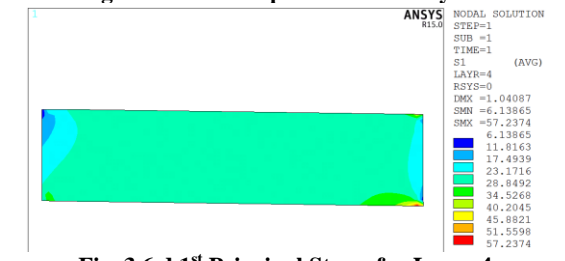


Fig. 3.6.d 1<sup>st</sup> Principal Stress for Layer 4

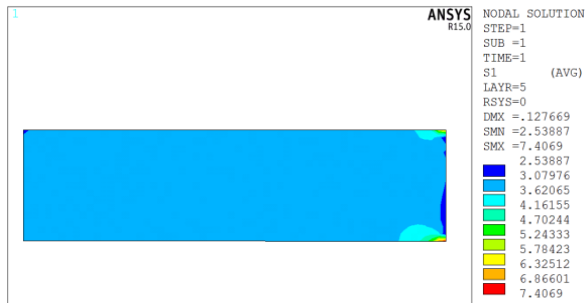


Fig. 3.6.e 1<sup>st</sup> Principal Stress for Layer 5

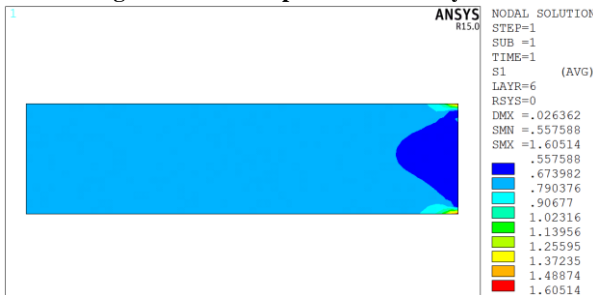


Fig. 3.6.f 1<sup>st</sup> Principal Stress for Layer 6

IV. RESULTS AND DISCUSSION

The results obtained from FEA and Mathematical modeling discussed for corresponding fiber angle orientation i.e. Layer from 1 to 6. The overall dimensions selected for laminate are 300\*75 in millimetres. The stresses developed in the composite laminate for material Glass/Epoxy are estimated. The optimization solution is obtained by using FEA results from Fig. 3.6.a to 3.6.f. The maximum and minimum stress values for each layer are calculated by FEA.

For Layers 1 and 2, stresses values are from 8.96091 to 25.65 MPa and from 1.27955 to 10.4092 MPa i.e. fiber angle orientation 15° and 30° respectively. Also layers 3 and 4, stresses values are from 6.49669 to 61.6155 MPa and 6.13865 to 57.2374 MPa i.e. angles 45° and 60° respectively. Thus from layer 3, the maximum stress value gives the optimized value and suitable for this laminate. But for layer 4, the maximum value goes on decreasing. At the last Layer 5 and 6, stresses values are from 2.53887 to 7.4069 MPa and 0.557588 to 1.60514 MPa i.e. 75° and 90°. Thus the results by FEA and Mathematical modeling shows that the maximum stress values before and after layer 3 i.e. 45° are quite less. The comparative results between Mathematical and FEA are given in Table 1.1. For solving the current problem the validation is worked out by ANSYS R 15.0.

Table 1.1 Comparitive Results between Matematical and Analysis

1st Principal Stress (MPa)		
Layers	Mathematical Result	FEA Result
1	25.72	25.65
2	7.8	10.4
3	66.46	61.62
4	57	57.24
5	5	7.41
6	1.72	1.62

V. CONCLUSIONS

In this paper, design optimization of composite laminates is performed using finite element analysis. The main objective of design optimization in aerospace

composite structures is to minimize the weight of the laminate for given loading. For present work Glass/Epoxy composite material is used. As it has many applications some of them are as fighter airplane with fuselage, helicopters and tilt rotors, ice hockey sticks, artificial portable lungs, marine housings, automotive Corvette, car bumpers. The fibre angles are chosen 15°, 30°, 45°, 60°, 75°, 90° as the six layers and Pincipal Stresses for each layer is solved by analytical method. The Layer 3 has fibre angle 45° must have the better stiffness as compared with other layers. Thus we can say that the optimised fiber angle for the Glass/Epoxy material is 45°.

APPENDIX

Following code is for optimization is paste in ANSYS software to get the 1<sup>st</sup> Principal Stresses for each layer.

```

/TITLE, COMPOSITE PLATE ANALYSIS
/prep7
ET,1,SHELL181
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,1.25e5
MPDATA,EY,1,,8600
MPDATA,EZ,1,,8600
MPDATA,PRXY,1,,0.27
MPDATA,PRYZ,1,,0.4
MPDATA,PRXZ,1,,0.27
MPDATA,GXY,1,,4700
MPDATA,GYZ,1,,3100
MPDATA,GXZ,1,,4700
*DO,J,1,9,1
/FILNAME,COMPOSITE_ANALYSIS%J%,0
T=SUM2+0.1
I=SUM+10
sect,1,shell,,Glass_epoxy
secdata, T,1,I,3
secdata, T,1,-I,3
secdata, T,1,I,3
secdata, T,1,I,3
secdata, T,1,-I,3
secdata, T,1,I,3
secoffset, TOP
seccontrol,,,,, , ,
SUM=I
SUM2=T
K, , , ,
K, ,300,,
K, ,300,75,,
K, ,0,75,,
FLST,2,4,3
FITEM,2,1
FITEM,2,2
FITEM,2,3
FITEM,2,4
A,P51X
FLST,5,2,4,ORDE,2
FITEM,5,1
FITEM,5,3
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL, ,_Y
!*
    
```



```

LESIZE,_Y1,,20,,,,,1
!*
FLST,5,2,4,ORDE,2
FITEM,5,2
FITEM,5,4
CM,_Y,LINE
LSEL,, , ,P51X
CM,_Y1,LINE
CMSEL,,_Y
!*
LESIZE,_Y1,,10,,,,,1
!*
MSHAPE,0,2D
MSHKEY,1
!*
CM,_Y,AREA
ASEL,, , , 1
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
!*
AMESH,_Y1
!*
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
!*
FLST,2,11,1,ORDE,4
FITEM,2,1
FITEM,2,32
FITEM,2,52
FITEM,2,-60
!*
/GO
D,P51X,,,,,ALL,,,,,
FLST,2,11,1,ORDE,3
FITEM,2,2
FITEM,2,22
FITEM,2,-31
!*
/GO
F,P51X,FZ,100
/AUTO,1
/REP,FAST
/UI,MESH,OFF
/AUTO,1
/REP,FAST
/DIST,1,1.08222638492,1
/REP,FAST
/DIST,1,1.08222638492,1
/REP,FAST
/DIST,1,1.08222638492,1
/REP,FAST
/DIST,1,1.08222638492,1
/REP,FAST
/DIST,1,1.08222638492,1
/REP,FAST
/RELOT,RESIZE
FINISH
/SOL
!*
ANTYPE,0
ANTYPE,0
NLGEOM,1
NSUBST,100,0,0

```

```

NCNV,0,0,0,0,0,0
AUTOTS,0
/STATUS,SOLU
SOLVE
SAVE,COMPOSITE_ANALYSIS%J%,DB, ,ALL
SAVE,COMPOSITE_ANALYSIS%J%,RST,
*ENDDO
FINISH
/POST1
!*
/EFACET,1
PLNSOL, S,EQV, 0,1.0

```

!PLEASE COPY THE ABOVE CODE INTO ANSYS AND  
INGORE ANY WARNINGS (IF ANY)

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## AUTHORS PROFILE

**Mr. Sharad D. Pawar**, completed Under Graduate B.E. Mechanical in Academic Year 2011 and in the same year admitted to Post Graduate M.E. Design from Solapur University, Solapur, UG Project Title is "Body Design & Optimization of Foldable 3-Wheeler with 2-Stroke Engine", working as a Assistant Professor at "Fabtech Technical Campus, College Of Engineering And Research, Sangola, since last 02 year."

**Dr. Abhay A. Utpat**, completed Under Graduate B.E. Mechanical in Academic Year 2000 from Shivaji University, Kolhapur and completed Post Graduate M.E Metallurgy in 2005 from Pune University, Pune. Also completed Ph.D. in 2012 from Pune University, Pune. He has 13 year experience in teaching field

