International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878 (Online), Volume-2 Issue-5, November 2013

GPS Aided Inertial Navigation System

Mukesh Kumar Maheshwari, Shah Najmus Saqib Mahmood, Md Hussain, Aurangzeb Rashid Masud

Abstract*— *This Paper will focus on the integration of GPS and Inertial Navigation Systems (INS). The measurements are acquired through Inertial Measurement Unit (IMU), which determine the position, velocities and rotations at roll, yaw and pitch axis respectively. Usually IMU's are very expensive, but here we have used low cost IMU and by low cost it means low performance as well. This is the main reason of introducing GPS to provide stability and feedback to the system. It is observed from the results that the errors of GPS are above 8 meters. These errors are reduced up to 2-3 meters by combining Inertial Navigation System with GPS.

***Index Terms* —GPS, INS, Kalman Filter.**

I. INTRODUCTION

Navigation means having the ability to determine one's position on land, sea or in air at any instant. In early days it was done with the help of sun, moon and magnetic compass. Nowadays navigation systems are widely used providing continuous information of position, velocity and orientation of controlled and autonomous vehicles. In many applications only the initial position of object is known, but subsequently its position cannot be tracked with respect to the reference point [1]. Navigation systems have wide application in agriculture, mining, automobile, aviation industries etc.

It is difficult to determine the one's position accurately of with the help of conventional systems; such as estimating the position of a submarine due to limitation of radio waves propagation under water. Aircraft is also one of the important applications because they travel at are higher attitude and must maintain a certain level of flight with ground reference. Similarly missiles are operated in environment where radar systems are jammed. Global Positioning System (GPS) is one of the famous technologies used for navigation but it has errors above 8 meters due to variation in atmospheric conditions and hence in RF wave propagation [2]. These problems can be rectified by GPS aided inertial navigation system.

This paper focus on low cost inertial GPS aided inertial navigation system. Section II covers the concept of Inertial Navigation system, integration of GPS, and INS and Kalman filtering to reduce the errors. In section III simulation results are discussed and in finally section conclusions are made.

Revised Manuscript Received on 30 November 2013.
* Correspondence Author
Mukesh Kumar Maheshwari, Lecturer, Department of Electrical Engineering Bahria University Karachi Campus, Pakistan.
Shah Najmus Saqib, Assistant Professor, Department of Electrical Engineering Bahria University Karachi Campus, Pakistan.
Muhammad Hussain, Lecturer, Department of Electrical Engineering Bahria University Karachi Campus, Pakistan.
Aurangzeb Rashid Masud, Lecturer, Department of Electrical Engineering Bahria University Karachi Campus, Pakistan.

II. SYSTEM DESCRIPTION

A. *Inertial Navigation System*

Inertial Navigation systems are self-contained and unlike GPS they can provide estimation of position, velocity and altitude at high rate. These measurements are acquired through Inertial Measurement Unit (IMU) which consist of three accelerometers and three gyroscopes at pitch, roll and yaw axis. The position, velocity and attitude can be known by integrating the acceleration and angular rate. However, low frequency noise and sensor biases are also amplified because of the integrative nature of INS [3].

B. *GPS and INS Integration*

The information of the navigation system is obtained from the sensors which can be divided in to two categories 1) Dead reckoning sensors 2) External sensors. Dead reckoning sensors are of (IMU) are robust, and independent and provide data at high frequency but a drawback of producing errors with time. External sensors are part of (GPS) that provide absolute values (information) but are limited to low frequency operation works at low frequency. The idea purpose is to combine the INS raw data with the GPS to obtain high data rates with better accuracy and smaller position error than the compared to standalone GPS receiver. Fig (a) shows the inputs and outputs of the system [**1**]

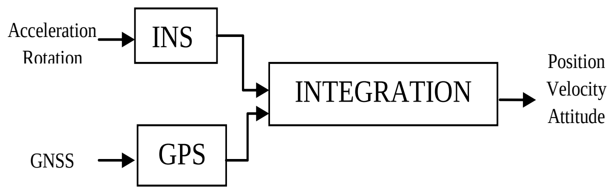

Fig (a): Inputs and Outputs of the system.

The data of INS and GPS are in different coordinate systems so before processing the data needs to be transformed in a common coordinate frame. In INS acceleration and angular rates are measured with respect to the inertial frame of reference i.e. body frame, while GPS provides data in the Earth-Centered Earth-Fixed Frame (ECEF Frame) [4].

i) *Data from sensors*

The data from the sensors is usually corrupted because of biasing errors and the noise measurements, so the error needs to be removed. The following equations show a mathematical approach for the correction of the measurement data before processing [6].

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

The velocities can be obtained by integrating the acceleration.

$$\mathbf{V_b} = \begin{bmatrix} \frac{1}{(1+S_{ax})} & 0 & 0 \\ 0 & \frac{1}{(1+S_{ay})} & 0 \\ 0 & 0 & \frac{1}{(1+S_{az})} \end{bmatrix} (\mathbf{V_b^-} - \mathbf{b_a \Delta t})$$

$$\eta = \begin{bmatrix} \frac{1}{(1+S_{gx})} & 0 & 0 \\ 0 & \frac{1}{(1+S_{gy})} & 0 \\ 0 & 0 & \frac{1}{(1+S_{gz})} \end{bmatrix} (\eta^- - \mathbf{b_g \Delta t})$$

Where b_a and b_g are the bias of the accelerometer and the gyros respectively and S_a and S_g are the scale factors of the accelerometers and gyros respectively while and Δt is the sampling time.

The transitions between the different frames can be acquired using a rotation matrix.

In order to represent matrix in different coordinate system coordinate transformation technique is used and the most common is Euler angles.

ii) Euler Rotations

The Euler angles were developed to determine the orientation of the body in three dimension space. To transform the coordinates from body frame to navigation frame the Euler angles roll ($\mathbf{\Phi}$), pitch ($\mathbf{\Theta}$) and yaw ($\mathbf{\Psi}$) are used and is denoted by $\mathbf{R_n^b}$.

$$\mathbf{R_n^b} = \begin{bmatrix} \cos\Psi & -\sin\Psi & 0 \\ \sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & 0 & \sin\Theta \\ 0 & 1 & 0 \\ -\sin\Theta & 0 & \cos\Theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi & \cos\Phi \end{bmatrix}$$

$$\mathbf{R_n^b} = \begin{bmatrix} \cos\theta\cos\Psi & -\cos\Phi\sin\Psi + \sin\Phi\sin\theta\cos\Psi & \sin\Phi\sin\Psi + \cos\Phi\sin\theta\cos\Psi \\ \cos\theta\sin\Psi & \cos\Psi\cos\Phi + \sin\Phi\sin\theta\sin\Psi & -\sin\Phi\cos\Psi + \cos\Phi\sin\theta\sin\Psi \\ -\sin\theta & \sin\Phi\cos\theta & \cos\Phi\cos\theta \end{bmatrix}$$

Where, the θ is 'pitch', $\o$ is 'roll' and Ψ is 'yaw'. This sequence corresponds first to right handed rotation around the vehicle's z-axis (+ Ψ), followed by right handed rotation around the vehicle's y-axis (+ θ) and a right handed rotation around vehicle's x-axis (+ Φ). The Euler angles can be obtained from above equation by:

Φ (Phi) $= \text{a tan2}(\mathbf{R_n^b}32, \mathbf{R_n^b}33)$
Θ (Theta) $= -\sin(\mathbf{R_n^b}31)$
Ψ (Psi) $= \text{a tan2}(\mathbf{R_n^b}21, \mathbf{R_n^b}11)$

$\mathbf{R_n^b}ij$ represents i,jth element of the matrix $\mathbf{R_n^b}$, where i represent rows and j represent columns.

iii) Discretization

The new angles for 'pitch' Θ can be obtained through integration by [5].

$$\Theta_{i+1} = \int \Theta \, dt + \Theta_I$$

The angles for roll and yaw can be obtained similarly.

The acceleration can be obtained in a body frame $\mathbf{f_b} = [\mathbf{f_x}\ \mathbf{f_y}\ \mathbf{f_z}]$ and can be transformed in the navigation frame by.

$$\mathbf{f_n} = \mathbf{R_n^b}\ \mathbf{f_b}$$

For velocity and position, the acceleration needs to be integrated.

iv) Quaternion

In this approach the rotation from one frame to another can be accomplished by a single rotation about a vector denoted by 'q' and angle q. A quaternion is a hyper-complex number with four parameters, which are a function of this vector and angle [5]. Quaternion can be initialized through roll, pitch and yaw angles defined in Euler angles.

$$\mathbf{q(i)} = \begin{bmatrix} \frac{0.5 \times (\mathbf{R_n^b}(3,2) - \mathbf{R_n^b}(2,3))}{\sqrt{1+\mathbf{R_n^b}(1,1)+\mathbf{R_n^b}(2,2)+\mathbf{R_n^b}(3,3)}} \\ \frac{0.5 \times (\mathbf{R_n^b}(1,3) - \mathbf{R_n^b}(3,1))}{\sqrt{1+\mathbf{R_n^b}(1,1)+\mathbf{R_n^b}(2,2)+\mathbf{R_n^b}(3,3)}} \\ \frac{0.5 \times (\mathbf{R_n^b}(2,1) - \mathbf{R_n^b}(1,2))}{\sqrt{1+\mathbf{R_n^b}(1,1)+\mathbf{R_n^b}(2,2)+\mathbf{R_n^b}(3,3)}} \\ \mathbf{0.5} \times \sqrt{\mathbf{1} + \mathbf{R_n^b}(\mathbf{1},\mathbf{1}) + \mathbf{R_n^b}(\mathbf{2},\mathbf{2}) + \mathbf{R_n^b}(\mathbf{3},\mathbf{3})} \end{bmatrix}$$

The angular increment obtained from gyros can be used to update quaternion 'q' as

$$q_{k+1} = q_k + 0.5 \begin{bmatrix} c & s\Delta\eta_z & -s\Delta\eta_y & s\Delta\eta_x \\ -\Delta\eta_z & c & \Delta\eta_x & s\Delta\eta_y \\ \Delta\eta_y & -\Delta\eta_x & c & s\Delta\eta_z \\ -\Delta\eta_x & -s\Delta\eta_y & -s\Delta\eta_z & c \end{bmatrix} q_k$$

Where $\quad s = \left(\frac{1-\Delta\eta^2}{24}\right) + \left(\frac{\Delta\eta^4}{1920}\right)$

$$c = -\left(\frac{\Delta\eta^2}{4}\right) + \left(\frac{\Delta\eta^4}{192}\right)$$

$$\Delta\eta = \sqrt{\eta_x^2 + \eta_y^2 + \eta_z^2}$$

The four parameters can be obtained by acquiring unit vectors.

$$C_n^b = \begin{bmatrix} (q_4^2 + q_1^2 - q_2^2 - q_3^2) & 2(q_1q_2 - q_4q_3) & 2(q_1q_3 + q_4q_2) \\ 2(q_1q_2 + q_4q_3) & (q_4^2 - q_1^2 + q_2^2 - q_3^2) & 2(q_2q_3 - q_4q_1) \\ 2(q_1q_3 - q_4q_2) & 2(q_2q_3 + q_4q_1) & (q_4^2 - q_1^2 - q_2^2 + q_3^2) \end{bmatrix}$$

Now, obtain the acceleration in the body frame and evaluate in the navigation frame.

$$f_n = C_n^b f_b$$

The position and velocity can be measured by integrating the acceleration.

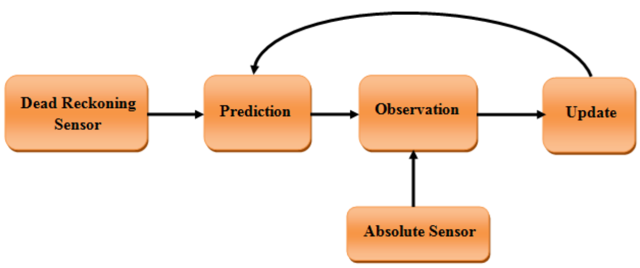

Fig (b): The filter predicts the state, while getting readings from the aiding absolute sensor and corrects the state [5].

C. Kalman Filter

A statistical filter and is simply a set of equations which combines information from separate sensors to optimally evaluate the pose of vehicle. The criteria of optimally is varied, but all work to minimize the errors in the estimated pose. Generally a filter implements a model which transforms the state of interest from time step to future point in time. In navigation system, the model is generally a kinematic representation of the vehicle. The information from dead rocking sensor is used to drive the model.
However error, accumulation associated with dead rocking sensors also causes unbounded errors growth on the output of filter. Thus an absolute sensor error is required to bind the errors. Fig (b) illustrate a simply layout of navigation system.

When there is no absolute fix from the aiding sensor, the filter is said to be "predicting" the state of interest. During this prediction stage the filter also evaluate the uncertainty in the predicted values. Once a fix occurs, the filter estimates the state of the vehicle by applying a weighting between the observation and prediction. This stage is called the "update". The Kalman filter for example, which is one of the most widely implemented fusion algorithms, evaluates this weighting by minimizing the Mean Square Error (MMSE) of the estimate [5-7].

The Kalman Filter is a filter which estimates the states of the system by noisy measurements. The filter has two major steps, prediction and update. First step utilize the previous state estimate information to predict the current state. In second step, the measurement information at current time step is used to correct the estimate, to get the accurate information.

The core elements of the Kalman Filter are; state matrix, covariance matrix, state model, observation vector and covariance, observation model, and the algorithm.

'State matrix' is based on the parameters of the system which the Kalman Filter will estimate. The 'error co-variance matrix' describes the uncertainties in the state and the correlation between the errors. 'State model' describes how the previous two matrices vary with respect to time. The 'observation vector' is the set of measurements described by some external system or aiding system. The 'observation model' describes how the observation vector varies as a function of state matrix. The Kalman Filter uses all these matrices to produce estimates for the state matrix [6]. The equations are provided below. For the complete derivation readers are referred to [8, 9].

A predicted state at discrete time can be described by

$$x_{k|k-1} = x_{k-1|k-1}F_k + B_k u_k \tag{1.1}$$

Where, $\mathbf{F_k}$ is the state transition matrix, $\mathbf{u_k}$ is the control matrix and $\mathbf{B_k}$ relates the control matrix to the states.

The predicted estimate error covariance can be described by

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k \tag{1.2}$$

The equation (1.1) and (1.2) needs to be evaluated every time the sample is obtained from the inertial sensors. When the observations are available from the external sensor at time 'k', the observation vector can be stated by,

$$Z_k = x_k H_k + v_k \tag{1.3}$$

Fig (c): INS/GPS Navigation System [3].

Where $\mathbf{H_k}$ is the observation model and $\mathbf{v_k}$ is the observation noise vector, which is assumed to be zero mean.

For the update time, when the values from the external sensor are available, the estimation can be obtained by

$$x_{k|k} = x_{k|k-1} + K_k(z_k - H_k x_{k|k-1}) \tag{1.4}$$

$(z_k - H_k x_{k|k-1})$ is the difference between the actual observation and predicted observation.

$$K_{k|k} = P_{k|k-1} H_k^T S_k^{-1} \tag{1.5}$$

Where S_k is the innovation covariance and is obtained by

$$S_k = H_k P_{k|k-1} H_k^T + R_k \tag{1.6}$$

The error covariance matrix can also be updated due to this observation

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \tag{1.7}$$

Where 'I' is the identity matrix.

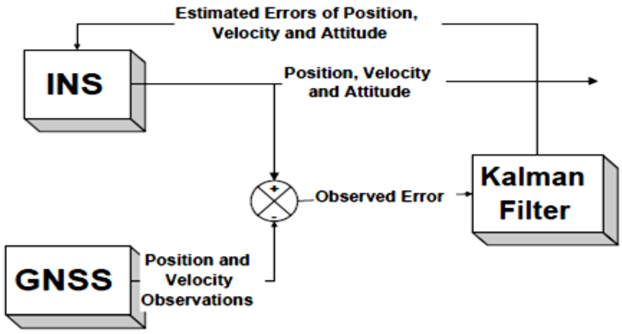

Fig (d): Velocities towards north, east and down.

III. RESULTS AND DISCUSSION

The above system is simulated in Matlab R2010 and Inertial Navigation System toolbox 2.0 is used to generate some functions for conversion of coordinate frames. Sensors and GPS data is entered using MAT file.

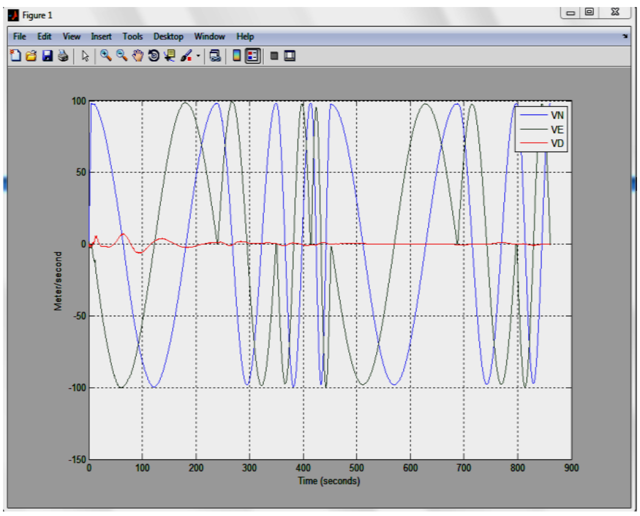

Figure (d) shows the velocities along north, east and down.

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

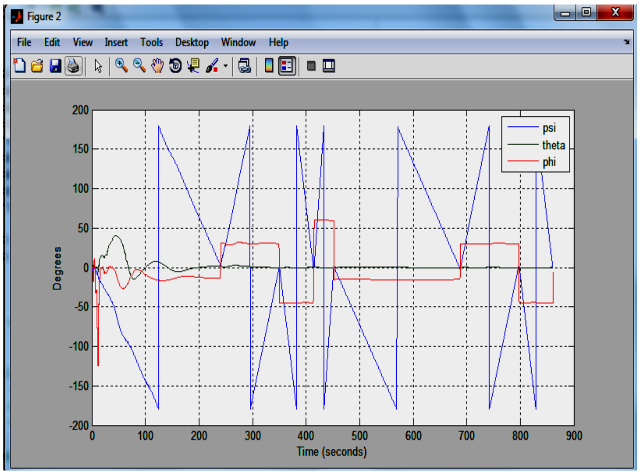

Fig (e): Psi, theta and phi angles with respect to time.

The velocity along down axis (red line) is zero, while velocity to north blue line and east are green line. In figure (e) blue lines represents the motion along vertical axis (yaw or Ψ), green line represents motion along lateral axis (pitch or θ) and red line represents the motion along longitudinal axis (roll or Φ).

There is a little variation along longitudinal and lateral axis initially but later on it settles down.

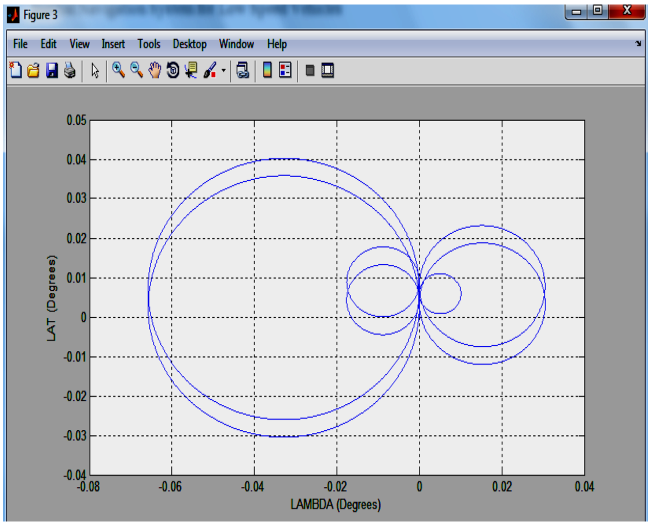

Fig (f): Plot between Latitude and longitude.

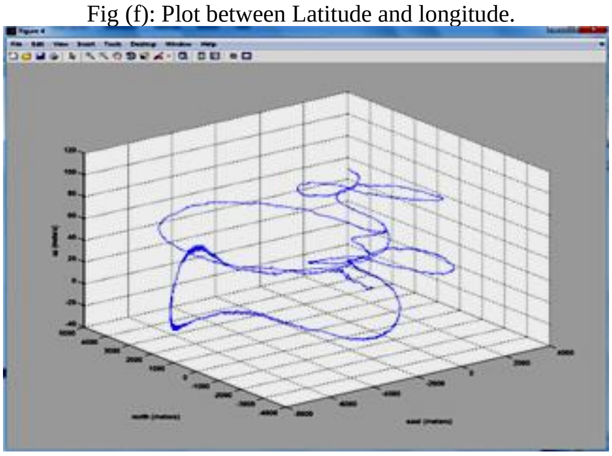

Fig (g): Position estimates of North, East and Up in meters

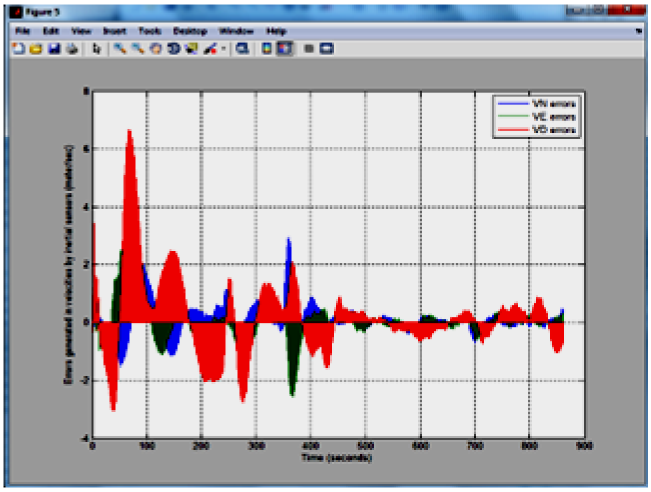

Fig (h): Errors in velocities generated by inertial sensors

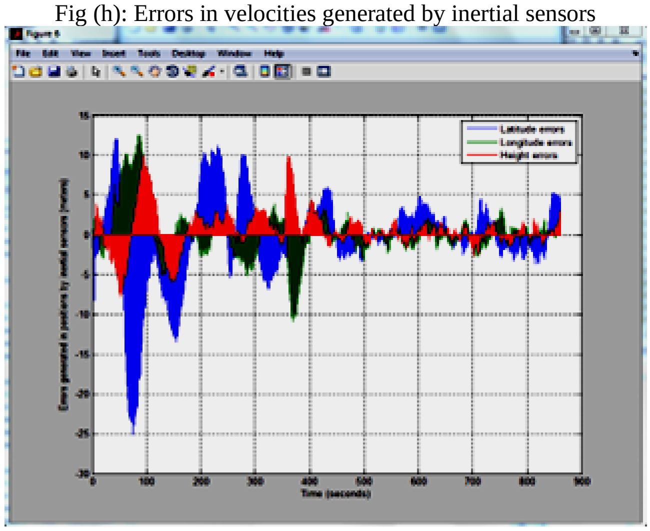

Fig (i): Errors in positions generated by internal sensors

Fig (f) represents changes in latitude and longitude axis. Fig (g) represent the motion along north, east and up in meters. Fig (h) shows the error in velocities. The errors in velocities are 6-7 meters initially. After 500 seconds there errors in velocities are reduce to less than 2-3 meter. Fig (i) represent the errors in position. Initially the errors in latitude are between -25 to 12 meters, at same time errors in longitude axis are between -10 to 12 meters and errors in height are between -7 to 10 meters. After 500 seconds the errors in latitude, longitude and in height are reduced to 3-4 meters.

IV. CONCLUSION

This paper has provided an overview of Inertial Navigation Systems and the benefits of Integrating GPS and INS to work together efficiently. The low cost IMU cannot work alone efficiently as there are high bias drifts in the sensors and hence it is unable to provide any reasonable information.. GPS can work alone but the data rates are very slow up to 1HZ i.e. we get data after 1 second. The INS works alone till it gets the update after every second from the GPS to calibrate its values.

The combination of data from GPS and INS has proved to be efficient as well as reasonably producing good results in comparison to that of standalone INS and GPS systems.

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

REFERENCES

1. Groves, P.D., 2008, Principles of GNSS, Inertial, Multi sensor Integrated Navigation System, Artech House, Boston London.
2. Global Positioning System Standard Positioning Service Performance Standard, Department of Defense United States of America, 4th Edition, Sep 2008,.
3. Xiaoying Kong, "INS algorithm using quaternion model for low cost IMU" Robotics and Autonomous Systems, Elsevier, Volume 46, Issue 4, 30 April 2004, Pages 221-246.
4. Schumacher, A., "Integration of GPS aided Strapdown Inertial Navigation System for land vehicles," MSc. Thesis, Stockholm, Sweden, March 2006
5. Sukkarieh, S., "Low Cost, High Integrity, Aided Inertial Navigation Systems for Autonomous Land Vehicles," Ph.D. Thesis, University of Sydney, March 2000.
6. Joshy, M.J., "Performance comparison of Extended and Unscented Kalman Filter implementation in INS-GPS integration," Master Thesis, Lulea university of technology, September 2009.
7. K. J. Walchko, and Paul Mason, "Inertial Navigation," 2002 Florida Conference on Recent Advances in Robotics, Miami, May 2002.
8. Li, Xiao-Rong, Bar-Shalom, Yaakov, "Estimation and Tracking – Principles, Techniques and Software. Artech House, Inc, 1993.
9. Maybeck, Peter S. "Stochastic Models, Estimation and Control, volume 1, 2", Academic Press, 1979.
10. http://www.gpsoftnav.com/inertial.html. Accessed on February 22, 2013.
11. Skog, Isaac. "A low-cost GPS Aided Inertial Navigation System for Vehicular Applications." KTH Signals Sensors and Systems (2005): 49.

AUTHOR PROFILE

Mukesh Kumar Maheshwari received B.E in Telecommunication Engineering from Mehran University of Engineering & Technology, Jamshoro Pakistan in 2008 and MSc. in Information & Communication Engineering from University of Leicester, England in 2009. He is working as Lecturer in Department of Electrical Engineering Bahria University Karachi Campus, Pakistan. His area of interest includes signal processing and wireless communication.

Shah Najmus Saqib received B.E in Electronics Engineering from Sir Syed University University of Engineering & Technology, Karachi Pakistan in 2002 and MSc. in Information & Communication Engineering from University of Hertfordshire, England in 2008. He is working as Assistant Professor in Department of Electrical Engineering Bahria University Karachi Campus, Pakistan. His area of interest includes control systems and radio communications.

Muhammad Hussain received B.E in Electronic Engineering from NED University of Engineering & Technology, Karachi, Pakistan in 2006 and M.E. in Telecommunication NED University of Engineering & Technology, Karachi Pakistan in 2009. He is working as Lecturer in Department of Electrical Engineering Bahria University Karachi Campus, Pakistan. His area of interest includes radio communication.

Aurangzeb Rashid Masud received B.S in Biomedical Engineering from S.S.U.E.T Karachi, Pakistan in 2000, and M.E in Microelectronics from N.E.D University Karachi, Pakistan. He is working as Lecturer in Department of Electrical Engineering Bahria University Karachi Campus, Pakistan. His area of interest is Microsystems designing & Signal Processing.

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)