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Effect of Time Delay on Robust PID Controllers for a Transfer Function

Mukul Gaur, Sachin Goyal, Sulata Bhandari

Abstract*: A controller designed for a nominal process model generally works fine for the nominal plant model, but may fail even by a slight change in it. Robust control deals with system analysis and control design for such imperfectly known process models. Robust control has been a recent addition to the field of control engineering that primarily deals with obtaining system robustness in the presence of uncertainties. A lot of research has been done and many approaches are available for robust design of the plants. In this paper, a graphical technique introduced in [1] to find all proportional integral derivative (PID) controllers that satisfy the robust stability constraint of a given single input-single-output (SISO) linear time-invariant (LTI) system with time delay[1], is followed and effects of change of time-delay in the nominal plant model is discussed.

Index Terms*: H∞ control, Robust stability, small gain theorem, time-delay.

I. INTRODUCTION

Since Proportional Integral Derivative (PID) controllers have become so important in process industry, there always has been a significant endeavor to obtain effective PID controller design methods, which will meet certain design criteria and provide system robustness. In modern control, the controller is the output of an optimization problem, and the complexity of the controller is directly linked with the complexity of the plant model: a complicated high–order plant will automatically lead to a complicated high–order controller, which is undesirable [7]. Robust control deals with system analysis and control design for such imperfectly known process models. Systems with delays abound in the world. One reason is that nature is full of transparent delays. Another reason is that time-delay systems are often used to model a large class of engineering systems, where propagation and transmission of information or material are involved. The presence of delays (especially, long delays) makes system analysis and control design much more complex [2]. Authors of [2] have shown that the first effective control method designed to deal with the time-delay systems was smith predictor method in which the minor feedback loop consisting of a predictor is introduced, then, the controller design problem for a time-delay system is converted to a design problem for a delay-free system. The disadvantage of smith predictor, not being applied on unstable system, motivated the modified smith predictor and finite spectrum assignment.

H∞ control was introduced in the late ’80s and is proved to be the most vital method in robust control.

As presented in [3], the modem robust control was actually introduced in two papers written in the early ’60s. One was a paper by Zames [4] written in 1963, which introduced the concept of the “small gain” principle, which plays such a key role in robust stability criteria. The other was a paper written by Kalman [5] in 1964, which demonstrated for SISO systems that optimal LQ state-feedback control laws had some very strong robustness properties, i.e. infinite gain margins and 60-deg phase margins .

The remainder of this paper is organized as follows: mathematical formulations will be done in Section II as presented in [1]. In Section III, this method will then be applied on D. C. motor with different time-delays included in the nominal plant model and the results are compared. Finally, Section IV concludes the results obtained in section III.

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* Correspondence Author
Mukul Gaur, Department of Electrical Engineering, PEC University, Chandigarh, India.
Sachin Goyal, Department of Electrical Engineering, PEC University, Chandigarh, India.
Prof. Sulata Bhandari, Department of Electrical Engineering, PEC University, Chandigarh, India.

II. DESIGN METHODOLOGY

Consider a SISO and LTI system with multiplicative uncertainty as shown in figure 1.

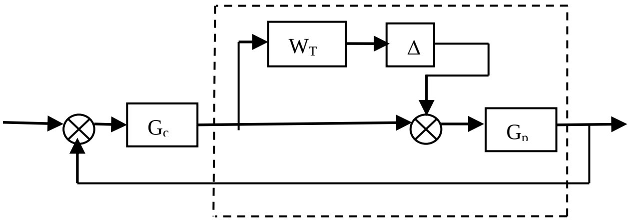

Figure 1: Plant with multiplicative uncertainty [1].

In block diagram, G_p is nominal plant, G_c is the PID controller, W_T is the multiplicative weight, G_Δ is the perturbed plant and Δ is any stable transfer function satisfying the constraint, $|\Delta(j\omega)| \leq 1 \quad \forall\omega$. The nominal plant transfer function can be represented as-

$$G_p(s) = G_o(s) * e^{-st}$$

where $G_o(s)$ is any transfer function and t is the time delay.

Controller transfer function is defined as-

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s$$

Converting the transfer functions in figure 1in frequency domain, we get-

$$G_p(j\omega) = Re(\omega) + jIm(\omega) \quad (1)$$

$$W_T(j\omega) = A_T(\omega) + jB_T(\omega) \quad (2)$$

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$$G_c(j\omega) = K_p + \frac{K_i}{j\omega} + K_d j\omega \qquad (3)$$

PID controller should be designed such that the nominal system remains stable and simultaneously satisfying the robust stability constraint-

$$\|W_T(j\omega)T(j\omega)\|_\infty \leq \gamma \qquad (4)$$

where $T(j\omega)$ is the complementary sensitivity function and $\gamma = 1$ [4]

$$T(j\omega) = \frac{G_p(j\omega)G_c(j\omega)}{1 + G_p(j\omega)G_c(j\omega)} \qquad (5)$$

The robust stability constraint can be represented in its magnitude and phase form as-

$$\left|W_T(j\omega)\,T(j\omega)\,e^{j\angle W_T(j\omega)T(j\omega)}\right| \leq \gamma \quad \forall\omega \qquad (6)$$

Putting T(jω) in robust stability constraint, we get-

$$\frac{W_T(j\omega)\,G_p(j\omega)\,G_c(j\omega)}{1 + G_p\,G_c(j\omega)} \leq \gamma \quad \forall\,\omega \qquad (7)$$

where, $\theta = -W_T(j\omega)T(j\omega)$

If equation (6) holds then equation (7) should be satisfied for some values of $\theta \in [0,2\pi]$. Hence, all PID controllers that satisfy (4), must lie at the intersection of all controllers that satisfy (7) for all $\theta \in [0,2\pi]$.

From equation (7), all PID controllers must satisfy-

$$P(\omega,\theta,\gamma) = 0 \qquad (8)$$

Where,

$$P(\omega,\theta,\gamma) = 1 + G_p(j\omega)G_c(j\omega) - \frac{1}{\gamma}W_T(j\omega)G_p(j\omega)G_c(j\omega)e^{j\theta}$$

Substituting (1), (2), (3) and $e^{j\theta} = \cos\theta + j\sin\theta$ in (8), we get-

$$P(\omega,\theta,\gamma) = 1 + \left((Re(\omega) + jIm(\omega))\left(K_p + \frac{K_i}{j\omega} K_d j\omega\right)\right) - \frac{1}{\gamma}\begin{pmatrix}(A_T(\omega) + B_T(\omega))(Re(\omega) + jIm(\omega)) \\ \left(K_p + \frac{K_i}{j\omega} K_d j\omega\right)(\cos\theta + j\sin\theta)\end{pmatrix}$$

Equation (9) can be reduced to the frequency response of the standard closed-loop characteristic polynomial as $\gamma \to \infty$.

Expanding equation (9) into its real and imaginary parts, we get-

$$\left.\begin{aligned} X_{R_p}K_p + X_{R_i}K_i + X_{R_d}K_d &= -\omega \\ X_{I_p}K_p + X_{I_i}K_i + X_{I_d}K_d &= 0 \end{aligned}\right\} \qquad (9)$$

where, the real components are-

$$X_{R_p} = \omega[\alpha Re(\omega) + \beta Im(\omega)]$$
$$X_{R_i} = -\beta Re(\omega) + \alpha Im(\omega)$$
$$X_{R_d} = \omega^2[\beta Re(\omega) - \alpha Im(\omega)]$$

and the imaginary components are-

$$X_{I_p} = \omega[-\beta Re(\omega) + \alpha Im(\omega)]$$
$$X_{I_i} = -\alpha Re(\omega) - \beta Im(\omega)$$
$$X_{I_d} = \omega^2[\alpha Re(\omega) + \beta Im(\omega)]$$

where,

$$\alpha = 1 - \frac{1}{\gamma}A_T(\omega)\cos\theta + \frac{1}{\gamma}B_T(\omega)\sin\theta$$
$$\beta = \frac{1}{\gamma}B_T(\omega)\cos\theta + \frac{1}{\gamma}A_T(\omega)\sin\theta$$

Now we will find the boundaries of (8) in the (K_p, K_i) plane with constant K_d.

Let us assume, $K_d = \bar{K_d}$. After setting the value of K_d in equation (9), it can be arranged as-

$$\begin{bmatrix} X_{R_p} & X_{R_i} \\ X_{I_p} & X_{I_i} \end{bmatrix}\begin{bmatrix} K_p \\ K_i \end{bmatrix} = \begin{bmatrix} -\omega - X_{R_d}K_d \\ -X_{I_d}K_d \end{bmatrix} \qquad (10)$$

Solving equation (10) for all $\omega \neq 0$ and $\theta \in [0,2\pi]$, we obtain the equations as-

$$K_p(\omega,\theta,\gamma) = \frac{-\alpha Re(\omega) - \beta Im(\omega)}{D(\omega)} \qquad (11)$$

$$K_i(\omega,\theta,\gamma) = \omega^2\bar{K_d} + \frac{\omega(\beta Re(\omega) - \alpha Im(\omega))}{D(\omega)} \qquad (12)$$

where,

$$D(\omega) = |G_p(j\omega)|^2\left(1 + \frac{1}{\gamma^2}|W_T(j\omega)|^2 - \frac{2}{\gamma}(A\cos\theta - B\sin\theta)\right)$$

$$|G_p(j\omega)|^2 = Re(\omega)^2 + Im(\omega)^2$$

$$|W_T(j\omega)|^2 = A_T(\omega)^2 + B_T(\omega)^2$$

Setting $\omega = 0$ in equation (12), we get-

$$\begin{bmatrix} 0 & X_{R_i}(0) \\ 0 & X_{I_i}(0) \end{bmatrix}\begin{bmatrix} K_p \\ K_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (13)$$

By the observation of equation (15), it can be concluded that $K_p(0,\theta,\gamma)$ is arbitrary and $K_i(0,\theta,\gamma) = 0$, unless Re(0) = Im(0) = 0, which holds only when $G_p(s)$ has a zero at origin [1].

Multiplicative weight is designed as-

$$|W_T(j\omega)| \geq \left|\frac{G_\Delta(j\omega) - G_p(j\omega)}{G_p(j\omega)}\right| \qquad (14)$$

OR

$$|W_T(j\omega)| \geq \left|\frac{e^{-j\omega t_\Delta} - e^{-j\omega t}}{e^{-j\omega t}}\right| \qquad (15)$$

III. EXAMPLE

Our objective is to find the set of PID controllers for different time-delays included in the nominal plant model and compare them. The feedback loop has an unknown communication delay between 0.05 and 0.15 seconds.

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$t_\Delta \in [0.05, 0.15]$
Here, the nominal plant is taken as to regulate the position control of the D. C. motor with negligible inductance.
The nominal transfer function model of the D. C. motor with time-delay [1] is chosen as-

$$G_p(s) = \frac{65.5}{s(s+34.6)} e^{-st} \tag{16}$$

where, t=0, 0.05, 0.1, 0.2, 0.3.
Using the equations [11] and [12], all PID controllers for the D. C. motor model, are found such that they ensure robust stability in the (K_p, K_i) plane with constant K_d for different time delays in nominal model. Here, following [1], we consider $\overline{K_d} = 0.2$.
The nominal stability boundary can be obtained by setting $\gamma = \infty$ and all PID controllers that satisfy the robust stability constraint in (4) are found by setting $\gamma = 1$. The intersection of all regions inside the nominal stability boundary of the (K_p, K_i) plane is the robust stability region [1].

A. For t=0,

The multiplicative weight transfer function obtained is-

$$W_T(s) = \frac{3s}{1.012s + 18} \tag{17}$$

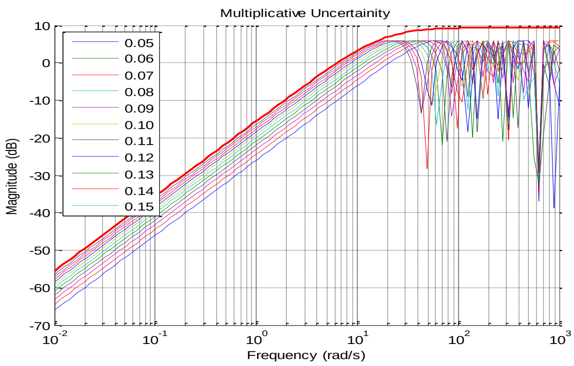

Figure 2: multiplicative weight design to bound the errors for different time delays

Figure 3 shows the robust stability region but the nominal stability boundary is not defined for the chosen transfer function with no time delay.
Choosing a random point in robust stability region, we get

$$G_c(s) = \frac{0.2s^2 + 3.744s + 9.41}{s} \tag{18}$$

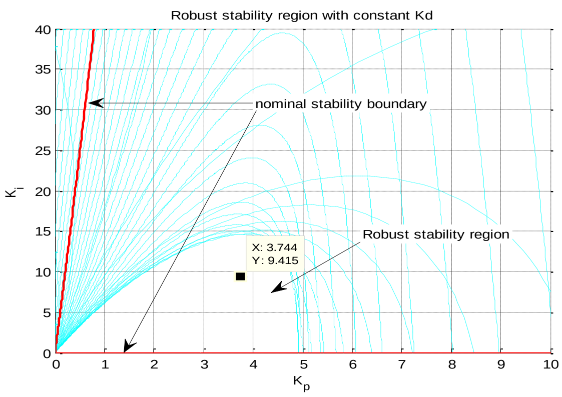

Figure 3: Nominal stability boundary and robust stability region in (K_p, K_i) plane with constant K_d

Now, putting (18) in equation (4), we get $\|W_T(j\omega)T(j\omega)\|_\infty = 0.8472$ which is less than 1, as shown in figure 5, hence, satisfying the robust stability constraint.

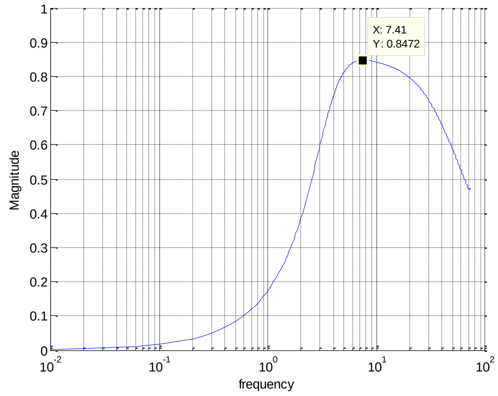

Figure 4: Magnitude of $W_T(j\omega)T(j\omega)$ for $G_c(s) = \frac{0.2s^2 + 3.744s + 9.41}{s}$

B. For t=0.05

The multiplicative weight transfer function obtained is-

$$W_T(s) = \frac{1.86s}{0.67s + 18} \tag{19}$$

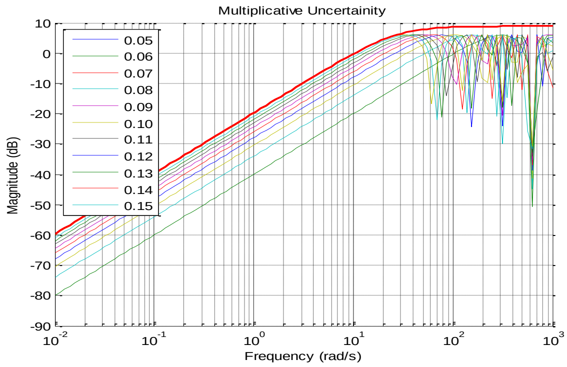

Figure 5: multiplicative weight design to bound the errors for different time delays

Figure 6 shows the robust stability region and nominal stability boundary is defined for the chosen transfer function with time delay, t=0.05.
Choosing a random point in robust stability region, we get

$$G_c(s) = \frac{0.2s^2 + 3.744s + 9.41}{s} \tag{20}$$

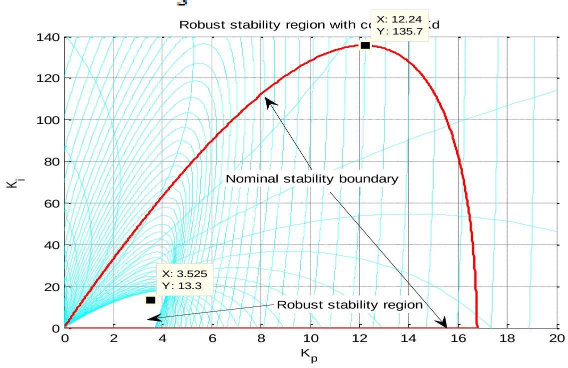

Figure 6: Nominal stability boundary and robust stability region in (K_p, K_i) plane with constant K_d

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Now, putting (20) in equation (4), we get $\|W_T(j\omega)T(j\omega)\|_\infty = 0.9079$ which is less than 1, hence, satisfying the robust stability constraint.
Comparing figure 6 with figure 3, it is observed that set of robust PID controller decreases due to the presence of time delay, while nominal boundary range increases.

C. For t=0.1

The multiplicative weight transfer function obtained is-

$$W_T(s) = \frac{s}{0.432s + 14} \tag{21}$$

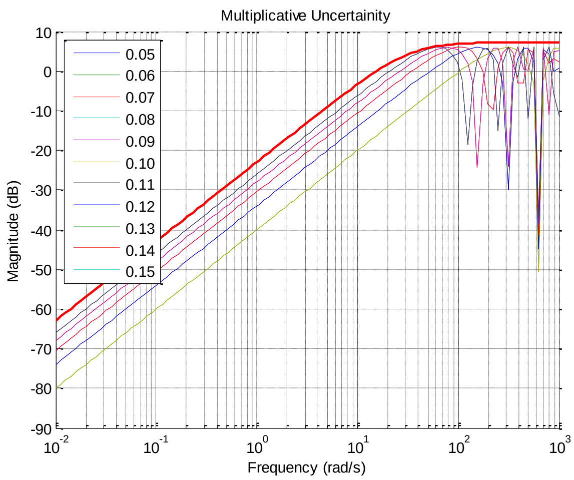

Figure 7: multiplicative weight design to bound the errors for different time delays

Figure 8 shows the nominal stability boundary and robust stability region.

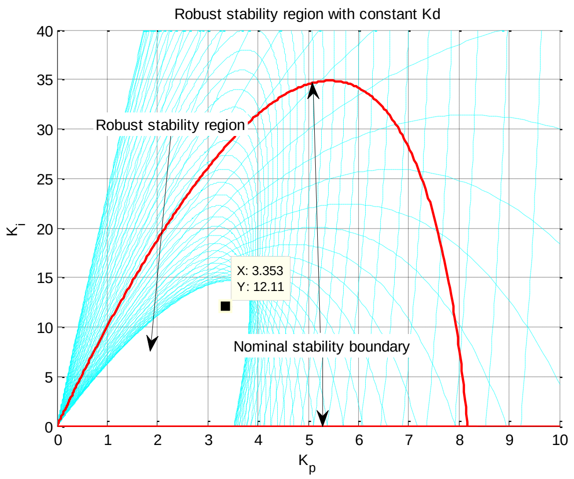

Figure 8: Nominal stability boundary and robust stability region in (K_p, K_i) plane with constant K_d

Figure 8 shows the robust stability region and nominal stability boundary is defined for the chosen transfer function with time delay, t=0.1.
Choosing a random point in robust stability region, we get

$$G_c(s) = \frac{0.2s^2 + 3.353s + 12.11}{s} \tag{22}$$

Now, putting (22) in equation (4), we get $\|W_T(j\omega)T(j\omega)\|_\infty = 0.596$ which is less than 1, satisfying the robust stability constraint.

Comparing figure 8 with figure 6, it is observed that range of nominal stability boundary has decreased significantly with a little effect on robust stability region.

(20)

D. For t=0.2

The multiplicative weight transfer function obtained is-

$$W_T(s) = \frac{2.4s}{0.969s + 14} \tag{23}$$

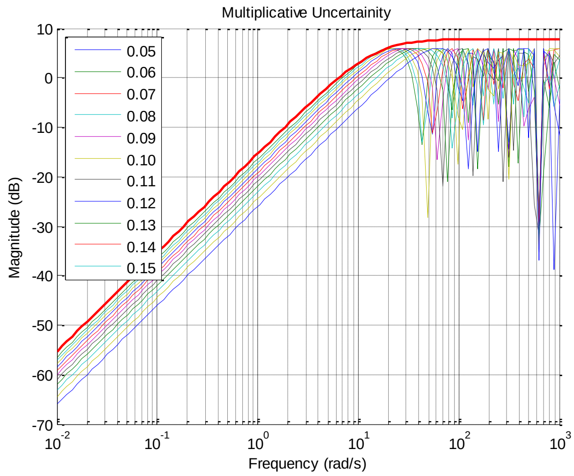

Figure 9: multiplicative weight design to bound the errors for different time delays

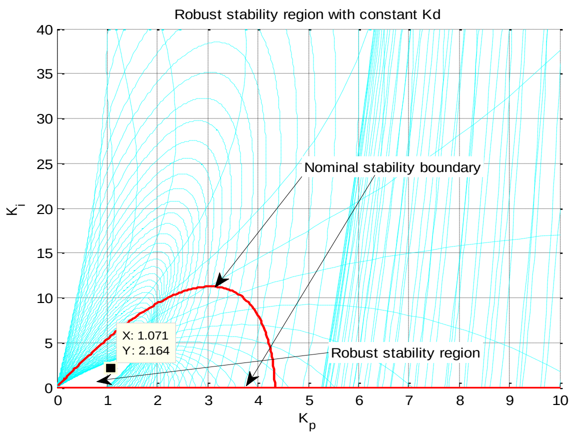

Figure 10: Nominal stability boundary and robust stability region in (K_p, K_i) plane with constant K_d

Figure 10 shows the robust stability region and nominal stability boundary is defined for the chosen transfer function with time delay, t=0.2.
Choosing a random point in robust stability region, we get

$$G_c(s) = \frac{0.2s^2 + 1.071s + 2.164}{s} \tag{24}$$

Now, putting (24) in equation (4), we get $\|W_T(j\omega)T(j\omega)\|_\infty = 0.9233$ which is less than 1, satisfying the robust stability constraint.
Comparing figure 10 with figure 8, it is observed that range of nominal stability boundary as well as robust stability region has decreased significantly.

E. For t=0.3

The multiplicative weight transfer function obtained is-

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$$W_T(s) = \frac{4.97s}{1.96s + 17} \quad (25)$$

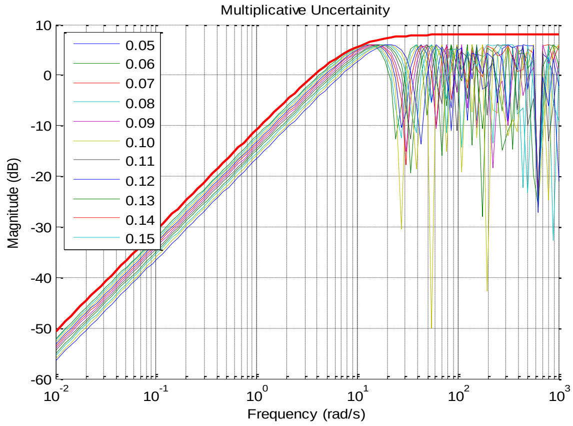

Figure 11: multiplicative weight design to bound the errors for different time delays

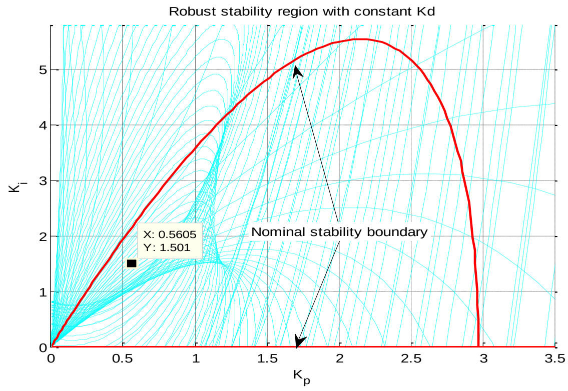

Figure 12: Nominal stability boundary and robust stability region in (K_p, K_i) plane with constant K_d

From figure 12, it is observed that the robust stability region has been almost diminished and range of nominal stability boundary is further decreased.

The above results can be compared and summarized as-

Time delay provided	Robust stability max. (K_p, K_i) range(approx.)	Nominal stability max. (K_p, K_i) range(approx.)
no time delay	(4,15)	Not defined
0.05	(4,18)	(12,135)
0.1	(3.5,14)	(5,35)
0.2	(1.3,3)	(3,12)
0.3	Not defined	(2.2,5.5)

IV. CONCLUSION

Following this graphical method for different time delays in nominal model, we observe that the range of robust stability region as well as nominal stability boundary decreases with increase in time delay. Hence, we can conclude that including the time delay in the nominal model, the conservativeness of the set of PID controllers can be reduced.

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AUTHOR PROFILE

Mukul Gaur is currently pursuing his M.E. in Electrical Engineering with the specialization in control systems from P.E.C. University of Technology (Formerly Punjab Engineering College), Chandigarh. He did complete his B.Tech degree from Giani Zail Singh college of engineering and technology, Bathinda, affiliated to Punjab Technical university, Jalandhar, in year 2010. He has two research papers published in International conferences and one in national conference. His special fields of interest are Robust Control systems, Renewable Energy systems and Instrumentation

Sachin Goyal currently pursuing his M.E. in Electrical Engineering with the specialization in control systems from P.E.C. University of Technology (Formerly Punjab Engineering College), Chandigarh. He received his B.Tech degree in Electrical and Electronics Engineering from GLA institute of technology and management, Mathura affiliated to UP Technical university, Lucknow in year 2008. He has two research papers published in International conferences and one in national conference. His special fields of interest are Control systems, Automation, Renewable Energy systems and Fuzzy logics.

Prof. Sulata Bhandari was born in India, on Apr. 26, 1964. She is an associate professor in Deptt. of Electrical Engg. PEC University, Chandigarh. She did her B.E. from Punjab Engineering College, Chandigarh in year 1985 and completed her M.E. in control system from Regional Engineering College (currently known as NIT Kurukshetra) in the year 1988. She is currently pursuing her Ph.D from PEC university of Technology, Chandigarh. She had a vast experience of teaching engineering graduates for more than 26 years and had guided a number of M.E. dissertations. She had published many research papers in national/international conferences. Her fields of Interest are measurement and Instrumentation, Non-conventional energy sources and Bio-Medical engineering. She is a member of IEEE and ITSE.

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