

# A New Approach to Analyze the Flow over Sharp Crested Curved Plan form Weirs

S. Kumar, Z. Ahmad, T. Mansoor, S. K. Himanshu

**Abstract-** Weirs have widely been used for the flow measurement, flow diversion and its control in the open channels. Generally they are used as normal weirs of various shapes like rectangular, triangular, trapezoidal, etc. pose problems of submergence upstream of the weir due to large afflux required to pass the discharge downstream. Several weirs of modified plan form like oblique weirs, diagonal weirs, Duckwill weirs, Labyrinth weirs etc. have successfully been used to control the afflux and to enhance their discharging capacity with minimum head over the weirs. Labyrinth weirs are folded in the plan view (i.e. the weir crest is not straight in plan form) to provide a longer crest length compared to a normal weir having the same lateral space to increase the discharge for a given operating head. For large reservoirs, the labyrinth weir is used as the overflow structure. It allows the overflow sill to be raised for the same maximum level of the water and flood, and thus, increase the storage capacity of the reservoir. This paper contains the results of experimental study carried out to analyse the flow characteristics of a sharp-crested curved plan-form weirs under free flow conditions in a rectangular channel with weir height around 0.10 m and vertex angles ranging from  $0^\circ$  (i.e. normal weir) to  $120^\circ$ . The equations for the discharge coefficient of curved weirs have been proposed. The results show that there is a gain of about 50% in discharge over a curved weir with vertex angle of  $120^\circ$  as compared to a normal weir. Finally, equations for discharge coefficient for all tested curved weirs in a free flow situation have been proposed.

**Key Words:** Sharp crested weir, Curved plan-form, Flow measurement, Coefficient of discharge, Open channel.

## I. INTRODUCTION

A weir is built across a river (or stream) in order to raise level of water on the upstream side and to allow the excess water to flow over its entire length to the downstream side. Sharp crested weirs are widely used for the purpose of flow measurement, flow diversion and water level control in hydraulics, irrigation, and environmental projects. Rectangular, triangular, cipolletti, circular and sutro are some of the important shapes of sharp crested weirs. All these weirs are generally aligned as normal weirs in which the direction of flow or sides of the straight channel is perpendicular to the length of weir (or axis of weir). Based on alignment, weirs can be classified as a normal weir in which weir axis is perpendicular to channel axis [1], a side weir in which weir axis parallel to channel axis [2] and a skew weir in which weir axis inclined to channel axis [3].

As projects are reassessed for safety, provision for an increased estimate of the probable maximum flood (PMF) has to be made in many cases. It is therefore necessary to provide more flood storage and/or larger capacity for spillways to pass the PMF safely. If the dam cannot adequately pass the updated flood, the structure requires modification by increasing the flood storage space, increasing the spillway capacity or using combinations of these two solutions.

An innovative and effective way of increasing the spillway capacity is to use a Labyrinth weir. The concept of the Labyrinth weir is to vary the plan shape of the crest to increase the effective crest length. This increases the discharge per unit width of the spillway for a given operating head. A curved weir in the form of a circular arch embedded between channel walls also offers more crest length for the passage of flow.

Flow over a sharp-crested weir ( $Q$ ) under free flow condition in a channel is expressed in terms of the following mathematical expression

$$Q = \frac{2}{3} C_d \sqrt{2g} L h^{\frac{3}{2}} \quad (1)$$

Where  $C_d$  = coefficient of discharge,  $L$  = crest length of the weir,  $g$  = acceleration due to gravity,  $h$  = head over the crest. The  $C_d$  depends on flow characteristics and geometry of the channel and the weir [4].

The experiments were conducted on skew weirs and an equation was proposed for the elementary discharge coefficient for sharp crested skew weirs over a big range of  $h/w$  from zero to infinity [3]. Using this equation, discharge over labyrinth weir tested by Hay and Taylor was computed and found satisfactory [5].

Tests were performed on various plan shapes in the form of labyrinth weirs and presented the results in the form of curves between the ratio of discharge over labyrinth weir to corresponding normal weir and  $h/w$  [5]. Tullis *et al.* [6] studied labyrinth weirs having trapezoidal plan form. They found that the coefficient of discharge depends on weir height, total head, weir wall thickness, crest shape, vertex configuration and the angle of the side legs. The Experiments were conducted on three submerged labyrinth weirs of different geometries with half-round crest shapes [7]. They described the submerged labyrinth weir head-discharge relationship using the dimensionless submerged head parameters and found that the relationship is independent of labyrinth weir sidewall angles.

Aeration performances of different plan forms of labyrinth weirs have been studied by various investigators [8], [9], [10]. They found that the geometry of labyrinth weirs provides increased sill length and often results in the over fall jets colliding with each other, both of which may lead to increased aeration.

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They found that the aeration efficiency of the labyrinth weirs is generally better than the equivalent-length normal weir and it increases as the weir included angle becomes smaller and also at lower over fall drop heights and higher discharges. The hydraulics of labyrinth weir using soft computing techniques was also studied in detail [11], [12].

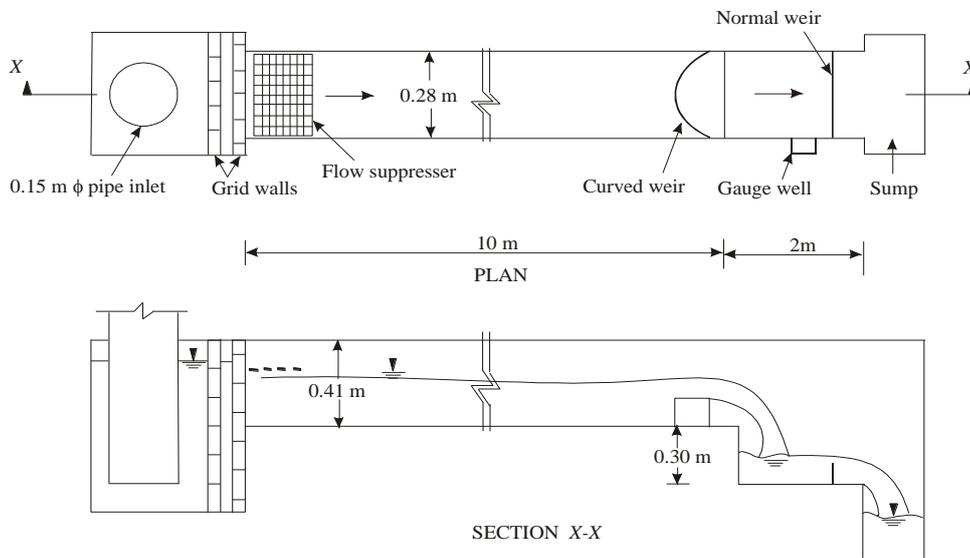
The experimental study was conducted on sharp-crested weirs of triangular plan form under free flow conditions in a rectangular channel [13]. It was found that the discharging efficiency of the triangular plan form weirs is more compared to the normal weir. It was also high for low vertex angle and low head over the crest of the weir. The sensitivity analysis, i.e., change of discharge due to unit change in head of the weir was also carried out and found that the weir is more sensitive at the low head and low vertex angle.

The following equation for  $C_d$  was proposed by conducting experiments over broad crested curved weirs [14]:

$$C_d = \left( 0.5 + 0.33 \frac{h}{w} + \frac{h}{L} \right)^{0.06} \quad (2)$$

Presented in this paper are results of the experimental study carried out to investigate the discharging capacity of a sharp-crested weir of curved plan form under free flow condition in a rectangular channel.

of experimental setup are shown in Fig. 1. Geometrically, the curved plan form represents an arc of a circle. Various curved plan form weirs of varying radii and vertex angles were fabricated of mild steel plates. The curved plan form reduces to a normal weir as the vertex angle approaches to zero and half circular plan form for vertex angle  $180^\circ$ . The weir was located at 9.30 m downstream from the head of the channel. Water was supplied to the channel through an inlet pipe from an overhead tank provided with an overflow arrangement to maintain the constant head. Discharge over these weirs was measured using a normal sharp crested weir fitted at a distance of 1.95 m from the curved weirs. The downstream bed was kept 0.30 m below the upstream bed in order to obtain sufficient range of  $h/w$  and to avoid submergence. The discharge is controlled by a valve. The water from the channel is collected in an underground well having a diameter of 3.65m. Depths were measured with a point gauge (0.1 mm accuracy) near the centre of the channel at 0.40 m upstream of the various weirs to avoid the curvature effect of water surface.



**Fig. -1 Layout of the experimental set-up**

**Table – I: Range of collected data**

S. no.	$\theta$ (deg.)	$w$ (m)	$h$ (m)	$Q$ ( $m^3/s$ )	No. of runs
1	00	0.1000	0.0318 - 0.0919	0.0030-0.0144	19
2	45	0.1029	0.0190 - 0.0841	0.0016-0.0140	18
3	60	0.1050	0.0157 - 0.0769	0.0014-0.0123	20
4	75	0.1031	0.0128 - 0.0781	0.0011-0.0129	23
5	90	0.1042	0.0181 - 0.0729	0.0019-0.0115	20
6	105	0.1087	0.0167 - 0.0749	0.0017-0.0125	23
7	120	0.1084	0.0204 - 0.0784	0.0022-0.0135	25

## II. EXPERIMENTAL WORK

The experiments were conducted in a horizontal, rectangular, prismatic channel plastered with cement having a width of 0.28 m and depth of 0.41 m. The schematic views

Head over the measuring weir was recorded in a well provided by the side of channel at the upstream of measuring weir. Ventilation holes were provided for the aeration of the nappe. Water was supplied to the channel through an inflow pipe from laboratory overhead tank provided with an overflow arrangement to maintain the constant head.

The experiments were performed for weirs of curved plan form of vertex angle  $\theta = 0^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ, 105^\circ$  and  $120^\circ$  and for varying discharges.

For each run, the head over the crest of the weir at about 4–5 times the head over the crest upstream of the weir was measured using point gauge. Discharge in the channel flowing over the weir was measured volumetrically in a sump provided at the tail of the channel. The ranges of the data collected in the present study are given in Table - I.

### III. ANALYSIS OF DATA

Data collected in the present study have been analyzed to obtain the functional relationship for coefficient of discharge for all tested curved weirs under free flow conditions in the form of Rehbock's (1929) equation, i.e.

$$C_d = a + b \left( \frac{h}{w} \right) \quad (3)$$

Where  $w$  is the weir height and  $a$  and  $b$  are coefficients to develop a discharge equation for the sharp-crested curved weir.

Variation of discharge with head over the crest for the curved weirs of different vertex angles is shown in Fig. 2. Fig. 2 clearly indicates that for the same value of  $h$ , discharge increases with the increase of vertex angle due to increases of the crest length of the weir.

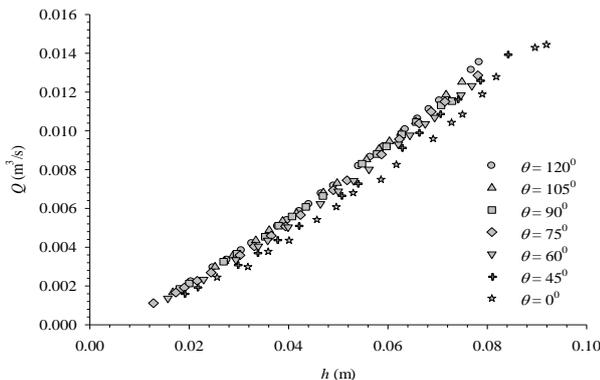


Fig. - 2 Variation of  $Q$  with  $h$  for weirs of different vertex angles.

For each data set, the  $C_d$  was computed using (1) for known value of discharge, head over the crest and the crest length for weirs of different vertex angles. Variation of  $C_d$  with  $h/w$  is shown in Fig. 3 for the weirs of different vertex angles. It can be noted that  $C_d$  decreases with an increase of vertex angle due to interference of the falling jets for high value of  $h/w$ .

However, for the low value of  $h/w$ , the interference of jets is not so severe, resulting in high  $C_d$ . The rate of decrease of  $C_d$  with  $h/w$  decreases with increase of vertex angle.

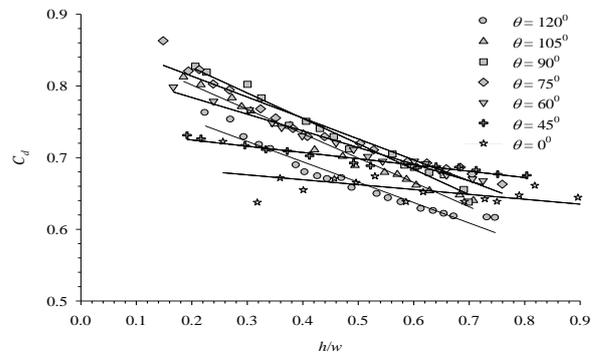


Fig. - 3 Variation of  $C_d$  with  $h/w$  for weirs of different vertex angles.

#### A. Discharge equation for curved plan-form weir

Invoking the Rehbock equation, the variation of  $C_d$  with  $h/w$  is fitted to linear equations for weirs of different vertex angles as follows:

$$C_d = 0.808 - 0.284 \left( \frac{h}{w} \right), \text{ for } \theta = 120^\circ, R^2 = 0.941 \quad (4a)$$

$$C_d = 0.869 - 0.339 \left( \frac{h}{w} \right), \text{ for } \theta = 105^\circ, R^2 = 0.983 \quad (4b)$$

$$C_d = 0.896 - 0.354 \left( \frac{h}{w} \right), \text{ for } \theta = 90^\circ, R^2 = 0.978 \quad (4c)$$

$$C_d = 0.872 - 0.292 \left( \frac{h}{w} \right), \text{ for } \theta = 75^\circ, R^2 = 0.940 \quad (4d)$$

$$C_d = 0.830 - 0.230 \left( \frac{h}{w} \right), \text{ for } \theta = 60^\circ, R^2 = 0.980 \quad (4e)$$

$$C_d = 0.742 - 0.087 \left( \frac{h}{w} \right), \text{ for } \theta = 45^\circ, R^2 = 0.949 \quad (4f)$$

A high correlation between  $C_d$  and  $h/w$  may be noted for all the weirs. Variations of constants ' $a$ ' and ' $b$ ' with vertex angle are shown in Fig. - 4. A second order polynomial has been fitted to the data for ' $a$ ' and ' $b$ ' as follows (angles are in radian):

$$a = (-0.074\theta^2 + 0.245\theta + 0.656), R^2 = 0.806 \quad (5a)$$

$$b = (0.049\theta^2 - 0.257\theta - 0.010), R^2 = 0.811 \quad (5b)$$

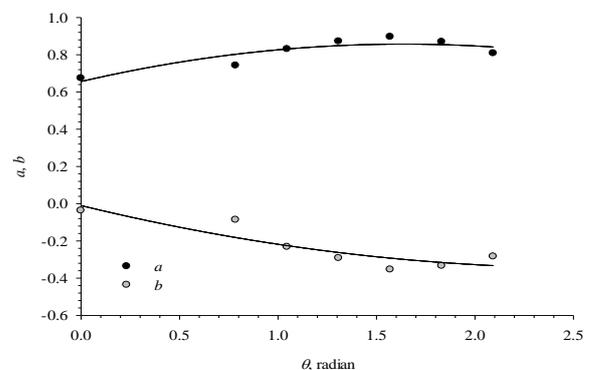


Fig. - 4 Variation of ' $a$ ' and ' $b$ ' with  $\theta$

## A New Approach to Analyze the Flow over Sharp Crested Curved Plan form Weirs

Out of 148 data sets for curved plan form weirs, 117 data sets were used to develop the relationship for coefficient of discharge. The generalized equation of  $C_d$  for the curved weir to be used in (1) for the computation of discharge can be written as:

$$C_d = \left( -0.074\theta^2 + 0.245\theta + 0.656 \right) + \left( 0.049\theta^2 - 0.257\theta - 0.010 \right) \left( \frac{h}{w} \right) \quad (6)$$

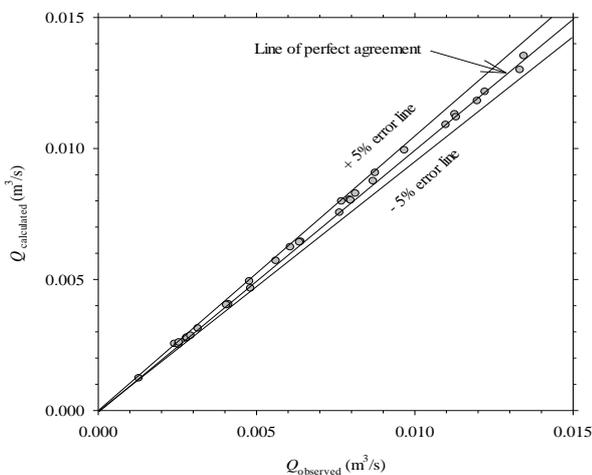
This equation is valid in the range  $0 < h/w < 0.90$  and  $0^\circ \leq \theta \leq 120^\circ$ .

### B. Validation of the proposed discharge equation

The remaining 31 data sets, not used in the derivation of (6), were used next to validate the proposed relationship for  $C_d$  i.e., (6). The computed discharge is compared with the corresponding observed ones in Fig. - 5, which shows that the computed discharge is within  $\pm 5\%$  of the observed ones for the weirs of all vertex angles studied herein. For a numerical measure for error between the observed and computed values, an average percentage error term  $e$  is defined as [15]:

$$e = \frac{100}{N} \sum_{i=1}^N \left[ \frac{Q_{computed} - Q_{observed}}{Q_{observed}} \right] \quad (7)$$

The average percentage error in the computation of discharge over the weir using (1) and (6) is found in the range 0%–5% for weirs of different vertex angles.

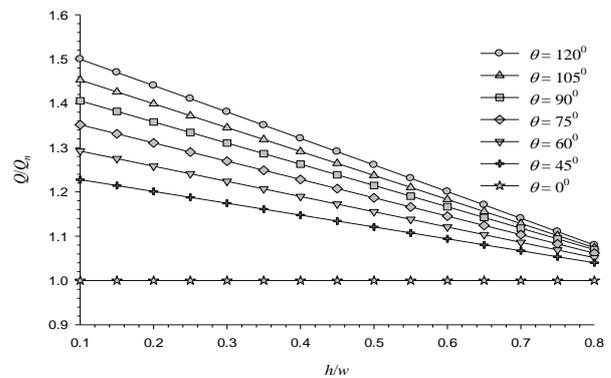


**Fig. - 5 Comparison of computed discharge using (1) and (6) with the observed ones.**

### C. Efficiency of the curved weir

An efficient weir passes more discharge for constant head and length of weir compared to the other. To examine the efficiency of the curved weir for different vertex angles, ratio of discharges over the curved weir and normal weir, i.e.,  $Q/Q_n$  is plotted with  $h/w$  in Fig. - 6.

The efficiency of curved weir is high for high vertex angle and decreases with increase of  $h/w$  due to interference of the jets downstream.



**Fig. - 6 Variation of  $Q/Q_n$  with  $h/w$  for the weirs of different vertex angles**

For  $h/w = 0.1$ , the weirs of vertex angle  $120^\circ$ ,  $105^\circ$ ,  $90^\circ$ ,  $75^\circ$ ,  $60^\circ$  and  $45^\circ$  are respectively 1.50, 1.45, 1.40, 1.35, 1.29 and 1.23 times more efficient than the normal weir. However, for  $h/w = 0.8$ , the efficiency of curved weir is low and even for  $\theta = 120^\circ$ , the efficiency is only 1.08 times the normal weir.

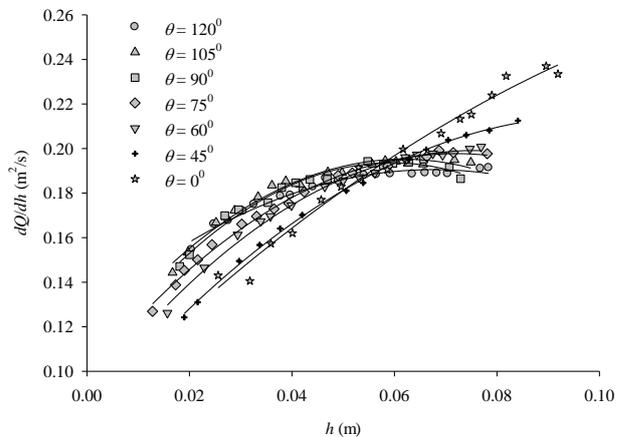
### D. Sensitivity analysis of the curved weir

Sensitivity analysis, i.e., change in discharge due to unit change in the head of water is carried out for the proposed discharge equation of the curved weir, which can be written as [16]:

$$Q = \frac{2}{3} \left( a + b \frac{h}{w} \right) \sqrt{2g} \frac{B}{\sin\left(\frac{\theta}{2}\right)} \left( \frac{\pi\theta}{360} \right) h^2 \quad (8)$$

The values of 'a' and 'b' can be obtained from (5a) and (5b) respectively. Differentiating  $Q$  with respect to  $h$  and arranging the terms, one can get

$$\frac{dQ}{dh} = \frac{3}{2} \frac{Q}{h} + \frac{Qb}{w \left( a + b \frac{h}{w} \right)} \quad (9)$$



**Fig. - 7 Sensitivity of the curved weir as function of head**

The larger value of  $dQ/dh$  implies higher sensitivity. Data collected in the present study was used to compute  $dQ/dh$  for different values of the central angles.

The variation of  $dQ/dh$  with  $h$  is depicted in the Fig. 7. A perusal of Fig. 7 reveals that the discharge through curved weir is more sensitive to the low head. As the head increases, the sensitivity decreases due to interference of the water jet downstream of the weir crest. Further, sensitivity is higher for the high central angle of the curved weir due to large weir crest length.

#### IV. RESULTS AND DISCUSSION

For the curved weirs an increase in discharge has been observed as compared with the normal weir operating under identical condition. It is evident from Fig. - 6 that for  $h/w = 0.1$ ,  $Q/Q_n$  attains a maxima at 50% more than normal weir for the last tested angle i.e.  $\theta = 120^\circ$ .

#### V. CONCLUSIONS

These weirs can be used easily and precisely as a discharge metering device. Generalized equation for  $C_d$  for curved weir has been obtained, which can be used to find  $C_d$  for any value of  $\theta$  in the corresponding specified range and hence the discharge. Larger flow area on crest is available for the same head relative to the conventional normal weir and this reduces afflux on the upstream of the weir. Since the discharging capacity of these weirs is more, the requirement of free board in channels gets reduced and hence sections can be designed more economically. Also with its simple geometric shape it is easy to design and fabricate high discharging weirs, even in existing channels. Curved weir with vertex angle  $\theta = 90^\circ$  is found to be the optimal plan shape. In light of the above discussion, it is concluded that by using curved weir, we get significant increase in the discharge with lesser complications in design and placement.

#### NOTATIONS

- $B$  = Channel width
- $C_d$  = Coefficient of Discharge for V- shaped Weir
- $C_{dn}$  = discharge coefficient of normal weir
- $g$  = Acceleration due to gravity
- $h$  = head over the weir
- $H$  = Total head Upstream of the weir
- $l$  = Effective crest length
- $L$  = Channel length
- $Q$  = Discharge over curved Weir
- $Q_o$  = Observed Discharge
- $Q_c$  = Computed Discharge
- $Q_n$  = Discharge over corresponding normal weir
- $Q_s$  = Discharge over skew weir
- $w$  = weir height above the bed of channel
- $Y$  = Channel depth
- $\theta$  = Vertex angle
- $a, b$  = coefficients

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