Modified Disc Model for Over-Speed Burst Margin with Thermal Load and Disc Speed Corrections and Compared with FE Model

Maruthi B H, K M Narayanappa, M Krishna,Venkatarama Reddy

Abstract : The present work was focused on modification of the disc model for over speed burst margin with thermal load and disc speed correction and verify the same with FE model. Hoop stress, radial stress and burst margin were carried out at different speed and thermal loading conditions using both finite element and mathematical model. Investigations are performed based on non-linear problem employing linear analysis tool ANSYS 12.0. A non-linear finite element method was utilized to determine the stress state of the disc / blade segment under operating conditions. In both cases (FE and mathematical model) the numerical burst rotation rate, associated with the loss of stability of the structure, is found to be in good agreement with the each other.

Keywords: Modified Model, Gas Turbine, Over-speed, thermal load.

I. INTRODUCTION

Theoretical investigation of the stresses in solid and annular disks rotating at high speeds has been receiving widespread attention due to a large number of applications in engineering such as turbine disc, flywheel etc. [1-3]. Gas turbine discs work in the hot section (650°C) of the engine and it is one of the most expensive parts [4]. All these conditions induce stress over 100 MPa in rim feature. Therefore, mechanical design of disks involves the evaluation of centrifugal and thermal stresses [5]. Non-linearlities originate from (i) the material behavior in different temperature, (ii) geometrical changes accounted for by the strain formulation.These results have been used in Ahmet [6], Mohammad et al.[7] to suggest that burst occurs when the mean hoop stress on a disk section becomes equal to the nominal tensile strength of the material, determined from an uniaxial tensile stress. This criterion shows good correlation with experimental results for rotating rings, but for solid or bored disks NIe & Batra [8] wonders if this criterion gives a precise estimate of the maximum angular velocity [9], and Manavi recommends the use of a burst factor between both aforementioned stresses[10]. The certification of gas turbine engine requires demonstrating the integrity of disks at higher rotation speeds than the maximum rotation speed reachable in service. The determination of the burst speed by analysis can help to reduce the number of tests required for the certification. This prediction can be established by non-linear stability analyses by using analytical and finite element simulations. In this work, loss of uniqueness and loss of stability criteria from[11]. A general theory of uniqueness and stability in elastic-plastic solids are applied[12]. The loss of stability criterion is restricted to the case of rotating disks and compared to several simple widely used material based failure criteria. The main objective of the present study was to evaluate hoop stress, radial stress and burst margin of a gas turbine disc under different speeds in thermal conditions environment using both mathematical and finite element model (ANSYS).

II. ANALYTICAL DEVELOPMENT

A. Consideration of centrifugal reaction

Stresses produced in a disk rotating at high speed are important in many practical applications. Among which the design of disk wheel in gas turbines and steam turbines are considered. The stresses due to tangential forces being transmitted are usually small and large stresses are due to centrifugal forces of the rotating disk.

Let the centrifugal force in radial direction is

$$R_r = \rho\omega^2 r, \quad R_\theta = 0 \qquad (1)$$

We know that, the equilibrium equation for rotating disk is

$$\frac{\partial\sigma_r}{\partial r} + \frac{1}{r}(\sigma_r - \sigma_\theta) + R_r = 0 \qquad (2)$$

Where σ_r = Radial stress σ_θ = Tangential Stress

Substituting the centrifugal force to above equation, we get

$$\frac{\partial\sigma_r}{\partial r} + \frac{1}{r}(\sigma_r - \sigma_\theta) + \rho\omega^2 r = 0 \qquad (3)$$

$$\frac{\partial(r\sigma_r)}{dr} - \sigma_\theta + \rho\omega^2 r^2 = 0 \qquad (4)$$

$$\sigma_\theta = \frac{d}{dr}(r\sigma_r) + \rho\omega^2 r^2 \qquad (5)$$

The strain components are given by

$$\varepsilon_r = \frac{du}{dr} \quad and \ \varepsilon_\theta = \frac{u}{r} \qquad (6)$$

$U = r\,\varepsilon_\theta$

$$\frac{du}{dr} = \varepsilon_r = \frac{d}{dr}(r\varepsilon_\theta)$$

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For Hooke's Law with $\sigma_z = 0$

$$\varepsilon_r = \frac{1}{E}(\sigma_r - \mu\sigma_\theta) \quad (7)$$

$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \mu\sigma_r) \quad (8)$$

Then

$$\frac{1}{E}(\sigma_r - \mu\sigma_\theta) = \frac{1}{E}\left[\frac{d}{dr}(r\sigma_\theta - \mu r\sigma_r)\right] \quad (9)$$

Put y = $r\sigma_r$ in equation (5) and substitute in equation (9)

$$r^2\frac{d^2y}{dr^2} + r\frac{dy}{dr} - y + (3+\mu) + \rho\omega^2 r^3) = 0 \quad (10)$$

The solution of the above differential equation is

$$y = Cr + \frac{C_1}{r} - \left(\frac{3+\mu)}{8}\right)\rho\omega^2 r^3 \quad (11)$$

From y = r σ_r and (5) get

$$\sigma_\theta = C + \left(\frac{C_1}{r^2}\right) - \left(\frac{3+\mu}{8}\right)\rho\omega^2 r^2 \quad (12)$$

$$\sigma_r = C - \left(\frac{C_1}{r^2}\right) - \left(\frac{3+\mu}{8}\right)\rho\omega^2 r^2 \quad (13)$$

The constants of integration are determined from the boundary condition.

Consider the hallow disk with central hole of inner radius 'a' and outer radius 'b'.

Applying initial boundary condition to eq. (12) and eq. (13) i.e.

At r = a, σ_r = 0, and

r = b, σ_r = 0

We can find the C and C_1.

$$C = \left(\frac{3+\mu}{8}\right)\rho\omega^2(b^2 + a^2) \quad (14)$$

$$C_1 = -\left(\frac{3+\mu}{8}\right)\rho\omega^2 b^2 \quad (15)$$

Substitute the above constant is eq.(12) and eq. (13), we get The Radial Stress for the hollow disk is

$$\sigma_r = \left(\frac{3+\mu}{8}\right)\rho\omega^2\left(b^2 + a^2 - \left(\frac{a^2b^2}{r^2}\right) - r^2\right) \quad (16)$$

The Tangential stress for hollow disk is

$$\sigma_\theta = \left(\frac{3+\mu}{8}\right)\rho\omega^2\left(b^2 + a^2 + \left(\frac{a^2b^2}{r^2}\right) - \left(\frac{1+3\mu}{3+\mu}\right)r^2\right)$$
(17)

The maximum radial stress at $r = \sqrt{ab}$ is

$$(\sigma_r)_{max} = \left(\frac{3+\mu}{8}\right)\rho\omega^2(b-a)^2 \quad (18)$$

Maximum tangential stress at r = a i.e. $(\sigma_\theta)_{max}$ is

$$(\sigma_\theta)_{max} = \left(\frac{3+\mu}{4}\right)\rho\omega^2\left(b^2 + \left(\frac{1-\mu}{3+\mu}\right)a^2\right) \quad 19)$$

The displacement u, for all the cases considered can be calculated as below

$$u_r = r\varepsilon_\theta = \frac{r}{E}(\sigma_\theta - \mu\sigma_r) \quad (20)$$

The equations (18), (19) and (20) give maximum radial stress, maximum tangential stress and deformation of the disc for lower speeds.

The turbine disc is working both high temperature and high speed conditions; hence this model needs to modify for both thermal and speed corrections.

B. Thermal Correction

Turbine disc works at different thermal environment form room temperature to its working temperature. There is no scope thermal stress in the disc due to free expansion of turbine disc. But the thermal loading indirectly influence on induced stresses. The values of Young's modulus, CTE and dimension of disc change with changing temperature. Only change of Young's modulus can be included in the equation 20.

The A is area of disc at room temperature, A' changed area at working temperature. Due to temperature variation only changes the outer radius but not inner radius because the inner radius is firmly fixed to rotating shafts. The value of 'b' changes to 'b'' and 'a' remains same at any temperature and even speeds.

$$A' = A + A\,\alpha\,\Delta t = A(1 + \alpha\,\Delta t) \quad (21)$$

$$\pi(b'^2 - a^2) = \pi(b^2 - a^2)(1 + \alpha\Delta t) \quad (22)$$

$$(b'^2 - a^2) = (b^2(1 + \alpha\Delta t) - a^2(1 + \alpha\Delta t)) \quad (23)$$

$$b'^2 = b^2 + \alpha\Delta t\,(b^2 - a^2) \quad (24)$$

$$b' = \sqrt{b^2 + \alpha\Delta t\,(b^2 - a^2)} \quad (25)$$

The modified equation of radial, tangential and deformation of disc are given

$$(\sigma'_r)_{max} = \left(\frac{3+\mu}{8}\right)\rho\omega^2(b' - a)^2 \quad (26)$$

$$(\sigma'_\theta)_{max} = \left(\frac{3+\mu}{4}\right)\rho\omega^2\left(b'^2 + \left(\frac{1-\mu}{3+\mu}\right)a^2\right) \quad (27)$$

The displacement u', for all the cases considered can be calculated as below

$$u'_r = \frac{r}{E'}(\sigma'_\theta - \mu\sigma'_r) \quad (28)$$

Where E' change in Young's moudulus with temperature and also r = b'.

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C. Speed Correction

The deformation (u_r) changes with speed of the disc then the value of b also changes continuously along with radial and tangential stresses.

b” = b’ + u’$_r$

The equation of radial, tangential and deformation of disc are modified with speed correction.

$$(\sigma''_r)_{max} = \left(\frac{3+\mu}{8}\right)\rho\omega^2(b''-a)^2 \quad (29)$$

$$(\sigma''_\theta)_{max} = \left(\frac{3+\mu}{4}\right)\rho\omega^2\left(b''^2+\left(\frac{1-\mu}{3+\mu}\right)a^2\right) \quad (30)$$

$$u''_r = \frac{r}{E}(\sigma''_\theta - \mu\sigma''_r) \quad (31)$$

The above equations are solved by using software.

C. Modification of Burst Margin Equation

The Burst Margin, M_b, may be computed using the equation shown below. This margin is used to evaluate the integrity of a rotating disk (e.g., a burst margin of less than one would result in an unacceptable design). The burst Margie is given

$$M_b = \sqrt{\frac{UTS}{\sigma''_\theta}} \quad (32)$$

$$M_b = \sqrt{\frac{(UTS)4g}{(3+\mu)\rho\omega^2\left(b''^2+\left(\frac{1-\mu}{3+\mu}\right)a^2\right)}} \quad (33)$$

The equation (33) can be predicting the burst speed and which can be evolved by using MS-Excel sheet.

III. FINITE ELEMENT MODEL OF TURBINE DISC

Parametric geometry models of disc were made, using the Ansys program [13]. A typical symmetrical model of rotor disc as shown in Fig. 1 and the same was considered predicting maximum speed before disc burst. The FE model of disc presented in Fig. 2 consists of 12,534 nodes of plane 182 type elements. A rotating hot section component in a turbine engine is in general subjected to a combination of surface loads, centrifugal loads and thermal loads. The centrifugal loads arising from the mass of the rotated disc and blades are usually the most critical loads acting on a turbine disc.

This load was determined through finite element calculation after defining the axis of symmetry, the rotational speed and the disc and blade material density. In this analysis, the operational turbine speed between 10,000 to 22,000 rpm was applied. Computations for the rotational speed range of 10,000–22,000 rpm additionally were performed for analysis of phenomena occurring in the turbine during excessive speed. The blade load was distributed on the nodes (load / number of nodes) in the simplified procedure imposed on disc as shown in Fig. 2(a). The thermal loads at bore temperature of 450 °C and at rim of the disc temperature of 600 °C were considered for analysis. The turbine disc is manufactured out of INC0 718 material. This alloy is a precipitation-hardened nickel-base super alloy with good strength, ductility, and fracture toughness. These properties along with good weldability and formability account for its wide use in aerospace applications. The mechanical properties of the disc material are given in the Table 1.

Table 1. Properties of turbine disc blades

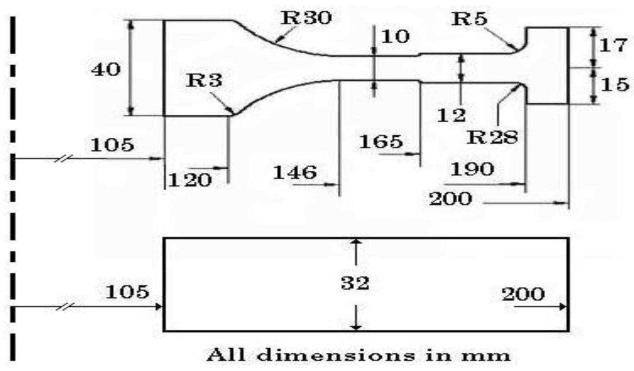

Fig. 1 Cross-section of a typical rotor disc

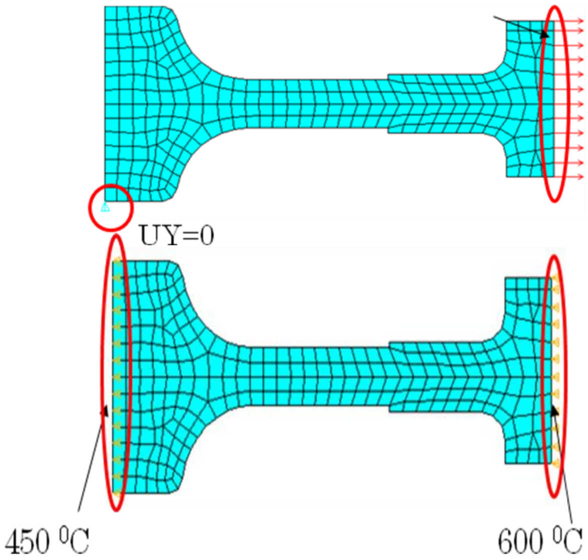

Fig. 2. Finite element mesh for turbine disc with load and thermal constraints

Property	Temperature °C					
	20	100	400	500	600	700
E, 10^9 Pa	209	195	183	178	170	157
CTE, $10^{-6/°C}$	12.2	12.8	13.9	14.0	14.5	15.0

IV. RESULTS AND DISCUSSION

A. Hoop stress prediction by FE

A preliminary hoop stress analysis was performed with the objective of obtaining the critical zones. Stress analysis is also useful to choose and validate elements and meshes. The first set of results was obtained with a solid disc of rectangular section with variable thickness. Non-linear elastic FE analyses were performed for the disc model. Simulation of hoop stress parameters have been calibrated from these curves as shown in Fig. 3.

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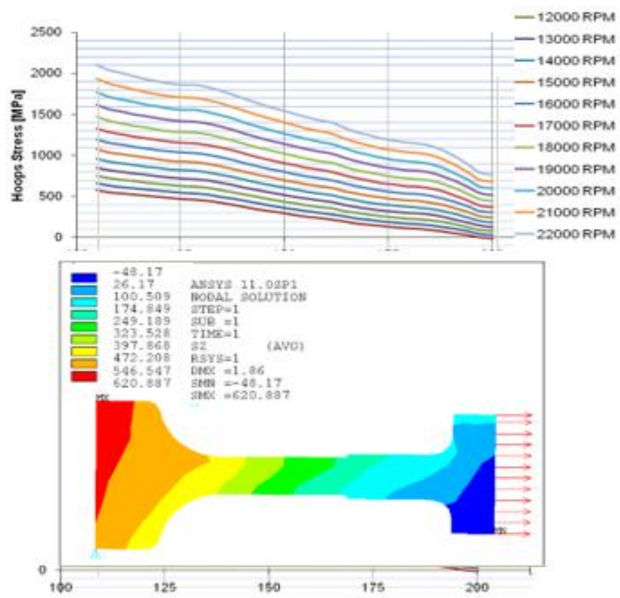

Fig. 3 Hoop stress distribution in the turbine disc as a function of radial distance at different speeds

These analyses simulated the steady state behavior of the disc under service conditions where the centrifugal load, thermal expansion and contract interactions between the disc and tolerances is shown in Fig. 3. For the nominal configuration the hoop stress of the disc occurs at the trailing edge where failure occurs. Many studies were performed to examine the effects of the design tolerances and boundary conditions imposed on the disc. For disc geometry, analytical solutions are available in the literature[14] and hence, a comparison could be made with the numerical data. Although results were found in lightly underestimate but better agreement with experimental curves the global stress level given in [15]. In all cases the maximum stress are seen in upper end of bore and then decreases with radial distance of the disc and minimum at lower end of outer rim. The remaining zones of disc hoop stress distribution between the maximum and minimum values. The graph shows stresses almost decaying linearly from bore to rim although variable thickness of the disc.

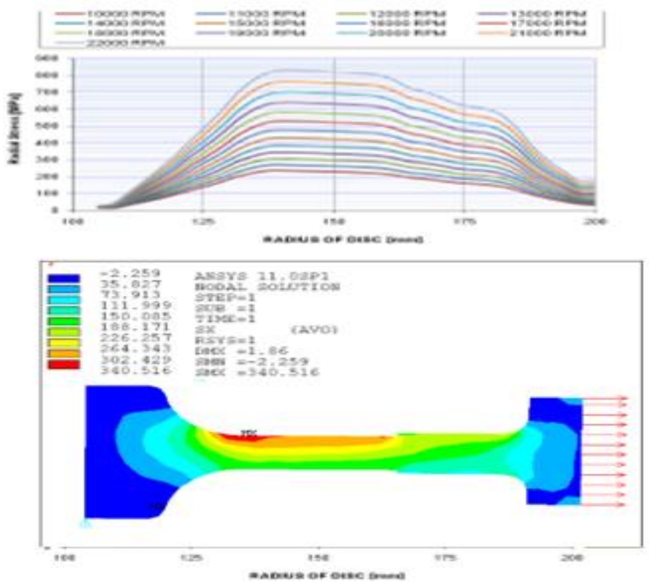

Fig. 4 Radial stress distribution in the turbine disc as a function of radial distance at different speeds

B. Radial stress prediction by FE

The radial simulation and their results are shown in Fig. 4. The overall maximum stress results indicate that the hoop stresses are higher than that of radial stress (more than two times) with the non linear elastic results. The radial stress is almost zero at inner bore and they increase with increasing radial disc distance, attained maximum is at the (R≈135 mm) and decrease. Note that the peak values of the stresses computed in the FE model are very close to the true value of radial stress given by [16].

C. Effect of speed on Maximum hoop and radial stresses

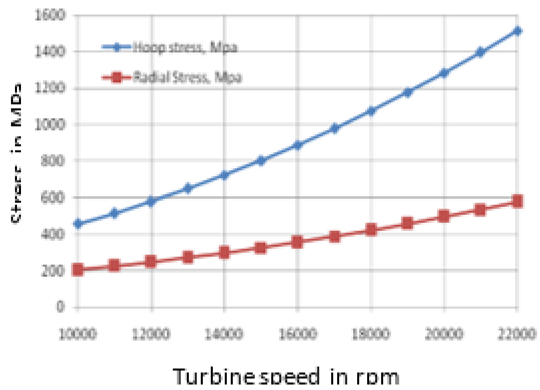

Fig. 5 Effect of turbine disc speed on Induced maximum hoop stress & radial stress

Fig. 5 shows the maximum hoop stress and radial stress are plotted for different operational speeds of 10k to 22k rpm are located on the inner side of the hub and in between turbine disc (R≈135 mm from the inner side of the hub) respectively. The hoop stress as well as radial stress increase with increasing speed of the turbine disc. But the nature of hoop stress increases exponentially but radial stresses increases almost linearly. The maximum radial stresses did not cross the ultimate tensile strength of material at any speeds. But maximum hoop stresses crosses the ultimate tensile strength of the materials of 1108 MPa at nearly 17000 rpm.

D. Effect of speed on deformation of the disc

Fig. 6 shows the predicated deformation as a function of turbine speed. The deformation increases with increasing speed in exponentially. The maximum contributor for this behavior is centrifugal reaction due to the disc blade[17]. The other parameters such as thermal load also has little contribution can be seen [18]. For disk made of incompressible material, radial as well as hoop stresses, is maxiumum at internal surface as compared to disk made of compressible material. The displacement is maximum at external surface for incompressible material. The displacement is maximum at internal surface while hoop stresses and displacement is maxiumum at external surface.

E. Bursting speed computed by Analytical method

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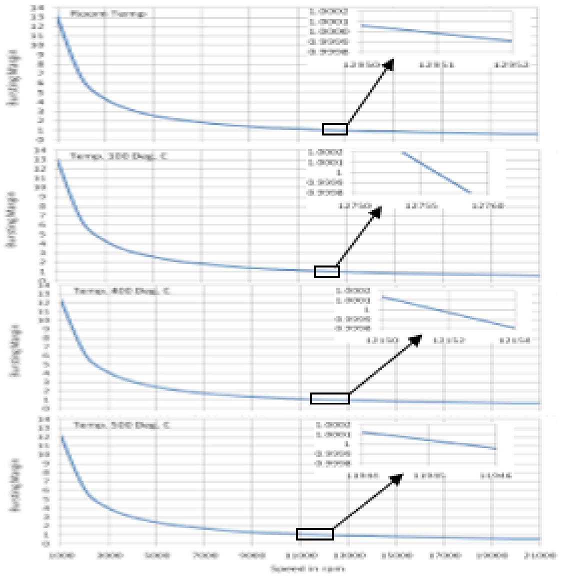

Fig. 7. Predicting bursting margin of turbine disc for various disc speed at different thermal load using modified model.

Fig. 7 shows the plot of burst margin as a function disc speed for different temperature expose to gas turbine disc. The graph shows the burst margin curve logarithmically decrease with turbine speed. The burst margin was computed using the equation (33).

This margin is used to evaluate the integrity of a rotating disc. A burst margin of less than one would result in an unacceptable design. It is shown that the uncertainty in statistical estimates for small samples can cause dramatic variations in predicted failure probability and that sample statistic uncertainty should be accounted for to ensure system reliability. The design intent is to have a 20 percent burst margin over the operating speed to provide a safe margin on hub burst. The requirement for adequate burst margin stems from the safety hazard involved if an uncontained failure occurs. The basic disk average tangential and radial stress must be low enough to provide this margin. The mathematical model predicted the speeds range between 11750 and 13,000 rpm for burst margin for different thermal loads.

F. Bursting speed computed by FEM

Once the stresses are known at each radial station, empirical design criteria may be calculated. Once common design criteria, the burst margin[19] is defined as

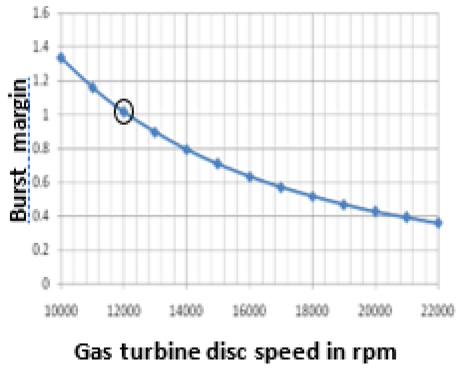

Fig.8. Predicting bursting margin of turbine disc for various disc speed using FE model.

$$Burst\ margin = \frac{0.47 \times \sigma_{ult}}{\sigma_{av.hoop}}$$

In this equation the average tangential stress in the disk is related to the ultimate strength of the chosen material. In a passing design the burst margin should always be larger than 1.0. It should be noted that this simple relation assumes that the material has the same ultimate strength at all radial stations. If the disk has a radial thermal gradient this assumption is not true and the burst margin criteria are no longer valid. Regardless, the burst margin still has its uses as an initial feasibility or sanity check of any disk design. In reality, a well designed disk should pass both the burst margin test and the more specific comparison of Von Mises stress vs. material yield strength at each radial station. Fig. 8 shows FE prediction of burst margin curves, the nature of curve as same as mathematical predicted curves, the burst margin of FE is 12,000 rpm. The FE predicated values lies between the mathematical model, i.e the FE values highly match with mathematical model.

V. Conclusion

The work served to demonstrate that modified mathematical model and FE allows automated analysis which can enable probabilistic simulation on a large scale. The final conclusion was demonstrated bellow

1. The hoop stress component is found to be highest at the inner surface but lowest at the outer surface
2. Radial stress components initially increases along the radial section then it decreases with radial distances.
3. Both modified model and FE analysis gives the burst margin decrease with decrease with increasing turbine disc speeds.
4. The deformation rotating disks increases with increasing speed of the rotating disc
5. FE and mathematical model predication limits the speed of gas turbine disc for 12000 rpm that means 120% of working speed.

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