

# Theoretical Investigation on Metamaterial Coupler: A Comparative Study

Sneha Radadia, Ved Vyas Dwivedi, Rachana Jani

**Abstract:** Two novel edge-coupler-line composite right/left-handed metamaterial couplers are presented in this paper, a symmetric “impedance coupler” and an asymmetric “phase coupler”, explained by a even/odd mode analysis. These two couplers are based on fundamentally different principles but exhibit the advantage of providing arbitrary coupling levels (up to quasi-complete coupling), where as conventional edge-coupled couplers are typically limited to less than 10-dB maximum coupling, while conserving the broad-bandwidth Benefit of their convention counterparts. The coupler is shown to exhibit broad bandwidth and tight coupling.

**Keywords:** Composite right-left-handed (CRLH) transmission lines (TLs), coupled lines, metamaterials.

## I. INTRODUCTION

The electromagnetic properties of a material can be described by its permittivity and permeability. The effect of induced electric and magnetic polarization is described by these two parameters. Material with simultaneous negative permittivity and permeability over a certain frequency band were first studied by Russian physicist viktar veselago in 1967[1]. This paper deals with novel concept inspired from metamaterial. Conventional coupled line coupler exhibit a broad band width, but can achieve only loose coupling levels[2], in contrast branch line or race rate coupler naturally achieve tight 3-dB coupling, but have the short coming of poor bandwidth[3]. The composite right/left-handed coupler presented in this paper is capable of arbitrarily tight coupling over a broad bandwidth. Metamaterial coupled line coupler are obtained by replacing one or both of the two lines constituting conventional coupled line coupler by CRLH TLs. In this paper two types of composite right/left-handed edge coupled coupler will be presented. Section-II describes the mathematical analysis design and application issues of symmetric impedance couple. Section-III describes the mathematical analysis, design and application issues of

Asymmetric Phase coupler, so termed because its coupling mechanism is based on the difference between the phase velocities of the  $c$  and  $\pi$  modes. Section IV present experimental result of coupling for the conventional and composite right/left-handed coupler. This coupler is constituted of two different transmission lines. Geometry and port designation of general coupled coupler shown in figure 1. If power is fed into port 1, the power is coupled only to ports 2 and 3. i.e., power flows in the forward direction of the auxiliary arm port 3 but no power couples to port 4, i.e., in backward direction. Similarly power fed in port 2 is coupled into ports 1 and 4 and not in port 3. All the four ports are matched, i.e., if three of them are terminated in matched load. The phase difference between the two output ports is  $90^\circ$ .

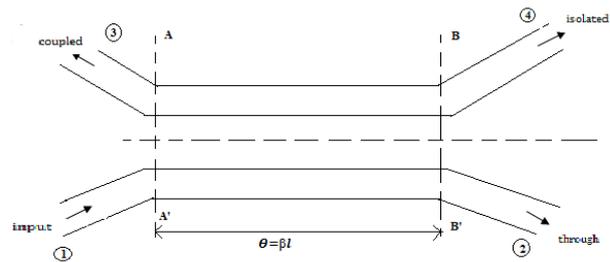


Figure.1 geometry and port designation of general coupled-line coupler[4]

## II. SYMMETRIC IMPEDANCE COUPLER

Symmetric impedance coupler constituted of two identical (symmetrical) TEM TLs, the field solutions may be represented by the superposition of an even (e) mode and an odd (o) mode, which are both also TEM.

### A. Electrical Behaviour

Figure2(a) shows a nine cell and figure2(b) shows the three cell CRLH microstrip coupler, consisting of two CRLH microstrip lines. Even and odd mode have the same propagation constant  $\beta_e = \beta_o = nK_0$  ( $n$  is the refractive index of the dielectric medium of the TL). In contrast, these two modes have different characteristics impedance  $Z_{ce} \neq Z_{co}$  because their equivalent TL capacitances are different[2]. The matching condition to port of impedance  $Z_c$  is mathematically express in (1).

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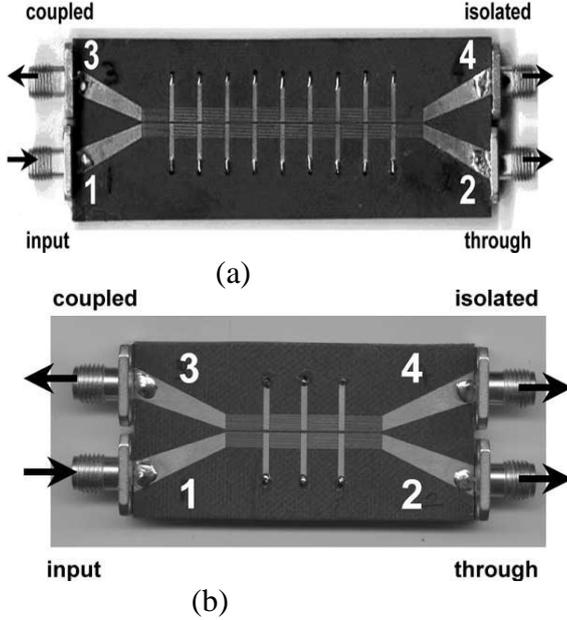
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$$Z_c = \sqrt{Z_{ce} Z_{co}} \quad (1)$$

From which the scattering parameters, referred to figure 1, are  $S_{11}=0, S_{41}=0$ ,

$$S_{21} = \frac{\sqrt{1-k^2}}{\sqrt{1-k^2} \cos \theta + j \sin \theta} \quad (2)$$

$$S_{31} = \frac{j k \sin \theta}{\sqrt{1-k^2} \cos \theta + j \sin \theta} = C_z \quad (3)$$



**Figure 2.** impedance coupling edge-coupled directional coupler, constituted of two interdigital/stub CRLH TLs with same unit cell as TL of figure 2a: (a):complete coupling nine-cell prototype; (b)3-dB three-cell prototype. in both cases, the line spacing  $s=0.3\text{mm}$ . [4]

Where  $\theta = \beta \ell$  is the electrical length of the coupler and  $k$  is the coupling factor because it corresponds to maximum coupling  $C_z$ , obtained for  $\theta = \pi/2$  or  $\ell = \lambda/4$ . The coupling coefficient is mathematically express in(4),

$$k = (Z_{ce} - Z_{co}) / (Z_{ce} + Z_{co}) \quad (4)$$

Where,  $Z_{ce}$  is characteristic impedance for even mode

$Z_{co}$  is characteristic impedance for odd mode

If the two lines are not perfectly TEM but quasi-TEM, we have  $\beta_e \approx \beta_o$ , so that  $\theta_e \approx \theta_o (= \theta)$  and, therefore, the above relations are still approximately valid.

### B. Mode Analysis

Let us consider the coupled line structure composed of two identical quasi-TEM microstrip CRTH TLs. shown figure 2 and corresponding to the even-odd models of figure 3. The even-odd models are to be identical to that of an isolated CRLH TL under the substitutions

$$L_R \rightarrow L_R + 2 L_m = L_{Re} \quad (\text{even}) \quad (5)$$

$$C_R \rightarrow C_R + 2 C_m = C_{Ro} \quad (\text{odd}) \quad (6)$$

The even –odd characteristic impedance are mathematically express in (7) and (8)

$$Z_{ce} = Z_L \sqrt{\frac{1 - (W/W_{se,e})^2}{1 - (W/W_{sh})^2}} \quad (7)$$

$$Z_{co} = Z_L \sqrt{\frac{1 - (W/W_{se,o})^2}{1 - (W/W_{sh,o})^2}} \quad (8)$$

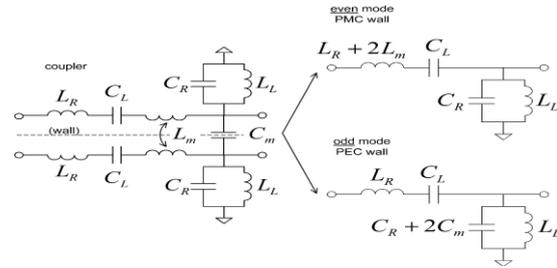
$$W_{se,e} = 1 / \sqrt{L_{Re} C_L} \quad (9)$$

$$W_{sh,e} = 1 / \sqrt{L_L C_R} \quad (10)$$

$$W_{se,o} = \sqrt{W_{sh} W_{se,e}} \quad (11)$$

$$W_{sh,o} = \sqrt{W_{sh,e} W_{se,e}} \quad (12)$$

Because each of the two lines in isolation is balanced with the parameters  $(L_R, C_R, L_L, C_L)$ , the even-odd equivalent lines, having different parameters, are necessarily unbalanced, which results in the emergence of even-odd gaps.



**Figure 3.** Circuit model for the CRLH coupler and corresponding even/odd-mode TL models. [4]

The impedance coupler operates in these gaps, which are designed to overlap each other. Since matching is obtained within the even-odd TL gaps, we need to generalize the expression of the Impedance Coupler coupling coefficient in(3)by changing  $\theta = \beta \ell$  into  $\theta = \gamma \ell = (\alpha + j\beta) \ell$ , where  $\alpha_e \approx \alpha_o \approx \alpha$  and  $\beta_e \approx \beta_o \approx \beta$ . The Impedance Coupler coupling coefficient  $C_z$  mathematically express in (13)

$$C_z = S_{31} = \frac{(Z_{ce} - Z_{co}) \tanh[(\alpha + j\beta) \ell]}{2Z_c + (Z_{ce} + Z_{co}) \tanh[(\alpha + j\beta) \ell]} \times \frac{Z_{ce}/Z_c - Z_{co}/Z_c}{2 + (Z_{ce}/Z_c + Z_{co}/Z_c)} \quad |\beta| \approx 0, \alpha \ell > 1 \quad (13)$$

Where,  $Z_{ce}$  is characteristic impedance for even mode

$Z_{co}$  is characteristic impedance for odd mode

$C_Z$  is coupling coefficient  
 $\alpha$  is attenuation constant  
 $\beta$  is phase shift constant

Where the approximation in the last term holds in the even-odd gaps ( $\beta \approx 0$ ).

**C. Characteristic Investigations**

If the length of the coupler is sufficient so that  $\alpha l > 1$ . By using (1) to eliminate either  $Z_{ce}$  or  $Z_{co}$  and also taking into account the fact that  $Z_{ci} = j \text{Im}(Z_{ci})$  ( $i=e,o$ ) (figure 4), this expression is further transformed, by defining  $\xi = Z_c / \text{Im}(Z_{co}) = \text{Im}(Z_{ce}) / Z_c$

$$C_Z \approx \frac{\xi + \xi^{-1}}{2j + (\xi + \xi^{-1})} \quad \text{With} \quad |C_Z| \approx \frac{\xi + \xi^{-1}}{\sqrt{4 + (\xi - \xi^{-1})^2}} = 1 \quad (14)$$

The complete backward coupling is achieved in this impedance coupler if its length is such that  $\alpha l > 1$  over a bandwidth which depends essentially on the even-odd bandwidth via the parameters in (4) and (5). Figure 4 illustrates the highly unusual behavior of the even-odd characteristic impedances.

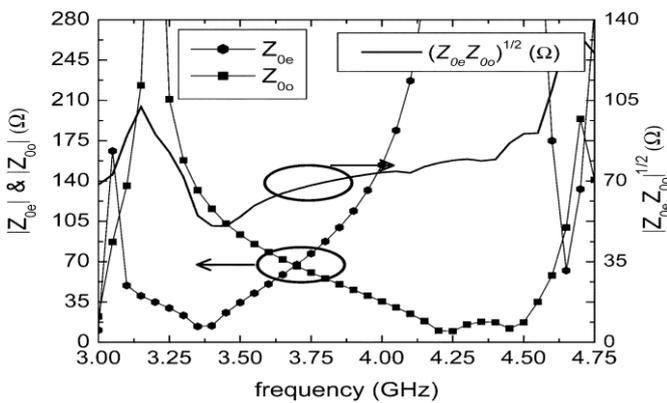


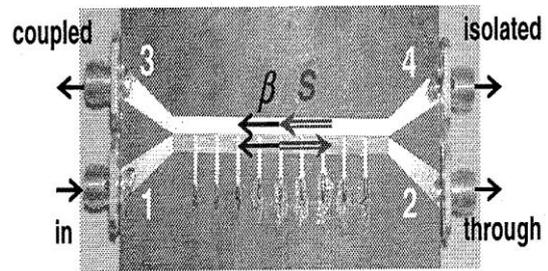
Fig. 4. Even/odd modes characteristic impedances for coupler of figure 1 left-hand axis :magnitudes of impedances. Right-hand axis: square root of product of impedances appearing in (1).

**III. ASYMMETRIC PHASE COUPLER**

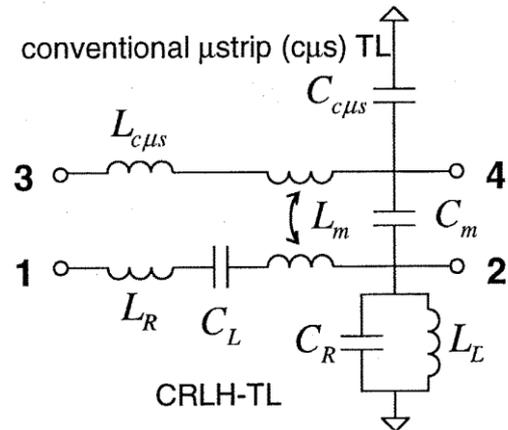
Asymmetrical coupler is constituted of two different transmission lines and its coupling mechanism is based on the difference between the phase velocities of the C and  $\pi$  modes [5].

**A. Electrical Behaviour**

In figure 5(a) mixed  $\mu s$ /CRLH backward wave coupled line directional coupler is shown. The CRLH line will be operating completely in its LH range for present coupler application, its RH components are essential for an accurate description of the behavior of the coupler [3]. Signal is injected at port 1 power propagated towards port 2, phase propagate backward toward port 1.



(a)



(b)

**Figure 5.** phase coupling edge-coupled directional coupler. (a) The 0-dB coupler prototype constituted of one conventional microstrip line and one interdigital CRLH TL operated exclusively in its LH range. (b) equivalent circuit model for the unit cell. [2]

**B. Mode Analysis**

In asymmetric coupler the even-odd analysis has to be replaced by the more involved c and  $\pi$  mode analysis even/odd mode analysis is shown in figure 5(b). If the spacing is this coupler is sufficiently larger so that  $Z_{oe} \approx Z_{oo}$  (due to negligible edge capacitance between the lines) and therefore  $C_Z = S_{31} \approx 0$  from (3), coupling is based on even-odd velocities and occurs at port 4 while becomes isolated (figure 1) we have in general for the coupling coefficient  $C_\beta = S_{41}$  [2]

$$S_{41} = -2j \frac{\sqrt{p}}{1+p} \exp\left(\frac{-j(\beta_c + \beta_\pi)l}{2}\right) \sin\left[\frac{(\beta_c - \beta_\pi)l}{2}\right] - j \exp\left(\frac{-j(\beta_c - \beta_\pi)l}{2}\right) \sin\left[\frac{(\beta_c - \beta_\pi)l}{2}\right] \quad \text{with } p = \frac{R_c}{R_\pi} \quad (18)$$

where  $\frac{R_c}{R_\pi}$  are the ratio of voltage on the two lines for the c and  $\pi$  modes [2].

Where the last expression is an approximation for the symmetric couple the two lines (A and B) tend to become identical. This equation reveals that maximum coupling occurs for the coupler length

$$l = \frac{\pi}{|\beta_c - \beta_l|} A \approx B \frac{\pi}{|\beta_c - \beta_l|} \frac{\lambda_0}{2\sqrt{\epsilon_{xx} - \sqrt{\epsilon_{xx}}}} \quad (16)$$

figure(5) show the CRLH of interest, which consists of a CRLH TL identical to that of figure 1 coupled of a conventional microstripline coupler, the coupler/isolated ports are inverted due to the propagation constant and pointing  $\vec{s}$  vector orientations shown in figure(5.a).in addition, we that the CRLH TL is operated Exclusively in its LH range. Since polarities isolated RH TL and in an isolated LH TL are opposite( $\beta_{LH} - \parallel \beta_{RH}$ ),the microstrip(RH) and CRLH(LH) TLs may be considered an approximation of the  $c/\pi$  equivalent TLs:  $\beta_{C\mu S} \rightarrow \beta_c \rightarrow \beta_\epsilon$  and  $\beta_{CRLH} \rightarrow \beta_\pi \rightarrow \beta_0$  while the small difference  $|\beta_\epsilon - \beta_0|$  leads to poor coupling in the conventional case, we have here  $\beta_0 \rightarrow |\beta_{CRLH}|$ ,so that the difference in the denominator of(16) is turned into a sum,

$$l_{max} = \frac{d}{\beta_{C\mu S} + |\beta_{CRLH}|} \quad (17)$$

Which shows that despite  $\beta_{C\mu S} \approx |\beta_{CRLH}|$  tight coupling can be achieved over a short length. complete power coupling, and hence arbitrary level of coupling by reducing the number of cells or increasing the spacing between the lines.

C. CHARACTERISTIC INVESTIGATIONS

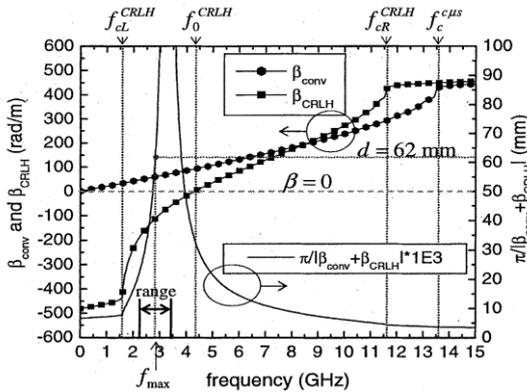


Figure 6 result obtained of circuit model of figure 5(b). Figure 6 shows the right-handed term of(17).maximum coupling is obtained when this quantity is equal to the length of the coupler d.maximum frequency obtained from(17), is 2.8 GHZ,is agreementwith fact that 2.8 GHZ lies in the center of the frequency range of the coupler.

IV. EXPERIMENTAL RESULT

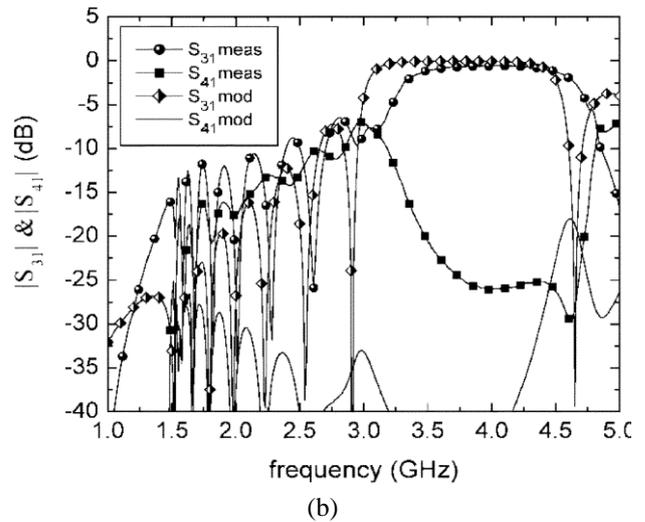
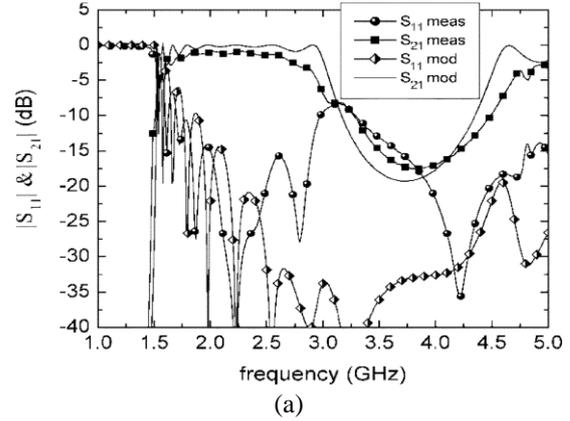
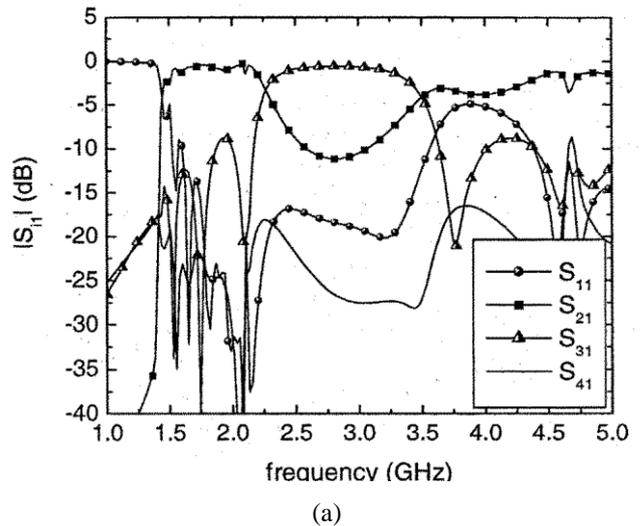


Figure 7 Circuit model simulated (using extracted parameters  $L_m=0.1nH$  and  $C_m=0.8pF$ ) and measured scattering parameters for quasi-0-dB coupler of figure 2.a: (a)  $S_{11}$  and  $S_{21}$  (b)  $S_{31}$  and  $S_{41}$  [3]



VI. CONCLUSION

In this work, the theoretical, mathematical and comparative investigations on metamaterial edge coupler are done. The working of this coupler has been explained by even/odd analysis. A novel CRLH coupler with broad bandwidth and tight coupling characteristic has been discussed and the results are reproduced. This study is highly useful to the future aspirants. Next phase of this research is to design, develop, test and measure the impedance coupler and phase coupler for 10db power.

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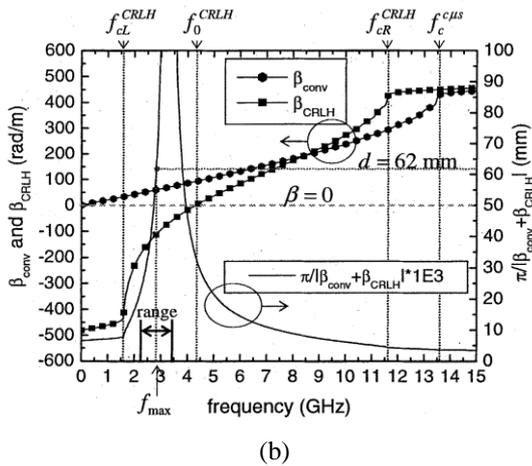


Figure 8. Result for PC of figure 5 (a) measured scattering parameters.(b) phase constants in isolated conventional and CRLH TLs and right hand term of (12).[3]

Figure 7 show the full wave simulation and measured scattering parameters for the coupler of figure 2(a) excellent agreement can be observed between experimental result and simulated result. The performances of the quasi-0-dB impedance coupler of figure 1.1a are presented in figure 2(a) close-to-zero coupling is achieved in the range from 3.2 to 4.6 GHz (36 percent) with a directivity of approximately 25dB. The coupler length is around 1.05λe (λe is the effective permittivity of the conventional microstrip on the same substrate). The impedance coupler shown in figure 2(b) is a 3-dB coupler with an amplitude balance of 2dB over a bandwidth of 50 percent from 3.5 to 5.8GHz and quadrature phase balance 90° ± 5° from 3.5 to 4GHz[4]

The performance of the PC shown in figure 5(a) is presented in figure 7a. figure 8(b) shows the propagation constant of isolated CμS and of CRLH lines constituting the coupler. Quasi-0-db coupling is achieved over the range from 2.2 to 3.8GHz (53 percent) with the excellent directivity of 30dB .

V. DISCUSSION ON RESULTS

Parameters	Symmetrical impedance coupler	Asymmetric phase coupler
Operating frequency range	3.2 to 4.6 GHZ	2.2 to 3.8 GHZ
Bandwidth	36%	53%
Directivity	25db	30db

In an Asymmetric coupler coupling is achieved over the range from 2.2 to 3.8GHZ ,which represent the broad fractional bandwidth 53% and 30 db directivity is achieved. In symmetric coupler operating frequency range is 3.2 to 4.6GHZ. which means that after rescaling, the asymmetric coupler is more compact than its symmetric counterpart. Asymmetric coupler does not exhibit 90° phase balance in contrast to the symmetric CRLH-CRLH coupler.