

# Composite Spur Gear Static Stress Analysis using First Order Shear Deformation



Shravankumar B. Kerur, Nagaraj Kantli

**Abstract:** Due to its benefits and superior capabilities, composite laminate is one of the most widely utilized materials today. With the use of this cutting-edge technology, the world will be less dependent on the use of traditional materials, particularly metal. In the current work, the behaviour of laminated composite gear with different fibre reinforcing configurations is covered. For the current FE formulation, first-order shear deformation theory (FSDT) is used. The focus of fibre orientation on gear stress behaviour is highlighted in the current work. In the current FE formulation, which is a new development in the field of composite gears and deflection behaviour of the composite gear, focus is placed on the accurate computation of material properties with respect to fibre orientation in gears.

**Keywords:** Fiber Reinforcement, FSDT, Composites.

## I. INTRODUCTION

Gears are a crucial component of power and motion transfer systems and have several uses in the wind turbine, aircraft, and automotive sectors. Computer printers, ATMs, and other mechanical equipment, for instance, all include one or more gear systems. In order to increase the robustness of the gear in service, many researchers are investigating the capabilities of polymer gear drives. Currently, power and motion transmission utilise thermoplastic polymer gear materials. Compared to metal gears, these polymer gears have a lower weight carrying capacity. An essential task that needs to be addressed is the improvement of the characteristics of current polymer gears. Accordingly, the goal of the current research is to increase the load carrying capacity of polymer gears by taking fibre reinforcement into account for the gears that are already made of polymer. Investigations are made into how well fibre reinforced composite gears operate when subjected to mechanical loading. The fibre reinforced composite materials are ideally suited for structural applications where high strength and low weight are required [1]. High-strength fibres and a weak matrix material make up composites. To create the required composite material, these fibres and matrix are blended with the proper volume/weight percentages of fibre and matrix.

When compared to conventional materials, one of the key advantages of composites is their ability to be manufactured to meet unique needs [2]. Applications for composite materials can be found in everything from small devices to spacecraft. The mechanical and aerospace industries have seen enormous growth in composites research and development. An good evaluation was provided by Vijayarangan et al.[3] who used the FE formulation to study the stress analysis of composite spur gear. The behaviour of metal matrix composites in comparison to traditional materials used in gear applications was attempted by Vijayarangan and Ganesan [4]. Vijayarangan and Ganesan [5] demonstrated that composite materials can take the place of traditional materials utilised in helical gear applications. The FE technique was proposed by Vijayarangan and Ganesan [6] for the strength analysis of composite gears. Using FSDT, Maiti & Sinha [7] investigated the deflection and free vibration behaviour of composite beams in a FE formulation. In light of Maiti and Sinha, composite gear is modelled as a cantilever beam. The goal of the current effort is to use FSDT to investigate the stresses that lead to composite gear failure.

## II. FORMULA FOR DISCRETE ELEMENTS

Fig. 1 displays the gear teeth model used for the FE study.

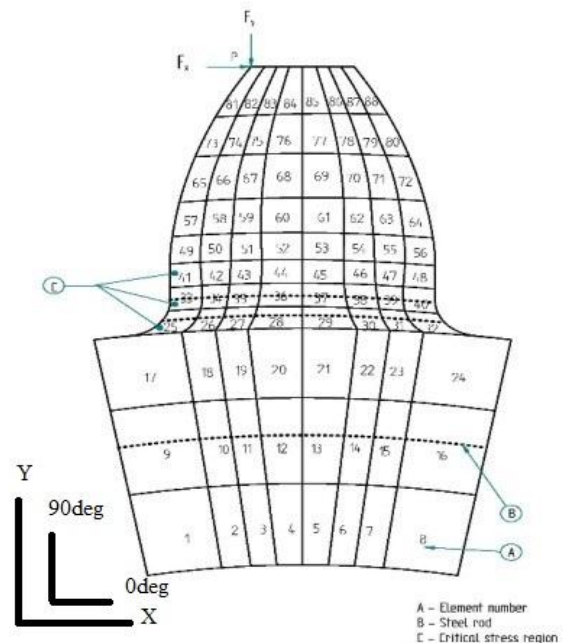


Figure 1: Gear Tooth sector with reinforcements

Fig. 2 displays a quadratic element with four nodes.

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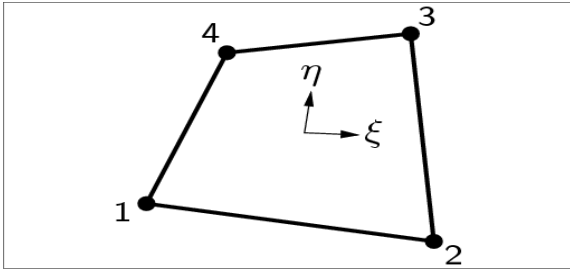


Figure 2: Four node element

These are the basis functions for an eight node element,

$$\begin{aligned}
 N_1 &= \frac{1}{4} * (1 - \zeta) * (1 - \eta) \\
 N_2 &= \frac{1}{4} * (1 + \zeta) * (1 - \eta) \\
 N_3 &= \frac{1}{4} * (1 + \zeta) * (1 + \eta) \\
 N_4 &= \frac{1}{4} * (1 - \zeta) * (1 + \eta)
 \end{aligned}
 \tag{1}$$

The formula for energy density ( $\Pi$ ) is,

$$\begin{aligned}
 \Pi &= \frac{1}{2} \int \sigma^T \varepsilon dv \\
 &= \frac{1}{2} \iiint \varepsilon^T D \varepsilon dx dy dz
 \end{aligned}
 \tag{2}$$

The strain vector  $\{\varepsilon\}$  is stated as,

$$\{\varepsilon\} = \left\{ \frac{du}{dx} \quad \frac{dv}{dy} \quad \frac{du}{dy} + \frac{dv}{dx} \right\}
 \tag{3}$$

The deflection was determined for the current analysis using the first order shear deformation theory, and the results are as follows:

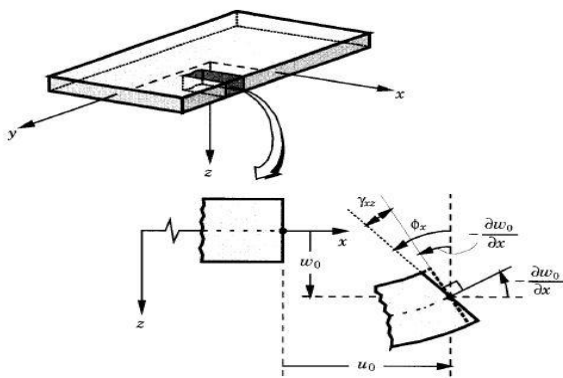


Figure 3: Edge Plate Geometries (Reddy [1])

$$\begin{aligned}
 u(x, y, z) &= u_0(x, y) + z\phi_x(x, y) \\
 v(x, y, z) &= v_0(x, y) + z\phi_y(x, y) \\
 w(x, y, z) &= w_0(x, y)
 \end{aligned}
 \tag{4}$$

$u_0, v_0, w_0, \phi_x$  and  $\phi_y$  are the middle-plane displacements and rotations. The relationship, which yields the element stiffness matrix,

$$[K_e] = \int_{A_e} [S]^T [ABD][S] dA
 \tag{5}$$

$$[ABD] = \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} & B_{xx} & B_{xy} & B_{xz} \\ A_{xy} & A_{yy} & A_{yz} & B_{xy} & B_{yy} & B_{yz} \\ A_{xz} & A_{zy} & A_{zz} & B_{zx} & B_{zy} & B_{zz} \\ B_{xx} & B_{xy} & B_{xz} & D_{xx} & D_{xy} & D_{xz} \\ B_{xy} & B_{yy} & B_{yz} & D_{yx} & D_{yy} & D_{yz} \\ B_{xz} & B_{zy} & B_{zz} & D_{zx} & D_{zy} & D_{zz} \end{bmatrix}
 \tag{6}$$

$[A]$  is the matrix of extensional rigidity,  
 $[B]$  is the matrix of bending-extensional connection and  
 $[D]$  is the bending matrix are given by

$$\begin{aligned}
 [A] &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} [Q_{xy}^k] dz \\
 [B] &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} [Q_{xy}^k] z dz \\
 [D] &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} [Q_{xy}^k] z^2 dz
 \end{aligned}
 \tag{7}$$

The matrix's lower rigidity  $[Q]$  is stated as,

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}
 \tag{8}$$

supplied by the stress results in composite lamina,

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}
 \tag{9}$$

Where

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}}, Q_{66} = G_{12}$$

Since the gear's fibres (glass fibres) are orientated in a radial orientation, determining the fibre angle  $\theta$  is important and is measured with respect to  $y$  - axis and is given as,

$$\theta = \tan^{-1} \left( \frac{yy}{xx} \right)$$

where  $xx$  and  $yy$  are the co-ordinates along  $x$  and  $y$ -axis of the geometric center of any element under consideration. Present paper considers five DOF per node i.e.,  $u, v, w, \phi_x$  and  $\phi_y$ .

Transformation of elastic properties ( $E_1, E_2, \nu_{12}, G_{12}, G_{13}, G_{23}$ ) are performed by the transformation matrix as,

$$[T] = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & mn & (m^2 - n^2) \end{bmatrix} \quad (9)$$

Where  $m = \cos \theta$  and  $n = \sin \theta$ , and after assembling all  $[K_e]$  matrices, the global mass and stiffness matrices are obtained as follows,

For bending analysis, the equation is  $[K]\{u\} = \{F\}$  (10)

Where  $\{u\}$  is the deflection vector,  $\{F\}$  is the force vector,  $[K]$  is the structural stiffness matrix.

### III. RESULTS AND DISCUSSION

The accuracy of the current FE code has been verified with results from the literature, which are shown in Tab. 1–3. It was found that the results of the current FE code validated well with those from the literature. The current FE code is additionally utilised for the study of various gear setups. The bending stress calculated for conventional gear utilising Lewis equation from the Design Data Handbook [8] and current FE code results are smaller than those of analytical values, as shown in Tab. 3.

**Table 1: Displacement validation study for composite beams**

a/h	Degree	4 node 5elements		8 node 5elements	
		Present FE Code	Maiti [7]	Present FE Code	Maiti [7]
60	0	2.8942	2.9265	2.9240	2.9265
	30	15.9093	17.1238	15.6671	17.1231
	45	26.6344	28.3620	26.5586	28.3620
	60	34.4019	36.0543	34.8721	36.0543
	90	39.5719	40.0831	39.9906	40.0831
20	0	2.9643	2.9840	2.9953	2.9840
	30	15.9929	17.3219	16.1405	17.3219
	45	26.7308	28.5228	27.1796	28.5226
	60	34.5098	36.1626	35.3154	36.1626
	90	39.6906	40.1845	40.1142	40.1845

**Table 2: Displacement validation study for composite beams**

Fiber orientation (Degree)	Deflections ( $W_0$ )		Deflections ( $W_0$ )	
	a/h = 100		a/h = 20	
	Present FEA	Reddy [1]	Present FEA	Reddy [1]
0	15.861	16.02	16.5522	16.6
90	395.9077	400	397.6357	401.5
(90/0) <sub>s</sub>	18.0067	18.18	18.9941	19.01

**Table 3. Analytical bending stress calculation comparison**

Analytical Results in $N/mm^2$	FE Results in $N/mm^2$	% of Error
88.0905	79.4783	10.8259

In the present investigation, four different cases are considered and are as given below,

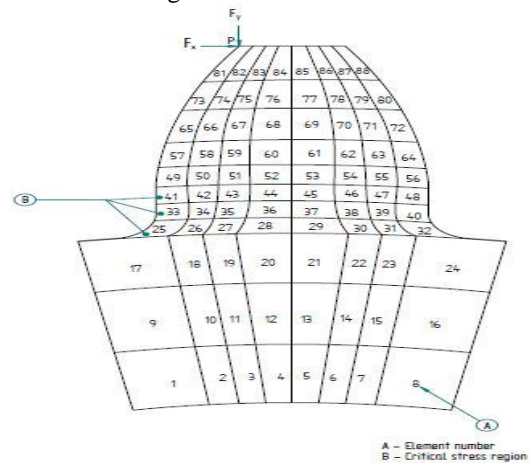
1. Whole gear made of isotropic material.

2. Reinforcing glass fibers in the gear rim and remaining portion of the gear tooth as isotropic.
3. Reinforcing glass fibers in the rim and the involute portion of the gear tooth.
4. Reinforcing glass fibers in the rim and the root section (critical section) of the gear tooth.

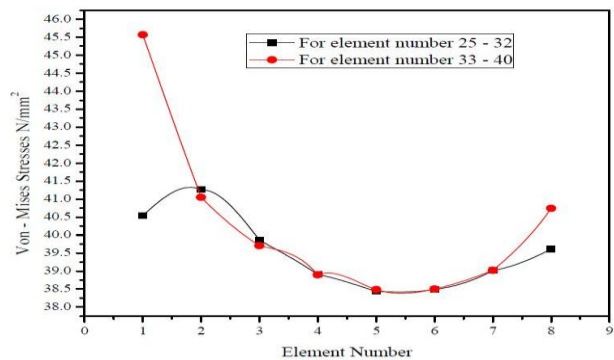
The engineering constants of plastic and glass fiber are considered as matrix and fibers of composite and properties are determined by using the rule of mixture. The determination of properties for the composite are as follows: Consider the volume fraction for fiber and matrix as 35% and 65% respectively. The engineering constants for glass fiber and plastic are  $E_f = 87$  GPa,  $E_m = 2.4$  GPa and  $\nu_{12} = 0.3$ ,  $G_f = 36$  GPa and  $G_m = 1.287$  GPa. The Material properties  $E_1$ ,  $E_2$  and  $G_{12}$  for the FRP are determined using the rule of mixture and are given by,  $E_1 = 32.01$  GPa,  $E_2 = 3.6383$  GPa and  $G_{12} = 1.9242$  GPa.

#### 1. Whole gear tooth as isotropic material:

For this case the gear tooth model for the analysis is shown in Fig. 4. The gear tooth model for the analysis is made of isotropic material and Von-Mises stresses for different elements at the root sections of the isotropic gear tooth are obtained and are plotted against element number at the root sections shown in Fig.5.



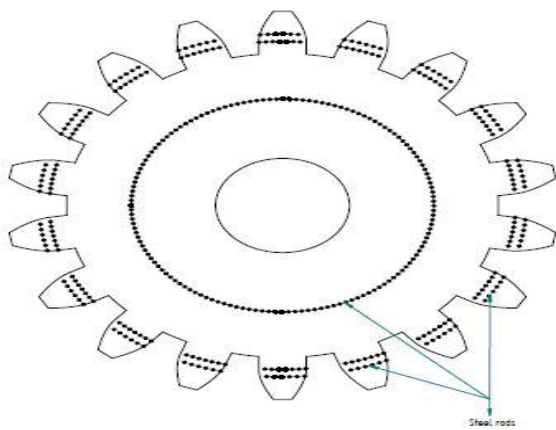
**Figure 4: Gear tooth sector used for the analysis**



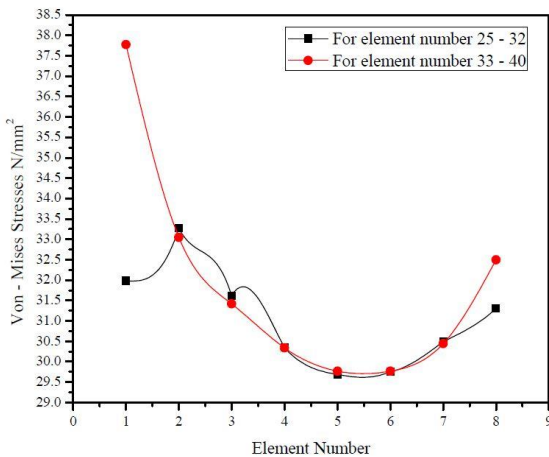
**Figure 5: Von-Mises Stress at critical section of the Isotropic Gear Tooth.**

**2. Reinforcing glass fibers in the rim and remaining portion of the gear tooth as isotropic:**

In the earlier case there were no reinforcing of glass fibers in the gear tooth. Hence the stresses are more at the critical section of the composite gear tooth, to reduce the stresses at critical root section of gear tooth we considered the second case of reinforcing the concentric glass fibers in the rim of the gear disc and is as shown in the Fig. 6. By reinforcing the glass fibers, stiffness/strength in the rim of the gear tooth can be increased and its effect on the stresses at the critical root section is studied. Glass fibers are reinforced from element number 9 -- 16 in the rim and its behavior is studied. Plot of Von-Mises stresses against element numbers at the critical root sections are shown in the Fig. 7.



**Figure 6: Gear disc with reinforced concentric glass fibers in the rim of gear tooth.**



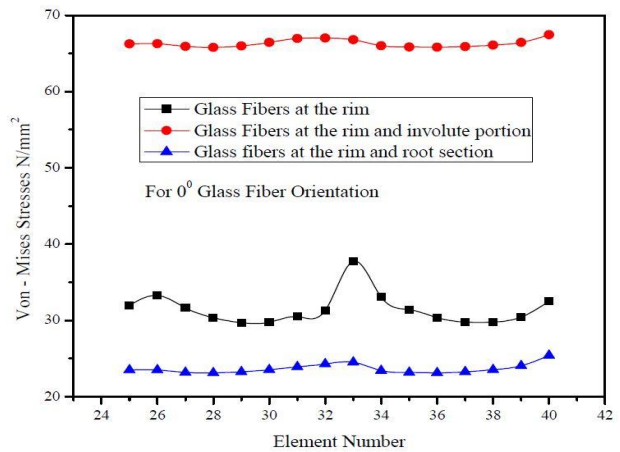
**Figure 7: Von-Mises Stress For critical section of the Gear Tooth.**

**3. Reinforcing glass fibers in the rim and in the involute portion of the gear tooth:**

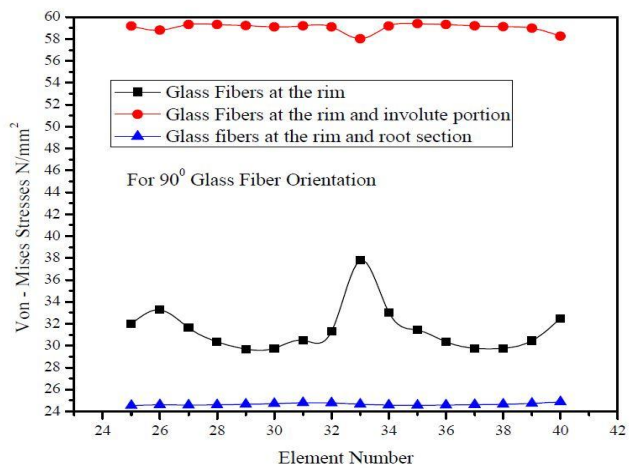
In this case, the reinforcement of glass fibers in the rim as well as in the involute portion of the gear tooth is considered. The reinforcement of glass fibers in the involute of the gear tooth are based on the orientation of the fiber and hence the stresses. The present FE formulation takes into account the effect of fiber orientation. From study it can be seen that Von Mises at the root section are less for 0° and 90° orientation of the fiber as shown in the Figs. 8 and 9.

**4. Reinforcing glass fibers in the rim and in the root (critical) section of the gear tooth:**

In this case glass fibers are reinforced in the rim and at the critical root section of the gear to reduce the stresses induced in root (fillet). The FE formulation is developed for accounting glass fibers reinforced in the rim and in the root section of the gear. Gear tooth model for this case is shown in the Figure 1. The values of Von Mises stresses for the root elements are plotted against the root section elements and are shown in the fig. 8 – 9. Fig. 8 and 9 represents the variation of Von – Mises stresses for composite gear at the root section for four different cases. It also represents that the stresses which are important and responsible for the failure of gear can be reduced by reinforcing the glass fibers in the gear.



**Figure 8: Von-Mises stresses at critical section of the gear tooth**



**Figure 9: Von-Mises stresses at critical section of the gear tooth**

**IV. CONCLUSION**

The following are the conclusions for the stress behaviour of fibre reinforced composite gear:

- It is discovered that composite materials may potentially be employed as a material for gear drives to convey power based on the numerical findings obtained.

- This study shows that the stresses are lower for  $0^{\circ}$  and  $90^{\circ}$  fiber orientations.
- The rigidity and strength can be increased by strengthening the glass fibres in the gear teeth.
- By creating numerical code, the FE formulation of composite gear is illustrated effectively. Additionally, the numerical code was effectively created to account for the reinforcement of fibres in the gear.

Future power transmission systems may incorporate fibre reinforced composite gears.

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